Zeitschrift: IABSE publications = Mémoires AIPC = IVBH Abhandlungen
Band: 30 (1970)

Artikel: On the problem of edge disturbances in lattice domes

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DOI: https://doi.org/10.5169/seals-23593

On the Problem of Edge Disturbances in Lattice Domes

Problèmes des perturbations aux bords dans les coupoles à treillis

Zum Randstörungsproblem in den Gitterkuppeln

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1. Introduction

The analysis of lattice shells has attained a considerable importance in recent years, particularly in connection with the design of large-span geodesic domes.

The purpose of the present paper is to present an approximate method of determining the magnitude of edge disturbances in such shells. Both rotationally symmetrical and non-symmetrical perturbations are dealt with, by assuming an analogue model for the structural behaviour of the shell lattice. By an appropriate reasoning, the constitutive equations of isotropic shell theory can be replaced by relationships associated with the properties of the lattice members and dimensions.

The treatment of rotationally symmetric perturbations is performed under the usual assumption that it is possible to neglect lower order, as compared to higher order derivatives of shell quantities. In the case of rotationally non-symmetrical perturbations, the shallow shell theory is used, which is also mathematically equivalent to the above assumption.

The results obtained in the present paper were part of the studies for the design of Manaus Dome, a geodesic dome with 300 m in diameter, to be built in the Amazon jungle in Brazil, by Companhia Tropical de Hoteis. The architect was Mr. Sérgio Bernardes and the structural engineer Mr. Paulo

Fragoso. The development of the methods of analysis of the dome structure was entrusted to the present author.

The examination of the theoretical assumptions and results was entrusted to Prof. W. ZERNA, at the Ruhr-Universität Bochum, Germany.

Following the requirements in the design, only triangular meshes in the lattice were considered.

2. The Analogue Model

In order to account for the lattice properties in the calculations, we must supply an analogue model for the bending and the extensional rigidities of the shell, in terms of the characteristics of the lattice members. We refer to Fig. 1 for notations. The moment of inertia and the cross section of a bar are J and F respectively and a is the height of an equilateral triangle.

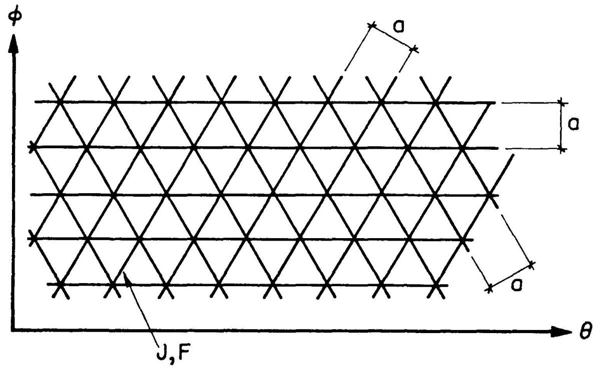

Fig. 1.

A meridional (ϕ) and parallel (θ) coordinate system is used. The stiffness properties of the lattice model are derived in a simple way, by subjecting the lattice to unit generalized deformations, as unit changes of curvature and twist and unit extensions and shears. The contributions of each group of bars to the shell stress resultants are collected together and the results referred to the unit length of the lattice plane. We refer to [2] and [3] for detailed demonstrations. If coupling of in-plane and bending effects is neglected, this analysis will lead to the following constitutive equations (see Fig. 2b):

$$M_\phi = D_\phi^\phi k_\phi + D_\phi^\theta k_\theta\,; \qquad M_\theta = D_\theta^\theta k_\theta + D_\theta^\phi k_\phi\,; \qquad M_{\phi\theta} = D_{\phi\theta}^{\phi\theta} k_{\phi\theta}\,. \tag{1}$$

$$\epsilon_\phi = \Delta_\phi^\phi N_\phi + \Delta_\phi^\theta N_\theta\,; \qquad \epsilon_\theta = \Delta_\theta^\theta N_\theta + \Delta_\theta^\phi N_\phi\,; \qquad \epsilon_{\phi\theta} = \Delta_{\phi\theta}^{\phi\theta} N_{\phi\theta}\,. \tag{2}$$

In these relationships k_ϕ, k_θ and $k_{\phi\theta}$ are changes of curvature and twist respectively and ϵ_ϕ, ϵ_θ, $\epsilon_{\phi\theta}$ the strains of the shell middle surface. The appropriate sign for the changes of curvature and twist must be inserted in (1) according to the circumstances.

