Approximate Analysis of Continuous Prismatic Shells

Calcul approché des voiles prismatiques à travées multiples

Näherungsverfahren für durchlaufende, prismatische Schalen

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Introduction

During the past few years several investigators have presented various techniques for the analysis of prismatic shell structures, and many computer programs have been developed for analyzing this type of simple span structure. An excellent review of the numerous papers which have been published in the literature on the analysis of simply supported prismatic shell structures can be found in the report of the ASCE Task Committee on Folded Plate Construction [1], and a comparison of simplified theories has been given by POWELL [2].

In contrast, the analysis of continuous prismatic shell structures has not received as much attention. However, this subject has recently been considered by several investigators, and a few methods of analysis have been presented. Approximate methods for the analysis of continuous prismatic shell structures have been discussed by YITZHAKI and REISS [3], and BEAUFAIT [4]. Procedures for obtaining more accurate solutions have been suggested by PULMANO and LEE [5], GOLDBERG, GUTZWILLER and LEE [6], SCORDELIS and LO [7], and LEE and MOUSA [8].

This paper presents an approximate method for the analysis of prismatic shell structures continuous over intermediate transverse diaphragms and simply supported at the two end diaphragms. The method is based on the ordinary theory for folded plates [9], and leads to improved accuracies in comparison with the procedure suggested by BEAUFAIT [4], as discussed by SCORDELIS and LO [7], by formulating the elastic curves of individual plates, in the beam action analysis, in terms of infinite trigonometric series. The suggested procedure is convenient for hand computations.

An example problem to illustrate the application of the method is solved, and the results are compared with those obtained by LEE and MOUSA [8]. As a further comparison, the proposed method is used in solving one of the illustrative examples given by BEAUFAIT [4], which was also analyzed by SCORDELIS and LO [7].

Assumptions. The method of analysis is based on the following assumptions: Each plate is rectangular and of constant thickness; the material is homogeneous, isotropic and elastic; the longitudinal strains vary linearly across the width of each plate; the principle of superposition holds; the longitudinal fold lines are continuous and fully monolithic along their entire length; plates are relatively long compared to their width, aspect ratio equal to or greater than three; the supporting diaphragms are infinitely rigid in their planes but flexible normal to their planes.

Sign Convention. The sign convention adopted in this analysis is summarized below for convenient reference.

Component	*Positive Direction*
Normal load component	Acting downward
Tangential load component	Acting from left to right
In-plane load	Acting from left to right
Transverse bending moment	Producing tensile stresses on bottom fiber
Longitudinal bending moment	Producing tensile stresses on right edge
In-plane displacement	Producing movement from left to right
Rotation	Clockwise
Normal stress	Tensile

Notation. The letter symbols adopted for use in this paper are defined where they first appear and are listed alphabetically in the Appendix.

Method of Analysis

The method of analysis entails the following steps:

a) Assuming non-yielding fold lines, the reactions of the longitudinal edges due to the applied loads are determined using a transverse slab action analysis.

b) The final in-plane loads, the redundant reactions, and the elastic curves for individual plates are expanded into Fourier series satisfying the boundary conditions at the two transverse end diaphragms; setting the displacements at the intermediate supports equal to zero gives the value of the redundant reactions.

c) Applying arbitrary rotation for each plate and assuming that these rotations and the corresponding transverse bending moments and fold line reactions vary along the span as the elastic curves due to applied loads, the corrected values of these rotations are determined for each term of the Fourier series by satisfying the compatibility conditions at the fold lines.

d) Superposition of steps a, b and c gives the total stresses and displacements in the structure due to applied loads.

e) Steps b, c and d are repeated for succeeding terms of the Fourier series until the necessary degree of accuracy is achieved.

Beam Action Analysis. The differential equation governing the displacement of a prismatic beam subjected to an arbitrary loading of intensity $p(x)$ is given by

$$\frac{d^4 y}{dx^4} = \frac{p(x)}{E\,I}, \tag{1}$$

in which E is the modulus of elasticity and I the moment of inertia of the beam. The general solution of Eq. 1 is

$$y(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3 + y_p(x), \tag{2}$$

in which $y_p(x)$ is the particular solution of Eq. (1) and c_1 to c_4 are integration constants.

A beam, simply supported on the two outer edges, continuous over t intermediate supports and subjected to the applied load $q(x)$, as shown in Fig. 1,

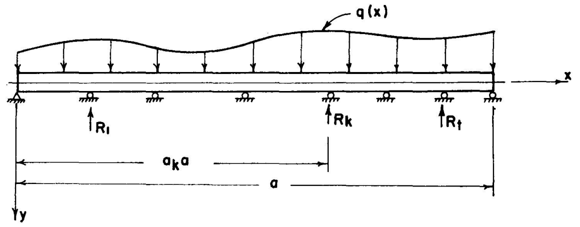

Fig. 1. Beam continuous over t intermediate supports and subjected to transverse load.

can be analyzed [10] as a simply supported beam subjected to the applied load $q(x)$ and the redundant reactions $R_1, R_2, \ldots R_t$, the latter assumed to be positive if acting upward. The applied load and the redundant reactions are approximated by Fourier series of the forms

$$q(x) = \sum_{m=1}^{\infty} A_m \sin\frac{m\,\pi\,x}{a}, \tag{3}$$

$$R_k = \sum_{m=1}^{\infty} A_m^k \sin\frac{m\,\pi\,x}{a} \quad (k = 1, 2, \ldots t), \tag{4}$$

in which

$$A_m = \frac{2}{a}\int_0^a q(x)\sin\frac{m\,\pi\,x}{a}\,dx, \tag{5}$$

$$A_m^k = \frac{2}{a} \lim_{r \to 0} \int_{(a_k a - r)}^{(a_k a + r)} \frac{R_k}{2r} \sin \frac{m \pi x}{a} dx = \frac{2 R_k}{a} \sin m \pi a_k, \tag{6}$$

in which a is the total longitudinal span, and a_k is the ratio of the distance between the end support and the redundant reaction k to the total longitudinal span length, as shown in Fig. 1.