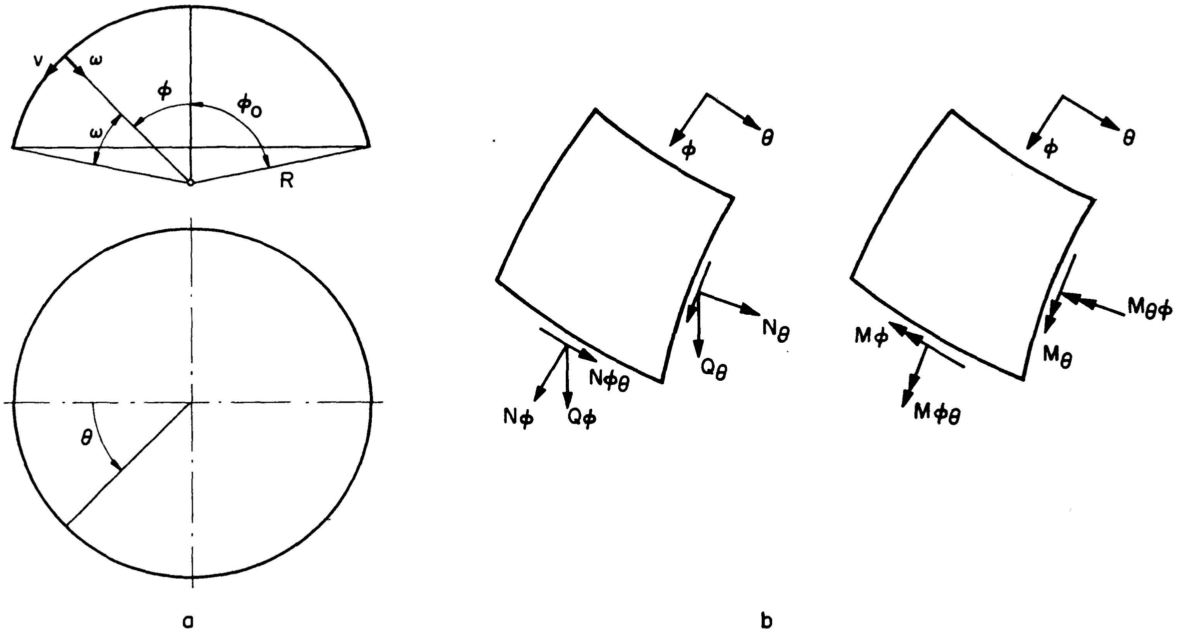

Fig. 2.

For the coefficients D_{ϕ}^{ϕ}, $D_{\phi}^{\theta} \ldots \Delta_{\phi}^{\phi} \ldots \Delta_{\phi\theta}^{\phi\theta}$, following results are found (Fig. 1):

$$D_{\phi}^{\phi} = D_{\theta}^{\theta} = \frac{3\,E\,J}{8\,a}(3+\mu); \quad D_{\phi}^{\theta} = D_{\theta}^{\phi} = \frac{3\,E\,J}{8\,a}(1-\mu); \quad D_{\phi\theta}^{\phi\theta} = \frac{3\,E\,J}{4\,a}(1+\mu) \tag{3}$$

where

$$\mu = \frac{G\,J_d}{E\,J} \tag{4}$$

($G\,J\,d$ = Saint Venant's torsional stiffness of a lattice member; $E\,J$ = Bending stiffness of a lattice member.)