The boundary conditions are given by

$$y(0) = y''(0) = y(a) = y''(a) = 0. \tag{7}$$

In view of Eq. (7), $y_p(x)$ can be conveniently taken in the form

$$y_p(x) = \sum_{m=1}^{\infty} B_m \sin \frac{m \pi x}{a}. \tag{8}$$

Referring to Eqs. (3) and (4), the total load $p(x)$ acting on the beam can be taken as

$$p(x) = \sum_{m=1}^{\infty} (A_m - \sum_{k=1}^{t} A_m^k) \sin \frac{m \pi x}{a}, \tag{9}$$

substituting Eqs. (8) and (9) into Eq. (1) gives

$$B_m = \frac{a^4}{E I \pi^4 m^4} (A_m - \sum_{k=1}^{t} A_m^k) \tag{10}$$

and substituting Eqs. (2) and (8) into Eq. (7) yields

$$c_1 = c_2 = c_3 = c_4 = 0. \tag{11}$$

Therefore, Eq. (2) takes the form

$$y(x) = y_p(x) = \sum_{m=1}^{\infty} B_m \sin \frac{m \pi x}{a}, \tag{12}$$

substituting Eq. (10) into Eq. (12) in view of Eq. (6) yields the elastic curve

$$y(x) = \frac{a^4}{E I \pi^4} \sum_{m=1}^{\infty} \frac{1}{m^4} \left(A_m - \frac{2}{a} \sum_{k=1}^{t} R_k \sin m \pi a_k \right) \sin \frac{m \pi x}{a}. \tag{13}$$

For the unyielding intermediate supports, the displacements at these supports vanish, i.e.,

$$y(x_l) = 0 \qquad (l = 1, 2, \ldots t), \tag{14}$$

in which x_l is the distance along the x-axis to the particular intermediate support l. Substituting Eq. (14) into Eq. (13) yields

$$\sum_{k=1}^{t} R_k D_{lk} = \frac{a}{2} D_l \qquad (l = 1, 2, \ldots t), \tag{15}$$

in which
$$D_l = \sum_{m=1}^{\infty} \frac{A_m}{m^4} \sin m \pi a_l, \tag{16}$$
$$D_{lk} = \sum_{m=1}^{\infty} \frac{1}{m^4} \sin m \pi a_l \sin m \pi a_k. \tag{17}$$

Eq. (15) gives a set of t simultaneous equations which can be solved for the redundant reactions $R_1, R_2, \ldots R_t$. Substituting the calculated reactions and the Fourier coefficient A_m given by Eq. (13) gives the elastic curves of individual plates.

Stress Relaxation. In general the longitudinal normal stresses caused by the above mentioned beam action at the common edges of any two adjacent plates are not the same. Therefore, to satisfy the continuity condition, shearing stresses equal in magnitude and opposite in direction are applied at the common edges as shown in Fig. 2a. The magnitude and direction of these shearing

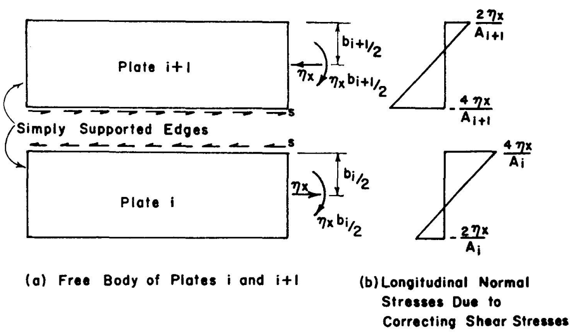

Fig. 2. Longitudinal normal stresses.

stresses are determined by a stress relaxation procedure [11]. Referring to Fig. 2b, it can be shown that the distribution factors are given by

$$d_i = \frac{4\,\eta_x/A_i}{(4\,\eta_x/A_i) + (4\,\eta_x/A_{i+1})} = \frac{A_{i+1}}{A_i + A_{i+1}}, \tag{18}$$
$$d_{i+1} = \frac{4\,\eta_x/A_{i+1}}{(4\,\eta_x/A_i) + (4\,\eta_x/A_{i+1})} = \frac{A_i}{A_i + A_{i+1}}, \tag{19}$$

in which d_i, A_i and d_{i+1}, A_{i+1} are the distribution factors and transverse cross-sectional areas of plate i and plate $i+1$, respectively; η_x is the normal force caused by the applied correcting shearing stresses. The carry over factors in this case are $-{}^1/_2$. It can be seen that the stress relaxation procedure is analogous to the moment distribution method in which the transverse cross-sectional areas correspond to the reciprocal of the stiffeness factors.

The corrected in-plane displacements due to the applied load of plate i at a reference transverse section are given by

$$\Delta_i = \frac{y_i}{M_{Li}} \frac{I_i(\sigma_1 - \sigma_2)}{b_i}, \tag{20}$$

in which b_i, I_i, M_{Li} and y_i are the width, the moment of inertia, the longitudinal bending moment and the in-plane displacements on the reference section before the correcting shear stresses are applied, of plate i, respectively; and σ_1 and σ_2 are the corrected longitudinal stresses at the right and left edges respectively.

Compatability Conditions. Considering the geometry of the transverse cross-section of a continuous prismatic shell structure, the rotation of plate i, ψ_i, is expressed in terms of the final corrected in-plane displacements as

$$\psi_i = f(\delta_1, \delta_2, \delta_3, \ldots), \tag{21}$$

in which ψ_i is the rotation of plate i at a reference transverse section; $\delta_1, \delta_2, \delta_3, \ldots$, are the final corrected in-plane displacements of plates $1, 2, 3, \ldots$ at the section.

$$\psi_2 = \frac{1}{b_2}\left(\frac{-\delta_1 + \delta_2 \cos \lambda_{12}}{\sin \lambda_{12}} + \frac{-\delta_2 \cos \lambda_{23} + \delta_3}{\sin \lambda_{23}}\right)$$

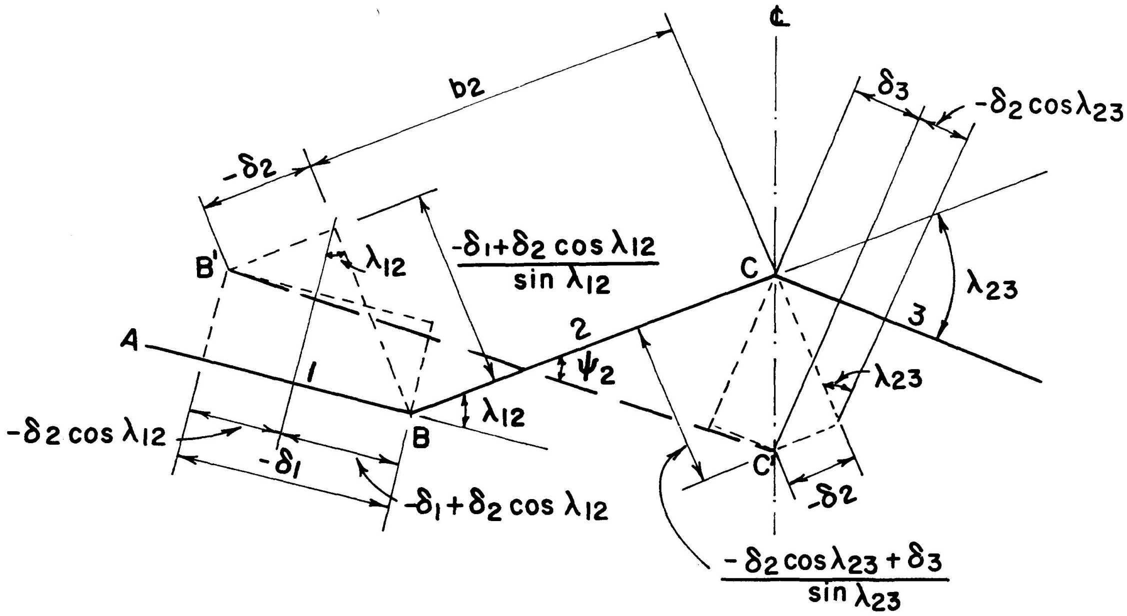

Fig. 3. Rotation of plate 2 in terms of in-plane displacements.

Eq. (21) is illustrated in Fig. 3. The plate rotation ψ_i can be expressed as

$$\psi_i = K_i \psi_i^0, \tag{22}$$

in which K_i and ψ_i^0 are the correction factor and an assumed arbitrary rotation of plate i respectively. A separate analysis for each plate is carried out for an assumed rotation ψ_i while the other plates undergo rigid body translation. The analysis is based on the assumption that the arbitrary rotation and the corresponding reactions at the fold lines vary, in the longitudinal direction, as the elastic curve, Eq. (13), due to the applied load for each term of the Fourier series.

The final corrected in-plane displacement δ_i is obtained by the superposition of the corrected in-plane displacements due to the applied load and those due to the plate rotations, i. e.,

$$\delta_i = \Delta_i + \sum_j \Delta_{ij} K_j, \tag{23}$$

in which Δ_i is the corrected in-plane displacements of plate i due to applied load, Δ_{ij} is the corrected in-plane displacement of plate i due to an arbitrary rotation of plate j, and j denotes the number of the plates excepting any cantilevered end plates. The displacement Δ_{ij} is computed by Eq. (20) in which, in this case, y_i, M_{Li}, σ_1 and σ_2 are those due to the assumed rotation of plate j.

Equating Eq. (21) to Eq. (22) yields

$$K_i \psi_i^0 = f(\delta_1, \delta_2, \delta_3, \ldots). \tag{24}$$

Eq. (24) gives a set of i simultaneous equations which satisfy the compatibility condition and, in view of Eq. (23), can be solved for the correction factors K_j.

The final longitudinal normal stresses and transverse bending moments for each term of the Fourier series are given by

$$\sigma_i = \sigma_i^1 + \sum_j K_j \sigma_{ij}, \tag{25}$$

$$M_{Ti} = M_{Ti}^1 + \sum_j K_j M_{Tij}, \tag{26}$$

in which σ_i^1 and M_{Ti}^1 are the corrected normal longitudinal stress and the transverse moment of plate i, respectively, due to the applied load; σ_{ij} and M_{Tij} are the corrected normal longitudinal stress and the transverse bending moment of plate i, respectively, due to an arbitrary rotation of plate j. The final stresses are the sum of all the stresses due to all the terms in the Fourier series.