$$\Delta_{\phi}^{\phi} = \Delta_{\theta}^{\theta} = \frac{a}{E\,F}; \quad \Delta_{\phi}^{\theta} = \Delta_{\theta}^{\phi} = \frac{a}{3\,E\,F}; \quad \Delta_{\phi\theta}^{\phi\theta} = \frac{8\,a}{E\,F}. \tag{5}$$

It can be checked that the coefficients (3) on the one hand and the coefficients (5) on the other hand, have the same structure as in the theory of isotropic uniform shells.

Nevertheless, the coefficients (3) are not related in a simple way to the coefficients (5), as in the constitutive equations of the isotropic shell theory, because in this case we have only two basic elastic constants.

The analogue shell thickness as evaluated by means of (3) is different from the membrane thickness as determined from (5). In particular, for the case of double-layer shells, the difference between bending and membrane thickness is very large and we may be led to considerable errors if we make estimates by means of the results from the isotropic shell theory, by using offhand analogies.

3. Axissymmetric Edge Disturbances

In the sequel, we shall summarize the basic relations for rotationally symmetrical edge perturbations [1]. The notation $(\ldots)'$ holds for $\frac{d}{d\phi}(\ldots)$.

a) *Equilibrium:*

$$\begin{aligned} -N_\theta \cos\phi + (N_\phi \sin\phi)' - Q_\phi \sin\phi &= 0, \\ N_\theta \sin\phi + N_\phi \sin\phi + (Q_\phi \sin\phi)' &= 0, \\ M_\theta \cos\phi + (M_\phi \sin\phi)' - Q_\phi R \sin\phi &= 0. \end{aligned} \tag{6}$$

b) *Strain displacement relations* (Fig. 2a)

$$\epsilon_\phi = \frac{1}{R}(v' - w)\ ; \qquad \epsilon_\theta = \frac{1}{R}(v \cot\phi - w)\ ; \qquad \chi = \frac{1}{R}(v + w'); \tag{7}$$

$$k_\phi = -\frac{1}{R}\chi'\ ; \qquad k_\theta = -\frac{\chi}{R}\cot\phi. \tag{8}$$

(χ = rotation of tangent to the meridian, positive if the center of curvature is raised.)

By eliminating the displacement components v and w in (7), the equations of compatibility

$$\chi = (\epsilon_\phi - \epsilon_\theta)\cot\phi - \frac{d\,\epsilon_\theta}{d\phi} \tag{9}$$

is found. Furthermore, the first two equations of (6) can be combined to yield

$$N_\phi = -Q_\phi \cot\phi; \quad N_\theta = Q'_\phi. \tag{10}$$

The equilibrium and the compatibility equations for symmetric edge disturbances are now readily derived.

The relevant equilibrium equation is the third of (6) and we eliminate the bending moments by means of the constitutive relations (1), combined with (3) and (8). The result is

$$\chi'' + [(1+\nu^*)\cot\phi - \nu^*\cot\phi]\chi' + [-\nu^* - \cot^2\phi]\chi + \frac{R^2 Q_\phi}{D_\phi^\phi} = 0 \tag{11}$$

where

$$\nu^* = \frac{D_\phi^\theta}{D_\phi^\phi} = \frac{1-\mu}{3+\mu}. \tag{12}$$

In the compatibility Eq. (9), we eliminate ϵ_ϕ and ϵ_θ by means of (2) and consider thereby (10), so that

$$\begin{aligned}\chi = &-(\Delta_\phi^\phi - \Delta_\theta^\phi)\cot^2\phi\, Q_\phi + (\Delta_\theta^\theta - \Delta_\phi^\theta)\cot\phi\, Q'_\phi + \Delta_\theta^\theta\, Q''_\phi \\ &-(1+\cot^2\phi)\,\Delta_\theta^\phi\, Q_\phi + \Delta_\theta^\phi \cot\phi\, Q'_\phi .\end{aligned} \tag{13}$$

In the relationships (11) and (13) we have a system of differential equations in the variables χ and Q_ϕ. As soon as these are determined, all other quantities are easily derived.