Procedure of Analysis

The procedure to be followed in the analysis of a continuous prismatic shell consists of the following steps:

1. The applied load acting on each plate is resolved into normal and tangential components.
2. Assuming non-yielding fold lines, a transverse section of unit width is analyzed as a continuous one-way slab subjected to the normal load component, and the reactions at the longitudinal edges are obtained.
3. The final in-plane load acting on each plate is the vectorial sum of the tangential component of the applied load in step (1) and the in-plane component of the forces equal in magnitude and opposite in direction to the reactions in step (2).

4. The final in-plane load, the intermediate reactions and the elastic curve are expanded into infinite trigonometric series. Eq. (5) is used to calculate the Fourier coefficient A_m for the applied load, the redundant reactions are determined by Eq. (15), and the elastic curve is defined by Eq. (13).

5. To satisfy continuity conditions, the longitudinal normal stresses at the common edges of any two adjacent plates are made equal by applying the stress relaxation procedure. The corrected in-plane displacements at the reference section, where the deflections are non-zero for all Fourier harmonics are determined by Eq. (20).

6. An arbitrary rotation is applied at the reference transverse section for each plate, excepting any cantilevered end plates, and the reactions at the fold lines are determined. The rotations, the resulting transverse bending moments and the reactions at the fold lines are assumed to vary along the span in the same manner as the elastic curve evaluated in step (4). The corrected in-plane displacements are determined at the reference section as discussed in steps (3) to (5).

7. To satisfy the compatability conditions, each plate rotation is expressed in terms of the final corrected in-plane displacements, which are the sum of the displacements due to the applied load and the displacements due to each of the assumed plate rotations multiplied by a corresponding correction factor, in accordance with Eq. (23). Equating each plate rotation to the arbitrarily chosen rotation multiplied by the correction factor leads to a set of simultaneous equations, Eq. (24), which, upon substitution of Eq. (23), can be solved for the correction factors K_j.

8. The stresses are evaluated as the sum of the stresses due to the applied loads and the stresses due to the different rotations, each multiplied by the appropriate correction factors obtained in step (7), usings Eqs. (25) and (26).

9. Steps (5) to (8) are repeated for each term of the Fourier series. The number of terms needed for a solution depends on the desired degree of accuracy. This point will be discussed later in the illustrative examples. It is pertinent to observe that the correction factors are independent of the reference transverse section taken in the calculation.

Illustrative Examples

Example 1

A continuous prismatic shell structure having the dimensions shown in Fig. 4 and subjected to a uniformly distributed live load of intensity 80 psf. of horizontal projection is analyzed to illustrate the application of the proposed method. All plates have the same thickness, $h = 4$ in. Due to symmetry only half of the shell need be taken into consideration.

A transverse section of unit width at $x = 21$ ft. is selected as the reference section. One-way slab action analysis due to the normal load component is shown in Table 1a applying a routine moment distribution procedure. The total in-plane loads per unit length, q_i, on plate i is shown in Fig. 5a.

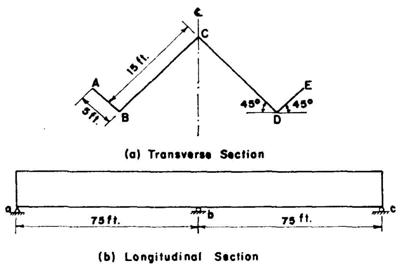

Fig. 4. Shell dimensions (example 1).

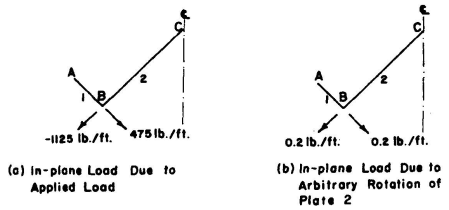

Fig. 5. Total in-plane load at $x = 21$ ft.

Table 1. Slab Action Analysis at x = 21 ft.
(Example 1)

a) Slab Action Analysis for Applied Load				b) Slab Action Analysis for Arbitrary Rotation of Plate 2			
Joint	*B*		*C*	Joint	*B*		*C*
D.F.	0	1	0	D.F.	0	1	0
F.E.M. (ft-lb)	500	—750	750	F.E.M. (ft-lb)	0	0	—3
Final Moment (ft-lb)	500	—500	875	Final Moment (ft-lb)	0	0	—3
Shear (lb)	—200	275	—325	Shear (lb)	0	0.2	—0.2

Substituting the uniform q_i into Eq. (5) yields

$$A_m = \frac{4\,q_i}{m\,\pi} \qquad (m = 1, 3, \ldots). \tag{27}$$

Substituting Eq. (27) into Eq. (15), taking only odd values of m in Eqs. (16) and (17) due to symmetry and solving for the redundant reaction R_1 lead to, using only the first three terms of the series,

$$R_1 = 0.6255\,q_i\,a, \tag{28}$$

where $q_i a$ is the total in-plane load of plate i; the exact value being $R = 0.625\,q_i a$. Substituting Eqs. (27) and (28) into Eq. (13) yields

$$y_i(x) = \frac{a^4 q_i}{E\,I\,\pi^4} \sum_{m=1,3,5}^{\infty} \frac{1}{m^4}\left(\frac{4}{m\,\pi} - 1.251 \sin\frac{m\,\pi}{2}\right) \sin\frac{m\,\pi\,x}{a}. \tag{29}$$

Differentiating Eq. (29) gives expressions for the longitudinal bending moment,

$$M_{Li}(x) = \frac{a^2 q_i}{\pi^2} \sum_{m=1,3,5}^{\infty} \frac{1}{m^2}\left(\frac{4}{m\,\pi} - 1.251 \sin\frac{m\,\pi}{2}\right) \sin\frac{m\,\pi\,x}{a}. \tag{30}$$

For $m=1$, Eqs. (29) and (30) give

$$y_i(x) = 0.02228 \frac{a^4 q_i}{E I \pi^4} \sin \frac{\pi x}{a}, \tag{31}$$

$$M_{Li}(x) = 0.02228 \frac{a^2 q_i}{\pi^2} \sin \frac{\pi x}{a}. \tag{32}$$

In view of Eq. (32) and using simple beam theory the longitudinal normal stresses at the fold lines are determined. Substituting the transverse cross-sectional areas of the plates into Eqs. (18) and (19) gives the distribution factors for the stress relaxation procedure. The corrected longitudinal normal stresses at the reference section, for the first term of the Fourier series due to applied load is shown in Table 2.

Table 2. Corrected Longitudinal Normal Stresses at $x=21$ ft. due to Applied Load for $m=1$ (Example 1)

Joint	A	B	B	C
D.F.	0	0.75	0.25	0
Long. Stresses (psi)	—51.37	51.37	13.52	—13.52
Dist.	0	—28.39	9.46	0
C.O.	14.20	0	0	—4.73
Dist.	0	0	0	0
Corrected Stresses (psi)	—37.17	22.98	22.98	—18.25

Substituting Eqs. (31) and (32) with $x=21$ ft., the corrected longitudinal normal stresses shown in Table 2, and the moment of inertia and width of each plate into Eq. (20), yield the corrected in-plane displacements due to applied load at the reference section for the first term of the Fourier series

$$\Delta_1 = 3948\,(10^3/E), \tag{33}$$

$$\Delta_2 = -9021\,(10^3/E), \tag{34}$$

in which Δ_i and E are in ft. and psf. respectively.

For the rotation of plate 2, assume, at the reference section,

$$\psi_2^0 = \frac{b_2}{E I_2}, \tag{35}$$

in which b_2 and I_2 are the width and moment of inertia of the transverse cross-section of plate 2 per unit length respectively. The moment distribution procedure for the assumed rotation and the resulting in-plane loads are shown in Table 1b and Fig. 5b respectively. The in-plane loads due to rotation of plate i, $q_{0i}(x)$, are assumed to vary along the span as the elastic curve due to the applied load. Hence $q_{0i}(x)$ is given by

$$q_{0i}(x) = \frac{q_{0i}^1}{y_i^1} y_i(x), \tag{36}$$

in which q_{0i}^1 is the in-plane load on plate i at the reference section due to the assumed rotation; y_i^1 is the in-plane displacement of plate i at the reference section due to the applied load. Substituting Eq. (31) into Eq. (36) yields, for $m=1$,

$$q_{0i}(x) = 2.349\, q_{0i}^1 \sin\frac{n x}{a}. \tag{37}$$

Applying the same procedure used perviously for the case of applied load, the corrected in-plane displacements due to rotation of plate 2 at the reference section, for $m=1$, are

$$\varDelta_{12} = 2.735\,(10^3/E), \tag{38}$$

$$\varDelta_{22} = 0.2363\,(10^3/E), \tag{39}$$

in which $\varDelta_{ij}$ and E are in ft. and psf. respectively.

Referring to Fig. 3 and Fig. 4a, it can be seen that $\lambda_{12}=\lambda_{23}=\pi/2$, $\delta_2=-\delta_3$ and

$$\psi_2^0 K_2 = -(\delta_1+\delta_2)/15, \tag{40}$$

in which δ_i is in ft. Substituting Eqs. (33), (34), (38) and (39) into Eq. (23) gives

$$\delta_1 = (3948+2.735\,K_2)(10^3/E), \tag{41}$$

$$\delta_2 = (902.1-0.2363\,K_2)(10^3/E), \tag{42}$$

in which δ_i and E are in ft. and psf. respectively. Next Eq. (35) yields

$$\psi_2^0 = 4.860\,(10^3/E), \tag{43}$$

in which E is in psf. Substituting Eqs. (41), (42) and (43) into Eq. (40) and solving for K_2 yields $K_2=-40.40$.

Finally, substituting the appropriate values of the stresses into Eqs. (25) and (26) gives the final longitudinal normal stresses and transverse bending moments at the reference section for $m=1$.

The elastic curve and the longitudinal bending moments given by Eqs. (29) and (30) respectively are obrained for the other harmonics of the Fourier series and the stresses are calculated in the same manner. The number of terms in the Fourier series used depends upon the desired accuracy. In the following, the series is truncated when the value of the last term is less than one percent of the partial sum up to the previous terms. A summary of the correction factors, the corresponding transverse bending moments and the longitudinal normal stresses are shown in Tables 3 and 4. Only four terms of the Fourier series are needed to achieve the desired accuracy. The correction factors have the same values along the span for each Fourier harmonic, hence the stresses along the span can be readily calculated.