In the case of isotropic shells, obviously $\nu^* = \nu$, the Poisson ratio, and $\Delta_\phi^\phi = \frac{1}{Eh}$, $\Delta_\theta^\phi = -\frac{\nu}{Eh}$, where h is the shell thickness.

Then, (11) and (13) simplify to

$$\chi'' + \cot\phi\,\chi' - (\nu + \cot^2\phi)\,\chi + \frac{R^2 Q_\phi}{D_\phi^\phi} = 0$$

with
$$D_\phi^\phi = \frac{E\,h^3}{12\,(1-\nu^2)}$$

and
$$E\,h\,\chi = Q''_\phi + \cot\phi\, Q'_\phi + (\nu - \cot^2\phi)\, Q_\phi$$

which are well known.

By neglecting first order derivatives and functional values as compared to second order derivatives in (11) and (13), these reduce to

$$\chi'' + \frac{R^2 Q_\phi}{D_\phi^\phi} = 0 ; \qquad Q''_\phi = \frac{\chi}{\Delta_\phi^\phi} . \tag{14}$$

With the notation
$$4\,k^4 = \frac{R^2}{\Delta_\theta^\theta D_\phi^\phi}$$

or, from (3) and (5),
$$k^4 = \frac{R^2}{\frac{3(3+\mu)}{2}\,\frac{J}{F}} . \tag{15}$$

Eqs. (14) are combined to yield

$$\chi^{\mathrm{IV}} + 4\,k^4 \chi = 0 ; \qquad Q_\phi^{\mathrm{IV}} + 4\,k^4\, Q_\phi = 0 . \tag{16}$$

With the usual coordinate transformation in order to measure the azimuthal angle from the edge (see Fig. 2a), the solutions of (16) which decrease from the edge are

$$Q_\phi = C\,e^{-k\omega} \cos(k\,\omega + \psi) ,$$

$$\chi = \frac{2\,a\,k^2}{E\,F}\, C\,e^{-k\omega} \sin(k\,\omega + \psi) \tag{17}$$

where C and ψ are amplitude and phase constants respectively. The expressions for the bending moments are now found right-away by means of the second of (17), (8) and (1). As a result,

$$M_\phi = \frac{3}{4}\frac{J k^2}{R F}(3+\mu) C e^{-k\omega}\left[\sqrt{2}\, k \cos\left(k\omega+\psi+\frac{\pi}{4}\right) - \frac{1-\mu}{3+\mu}\cot\phi \sin(k\omega+\psi)\right], \tag{18}$$

$$M_\theta = \frac{3}{4}\frac{J k^2}{R F}(3+\mu) C e^{-k\omega}\left[-\cot\phi\sin(k\omega+\psi) + \frac{1-\mu}{3+\mu}\sqrt{2}\, k\cos\left(k\omega+\psi+\frac{\pi}{4}\right)\right].$$

The membrane stresses are readily obtained from the first of (17), with (10):

$$N_\phi = -C\cot\phi\, e^{-k\omega}\cos(k\omega+\psi),$$

$$N_\theta = -C k\sqrt{2}\, e^{-k\omega}\sin\left(k\omega+\psi+\frac{\pi}{4}\right). \tag{19}$$

We shall now derive some formulas for edge thrusts and edge moments in the dome (Fig. 3), which are useful in accounting for boundary conditions along the foundation.