Table 3. Transverse Moment M_T at $x = 21$ ft.
(Example 1)

	m	K_2	Fold Line	
			B	C
Rotation of Plate 2	1	—40.40	0	—121.2
	3	—85.31	0	—255.9
	5	5.681	0	17.0
	7	—0.2079	0	—0.6
Applied Load			—500	—875.0
Final M_T (ft-lb)			—500	—1236

Table 4. Longitudinal Normal Stresses σ at $x = 21$ ft.
(Example 1)

m	K_2	Applied Load			Rotation of Plate 2			Final σ (psi)		
		Fold Line			Fold Line			Fold Line		
		A	B	C	A	B	C	A	B	C
1	—40.40	—27.2	23.0	—18.3	1.3	—0.4	0.1			
3	—85.31	—706.3	436.7	—346.8	24.9	—7.1	1.2			
5	5.681	126.3	—78.1	62.0	—0.0	—0.0	—0.0	—598.1	378.7	-305.3
7	—0.2079	—7.1	4.4	—3.5	0.0	0.0	0.0			

A comparison of the above results with those obtained by LEE and MOUSA [8] is shown in Figs. 6 and 7. In the latter, the plate and membrane analysis are treated as plate and plane stress problems and the exact distributions of the intermediate diaphragm reactions are approximated by uniform step functions expanded into double Fourier series using nine steps.

Example 2

A continuous prismatic shell structure having the dimensions shown in Fig. 8, with constant thickness $h = 4$ in., is analyzed by the proposed method for the case of a uniformly distributed live load of intensity 80 psf. of horizontal projection. A comparison with the results given by BEAUFAIT [4], and SCORDELIS and LO [7], is shown in Figs. 9 and 10. Beaufait's procedure is also based on the ordinary folded plate theory; the continuity condition at the fold lines is satisfied at the center of selected segments and the elastic curve is obtained from the corrected longitudinal moments at these locations. SCORDELIS

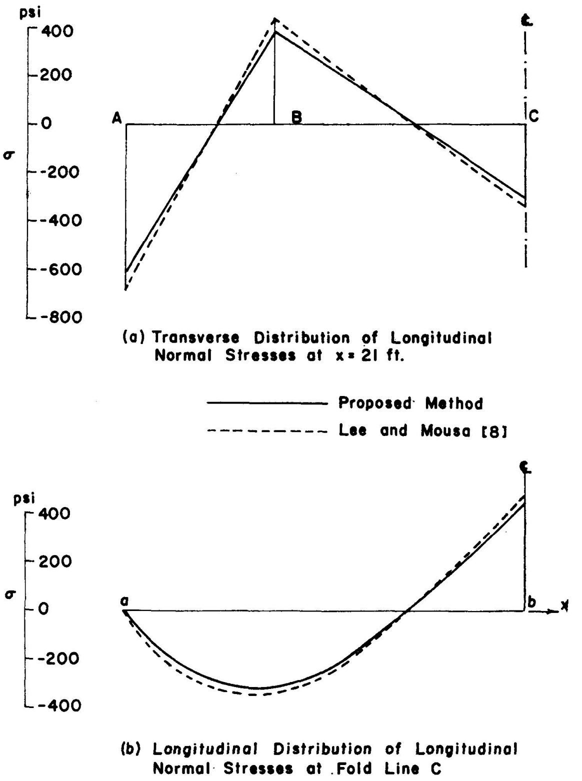

Fig. 6. Longitudinal normal stresses σ (example 1).

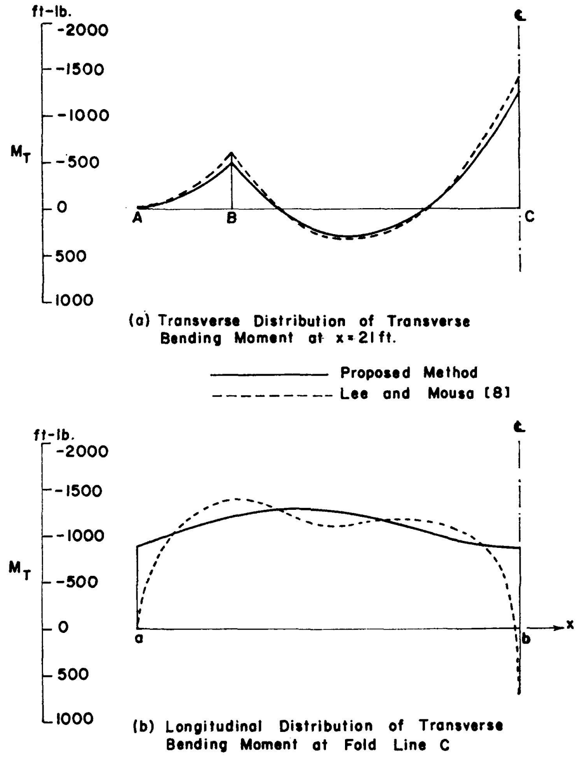

Fig. 7. Transverse bending moment M_T (example 1).