For the case of an edge thrust H (Fig. 3a), the boundary conditions are $M_\phi = 0$ and $Q_\phi = -H\sin\phi_0$ for $\omega = 0$ and therefore $\psi = \pi/4$ and $C = -\sqrt{2}\, H\sin\phi_0$. The stress resultants become now

$$Q_\phi = -H\sqrt{2}\sin\phi_0\, e^{-k\omega}\cos\left(k\omega+\frac{\pi}{4}\right)$$

$$M_\phi = \frac{3\sqrt{2}}{4}\frac{k^2 J}{R F}(3+\mu) H\sin\phi_0\, e^{-k\omega}\left[\sqrt{2}\, k\sin k\omega - \frac{1-\mu}{3+\mu}\cot\phi\sin\left(k\omega+\frac{\pi}{4}\right)\right],$$

$$M_\theta = \frac{3\sqrt{2}}{4}\frac{k^2 J}{R F}(3+\mu) H\sin\phi_0\, e^{-k\omega}\left[\cot\phi\sin\left(k\omega+\frac{\pi}{4}\right) + \frac{1-\mu}{3+\mu}\sqrt{2}\, k\sin k\omega\right], \tag{20}$$

$$N_\phi = \sqrt{2}\, H\sin\phi_0\cot\phi\, e^{-k\omega}\cos\left(k\omega+\frac{\pi}{4}\right),$$

$$N_\theta = 2H\sin\phi_0\, k\, e^{-k\omega}\cos k\omega.$$

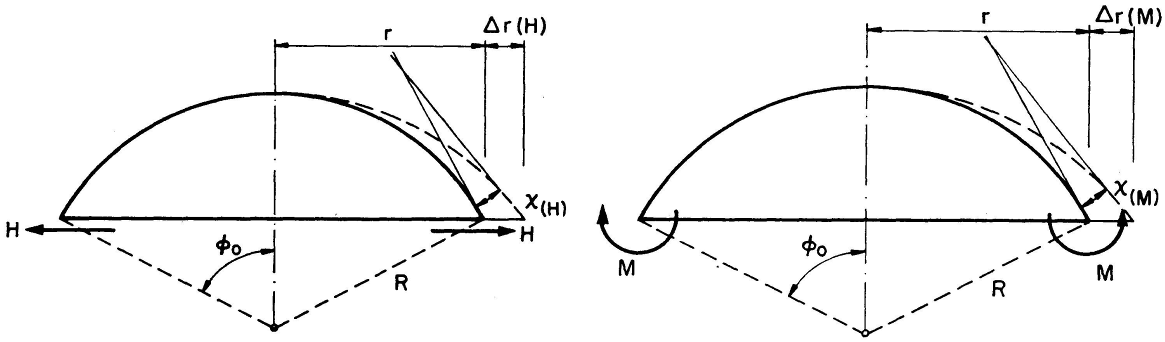

Fig. 3.

For the edge displacement, $\Delta_r(H) = R\sin\phi_0\,\epsilon_{\theta(\omega=0)}$ and, on account of (17) we find

$$\chi_{(H)} = -\frac{2H\sin\phi_0\, a k^2}{E F}; \qquad \Delta_{r(H)} = \frac{2a\sin^2\phi_0\, R k H}{E F}. \tag{21}$$

In the case of an edge moment (Fig. 3b), $M_\phi = M$ and $Q_\phi = 0$, for $=0$ so that

$$\begin{aligned}
Q_\phi &= \frac{4}{3}\frac{M R F}{k^3 J (3+\mu)} e^{-k\omega} \sin k\omega, \\
M_\phi &\simeq M e^{-k\omega} (\sin k\omega + \cos k\omega), \\
M_\theta &= M e^{-k\omega}\left[\frac{1}{k}\cot\phi \cos k\omega + \frac{1-\mu}{3+\mu}(\cos k\omega + \sin k\omega)\right], \\
N_\phi &= -\frac{4}{3}\frac{M R F}{k^3 J (3+\mu)} \cot\phi \, e^{-k\omega} \sin k\omega, \\
N_\theta &= \frac{4}{3}\frac{M R F}{k^2 J (3+\mu)} e^{-k\omega} (\cos k\omega - \sin k\omega)
\end{aligned} \tag{22}$$

and

$$\chi_{(M)} = -\frac{8}{3}\frac{M a R}{k J (3+\mu) E}; \qquad \Delta_{r(M)} = \frac{4}{3}\frac{M a R^2 \sin\phi_0}{k^2 J (3+\mu) E}. \tag{23}$$

A simple control may be obtained for the above results, by using the reciprocity principle. According to this principle, $|\chi_{(H)}| = |\Delta_{r(M)}|$ for $H = M = 1$, so that we must have

$$\frac{2 a k^2}{F} = \frac{4}{3}\frac{a R^2}{k^2 J (3+\mu)}.$$

If we substitute here k from (15), we conclude in fact that this is an identity and both sides will reduce to

$$\frac{2\sqrt{2}}{3}\frac{a R}{\sqrt{(3+\mu) J F}}.$$

4. Non-symmetrical Edge Disturbances

The treatment of non-symmetrical edge disturbances by accounting for the analogue model of the lattice structure is associated with considerable difficulties. Therefore an approximate analysis must be attempted.

It has been shown elsewhere [5] that the usual mathematical simplification of neglecting lower order derivatives in edge perturbation problems in shells of revolution is equivalent to the assumptions of the theory of shallow shells. The mathematical equivalence is particularly apparent in a tensor presentation of the theory.

Therefore, the shallow shell theory with an obvious modification for the analogue lattice model will be used here. WLASSOW [4] has shown that the equations of the shallow shell theory, in the case of unloaded isotropic uniform shells are

$$\frac{1}{E h}\nabla^2\nabla^2\Phi - \nabla_k^2 w = 0; \quad \nabla_k^2\Phi + \frac{E h^3}{12(1-\nu^2)}\nabla^2\nabla^2 w = 0, \tag{24}$$

where Φ and w are well known stress and normal displacement functions. $\nabla^2(\ldots)$ and $\nabla_k^2(\ldots)$ are certain operators in surface coordinates. The first equation is the compatibility equation and the second, the equation of equilibrium.

If we follow the derivation of (24) and observe the new expressions for the bending and the membrane coefficients (3) and (5) of the lattice model, we come to the conclusion that (24) must be modified to

$$\frac{a}{EF}\nabla^2\nabla^2\Phi-\nabla_k^2 w=0\ ;\quad \nabla_k^2\Phi+\frac{3\,EJ}{8a}(3+\mu)\nabla^2\nabla^2 w=0. \tag{25}$$

for lattice domes.

In spherical coordinates, with the notations $(\ldots)'=\frac{d}{d\phi}(\ldots)$ and $(\ldots)^{\cdot}=\frac{d}{d\theta}(\ldots)$, the operator ∇^2 is written as

$$\nabla^2(\ldots)=\frac{1}{R^2}[(\ldots)''+\cot\phi(\ldots)'+\text{cossec}^2\phi(\ldots)^{\cdot\cdot}]\text{ and }\nabla_k^2(\ldots)=\frac{1}{R}\nabla^2(\ldots).$$

The membrane stresses are derived from the stress function through

$$\begin{aligned} N_\phi &= \frac{1}{R^2}(\text{cossec}^2\Phi^{\cdot\cdot}+\cot\phi\,\Phi')\ ;\qquad N_\theta=\frac{1}{R^2}\Phi''\ ;\\ N_{\phi\theta} &= -\frac{1}{R^2}(\text{cossec}\,\phi\,\Phi'^{\cdot}-\cos\phi\,\text{cossec}^2\phi\,\Phi'). \end{aligned} \tag{26}$$

For the bending moments, we use again (1), along with (3), with the formulas

$$\begin{aligned} k\phi &= -\frac{1}{R^2}w''\ ;\quad k_\theta=-\frac{1}{R^2}(\text{cossec}^2\phi\,w^{\cdot\cdot}+\cot\phi\,w')\ ;\\ k_{\phi\theta} &= \frac{2}{R^2}\text{cossec}\,\phi\,(w'^{\cdot}-\cot\phi\,w^{\cdot}) \end{aligned} \tag{27}$$

for the changes of curvature and twist.