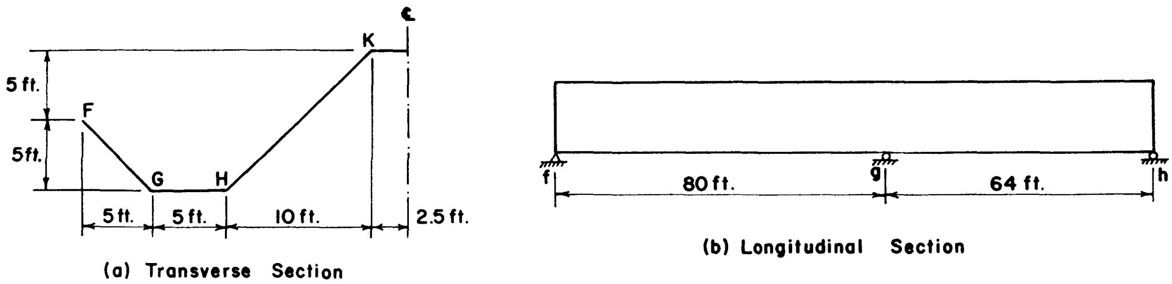

Fig. 8. Shell dimensions (example 2).

and Lo formulated a computer technique using plate bending and plane stress analysis and assuming that the reactions at the intermediate diaphragms vary linearly across the width of each plate. It is of interest to note that the variations of the normal and tangential component of the reactions across the width of each plate as shown by Lee and Mousa [8] deviate considerably from such an assumption.

Conclusions

This paper presents a simple procedure, which is convenient for hand computation, for the analysis of continuous prismatic shell structures. It can be seen that, except at the diaphragms, the transverse bending moments

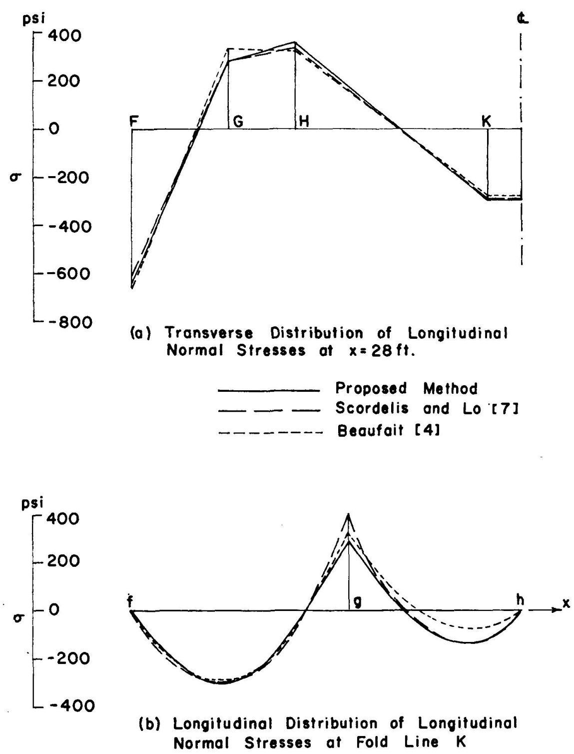

Fig. 9. Longitudinal normal stresses σ (example 2).

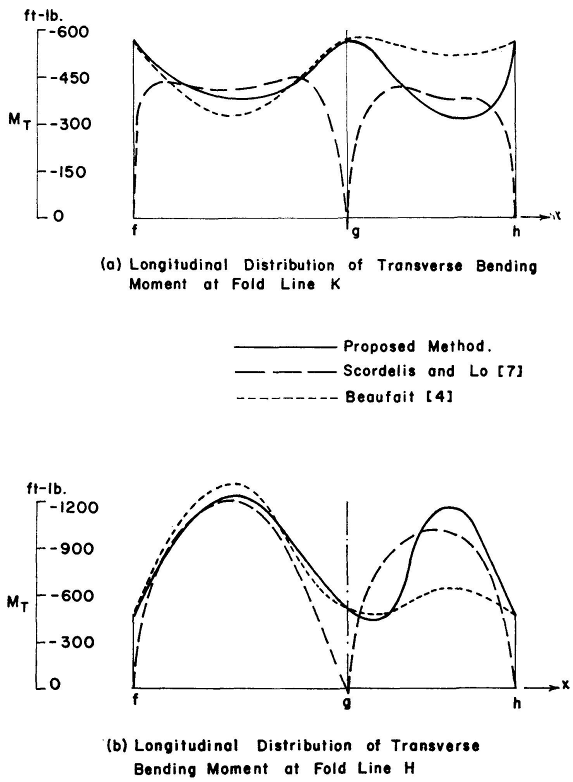

Fig. 10. Transverse bending moment M_T (example 2).

obtained by the proposed method of analysis are in good agreement with the results obtained by SCORDELIS and LO [7] and LEE and MOUSA [8]. Beaufait's results are in poorer agreement, especially at the smaller span. The agreement in the longitudinal normal stresses is better than the transverse bending moment, as expected.

Consideration of a unit strip in the analysis of the one-way slab action is justified by the fact that the variation of the transverse bending moment and the transverse normal stresses along the span, as shown by LEE and MOUSA [8], is very small except at the diaphragms.

The continuity condition need only be satisfied at one reference transverse section due to the fact that the reactions along the fold lines have the same variation along the span.

The compatibility condition is satisfied at all transverse sections if it is satisfied for a reference transverse section due to the assumption that the plate rotations, hence the corresponding reactions at the fold lines, have the same variation along the span as the elastic curves due to the applied load. Hence the correction factors K_j have the same values at all transverse sections.