We shall next obtain an approximate solution of Eqs. (25)

By expressing w and Φ as $w=\nabla^2\nabla^2 F$; $\Phi=\frac{EF}{a}\nabla_k^2 F$ in terms of an auxiliary function F, the first of (25) is satisfied identically. By observing that $\nabla_k^2(\ldots)=\frac{1}{R}\nabla^2(\ldots)$, the second equation reduces to

$$\nabla^2\nabla^2 w+\frac{8\,F}{3(3+\mu)R^2 J}w=0 \tag{28}$$

where now $\quad \nabla^2(\ldots)=(\ldots)''+\cot\phi(\ldots)'+\text{cossec}^2\phi(\ldots)^{\cdot\cdot}. \quad (29)$

After integrating (28), we determine Φ from the first of (25) which now may be written as

$$\nabla^2\left(\nabla^2\Phi-\frac{EF}{a}R\,w\right)=0\,. \tag{30}$$

By putting
$$\nabla^2 \Phi - \frac{E F}{a} R w = \psi \tag{31}$$
we see that
$$\nabla^2 \psi = 0 . \tag{32}$$
Thus, we first integrate (32) and substitute the result in (31), so that
$$\nabla^2 \Phi = \frac{E F}{a} R w + \psi . \tag{33}$$
The integration of (33) yields finally the stress function Φ. The differential Eq. (28) can be replaced by two second order differential equations
$$\nabla^2 \nabla^2 w + \mu w = 0 \tag{34}$$
with
$$\mu = \pm i R \sqrt{\frac{8 F}{3(3+\mu) J}} . \tag{35}$$
For real particular solutions, we need only to account for one sign in (35).

We assume the Fourier expansions
$$w = \sum_{n=1}^{\infty} w_n \cos n \theta ; \qquad \Phi = \sum_{n=1}^{\infty} \Phi_n \cos n \theta ; \qquad \psi = \sum_{n=1}^{\infty} \psi_n \cos n \theta \tag{36}$$
and then, from (34), we conclude that
$$w_n'' + \cot \phi \, w_n' + (\mu - n^2 \operatorname{cossec}^2 \phi) w_n = 0 . \tag{37}$$

We now make a simplifying assumption, in order to avoid complicate functions of mathematical physics in the integration of (37). The assumption is very appropriate for hemispherical large span domes, as in the case of Manaus Dome, in which the edge disturbances are highly localized. In (37) we shall neglect the variability of the coefficients in the differential equation and set them equal to their functional values at $\phi = \pi/2$. In other cases, the approximation should be tolerable with the numerical values of the coefficients at $\phi = \phi_0$.

Eq. (37) then simplifies to
$$w_n'' + (\mu - n^2) w_n = 0 \tag{38}$$
with μ defined by (35). The real form of the solution of (38) is shown to be
$$w_n = A_n e^{a_n \phi} \cos b_n \phi + B_n e^{a_n \phi} \sin b_n \phi + C_n e^{-a_n \phi} \cos b_n \phi + D_n e^{-a_n \phi} \sin b_n \phi$$
with
$$a_n = p_n \cos \omega_n ; \quad b_n = p_n \sin \omega_n \tag{39}$$
where
$$p_n = \sqrt[4]{n^4 + \mu_{Re}^2} ; \qquad \omega_n = \frac{1}{2} \arctan \frac{\mu_{Re}}{n^2} ; \qquad \mu_{Re} = R \sqrt{\frac{8 F}{3 (3+\mu) J}} . \tag{40}$$
For edge disturbances at $\phi = \pi/2$, we must retain only the solutions
$$w_n = A_n e^{a_n \phi} \cos b_n \phi + B_n e^{a_n \phi} \sin b_n \phi \tag{41}$$
and by testing this solution in the exact Eq. (37), we can make sure that the approximation is very appropriate.