A comparison of the stresses calculated by the proposed method with those obtained by the more exact method suggested by LEE and MOUSA [8], as shown in Figs. 6 and 7 and with those obtained by SCORDELIS and LO [7], as shown in Figs. 9 and 10, shows good agreement except at the supports. It is pertinent to note that the agreement between the proposed method and the solution by LEE and MOUSA [8] is better than that between the proposed method and the solution by SCORDELIS and LO [7]. The latter may be due to the assumption made by SCORDELIS and LO, in their analysis of linear variation of the components of intermediate diaphragm reactions along the width of each plate. It was mentioned previously that the linear distributions of the reaction components deviate considerably from the results obtained by LEE and MOUSA using uniform step functions with nine steps across each plate.

Appendix. Notation

The following symbols are used in this paper:

A_i	transverse cross-sectional area of plate i
A_m, A_m^1 to A_m^t	Fourier coefficients for load and reactions
a_1 to a_t	span ratio
a	longitudinal span
B_m	Fourier coefficient for elastic curves
b_i	width of plate i
c_1 to c_4	constants of integration
D_l, D_{lk}	series defined by Eqs. (16) and (17)
d_i	distribution factor of plate i
E	modulus of elasticity
K_i	correction factor of plate i
I_i	moment of inertia of plate i
$M_{Li}(x)$	longitudinal bending moment function of plate i
M_{Li}	longitudinal bending moment of plate i
M_{Ti}	final transverse bending moment of plate i
M_{Ti}^1	transverse bending moment of plate i due to applied load
M_{Tij}	transverse bending moment of plate i due to rotation of plate j
m	integer defining Fourier harmonic in x direction
$p(x)$	arbitrary load function
$q(x)$	in-plane load function due to applied load
$q_{0i}(x)$	in-plane load function on plate i due to rotation
q_i	uniformly distributed in-plane load on plate i due to applied load
q_i^1	in-plane load on plate i at reference section due to applied load
q_{0i}^1	in-plane load on plate i at reference section due to rotation
R_1 to R_t	intermediate reactions
r	infinitely small length increment
t	number of intermediate diaphragms
$y(x)$	in-plane displacement function
$y_p(x)$	particular solution for y

$y_i(x)$	in-plane displacement function of plate i
y_i^1	in-plane displacement of plate i at reference section
σ_i	final normal stresses of plate i
σ_i^1	normal stresses of plate i due to applied load
σ_{ij}	normal stresses of plate i due to rotation of plate j
η_x	resultant of longitudinal normal stresses at x
ψ_i	total rotation of plate i
ψ_i^0	assumed rotation of plate i
δ_i	final corrected in-plane displacement of plate i
Δ_{ij}	corrected in-plane displacement of plate i due to rotation of plate j
Δ_i	corrected in-plane displacement of plate i
λ_{ij}	angle between plate i and plate j defined in Fig. 3

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Summary

An approximate analysis of multiple span, multiple bay prismatic shell structures simply supported at the two end diaphragms and continuous over the intermediate transverse diaphragms is presented. The in-plane load, the intermediate diaphragm reactions and the elastic curve in the beam action analysis are expanded into Fourier series. The arbitrary plate rotations and the corresponding transverse bending moments and fold line reactions are assumed to vary along the span as the elastic curves due to the applied load for each term of the Fourier series. The actual plate rotations are determined from the compatability of fold line displacements at a reference transverse section. The number of terms in the Fourier series needed depends upon the desired accuracy of the solution. In general, four to five terms are sufficient for practical purposes and the proposed method is feasible for hand calculation.

Résumé

On présente une méthode de calcul approché des voiles prismatiques à travées multiples et à baies multiples simplement appuyées aux deux diaphragmes extrèmes et continus sur les diaphragmes transversaux intermédiaires. La charge dans le plan, les réactions des diaphragmes intermédiaires et la ligne élastique calculée comme pour une poutre ont été exprimé en séries de Fourier. Les rotations arbitraires de la plaque et les moments de flexion transversaux correspondants et les réactions des lignes de jonction sont supposés varier le long de portée comme les lignes élastique sous la charge appliquée pour chaque terme des séries de Fourier. Les rotations effectives de la plaque sont déterminées grâce à la loi de compatibilité des déplacements des lignes de pli par rapport à une section transversale de référence. Le nombre des termes nécessaires de la série de Fourier dépend de l'exactitude de la solution exigée. En général, 4 ou 5 termes sont suffisants dans la pratique et la méthode proposée est réalisable à la main.

Zusammenfassung

Für mehrfeldrige, mehrschiffige und prismatische Schalen, deren Endscheiben frei aufgelegt sind sowie über Zwischenquerscheiben durchlaufen, wird ein Näherungsverfahren vorgelegt. Die in der Fläche liegende Last, der Zwischenscheiben Auflagerkraft und die sich für einen Balken ergebende elastische Linie wurden in Fourier-Reihen entwickelt. Zur Variation längs der Spannweite sind die willkürlichen Plattenverdrehungen und die entsprechenden Quer-Biegemomente sowie Faltlinien-Auflagerkräfte so wie die elastische Linie, angemessen jedem Glied der Fourier-Reihe der angreifenden Last, angenommen. Die wirklichen Plattenverdrehungen sind durch die Verträglichkeit der Faltlinien-Verschiebungen eines nachgewiesenen Teils bestimmt. Es hängt von der gewünschten Genauigkeit ab, wie hoch die Zahl der benötigten Fourier-Glieder ist. Im allgemeinen dürften für die praktischen Fälle vier bis fünf Glieder genügen und es zeigt sich, daß das vorgeschlagene Verfahren für die Handrechnung durchführbar ist.

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