Table I. Fourier Coefficients for Bending Stress Resultants

	Stiffness	A_n	B_n
M_{ϕ_n}	$-\frac{3(3+\mu)EJ}{8aR^2}$	$e^{a_n\phi}\Big[\Big(a_n^2-b_n^2-\frac{1-\mu}{3+\mu}\Big)\cos b_n\phi$ $-2a_nb_n\sin b_n\phi\Big]$	$e^{a_n\phi}\Big[\Big(a_n^2-b_n^2-\frac{1-\mu}{3+\mu}n^2\Big)\sin b_n\phi$ $+2a_nb_n\cos b_n\phi\Big]$
M_{θ_n}	$\frac{3(3+\mu)EJ}{8aR^2}$	$e^{a_n\phi}\Big[\Big(n^2-\frac{1-\mu}{3+\mu}(a_n^2-b_n^2)\Big)\cos b_n\phi$ $+2\frac{(1-\mu)}{(3+\mu)}a_nb_n\sin b_n\phi\Big]$	$e^{a_n\phi}\Big[\Big(n^2-\frac{(1-\mu)}{(3+\mu)}(a_n^2-b_n^2)\Big)\sin b\phi$ $-2\frac{(1-\mu)}{(3+\mu)}a_nb_n\cos b_n\phi\Big]$
$M_{\phi\theta_n}$	$-\frac{3(1-\mu)EJ}{2aR^2}$	$e^{a_n\phi}(na_n\cos b_n\phi-nb_n\sin b_n\phi)$	$e^{a_n\phi}(na_n\sin b_n\phi+nb_n\cos b_n\phi)$
Q_{ϕ_n}	$-\frac{3(3+\mu)EJ}{8aR^3}$	$e^{a_n\phi}[((a_n^2-b_n^2)a_n-2a_nb_n^2-n^2a_n)\cos b_n\phi$ $+(-(a_n^2-b_n^2)b_n-2a_n^2b_n+n^2b_n)\sin b_n\phi]$	$e^{a_n\phi}[((a_n^2-b_n^2)a_n-2a_nb_n^2-n^2a_n)\sin b_n\phi$ $+((a_n^2-b_n^2)b_n+2a_n^2b_n-n^2b_n)\cos b_n\phi]$
Q_{θ_n}	$\frac{3(3+\mu)EJ}{8aR^3}$	$e^{a_n\phi}[((a_n^2-b_n^2)n+n^3)\cos b_n\phi$ $-2a_nb_nn\sin b_n\phi]$	$e^{a_n\phi}[((a_n^2-b_n^2)n+n^3)\sin b_n\phi+2a_nb_n\cos b_n\phi]$
$Q^*_{\phi_n}$	$-\frac{3(3+\mu)EJ}{8aR^3}$	$e^{a_n\phi}\Big[\Big((a_n^2-b_n^2)a_n-2a_nb_n^2-n^2a_n$ $+4\frac{(1+\mu)}{(3+\mu)}n^2a_n\Big)\cos b_n\phi+\Big(-(a_n^2-b_n^2)b_n$ $-2a_n^2+n^2b_n-4\frac{(1+\mu)}{(3+\mu)}n^2b_n\Big)\sin b_n\phi\Big]$	$e^{a_n\phi}\Big[\Big((a_n^2-b_n^2)a_n-2a_nb_n^2-n^2a_n$ $+4\frac{(1+\mu)}{(3+\mu)}n^2a_n\Big)\sin b_n\phi+\Big((a_n^2-b_n^2)b_n$ $+2a_n^2b_n-n^2b_n+4\frac{(1+\mu)}{(3+\mu)}n^2b_n\Big)\cos b_n\phi\Big]$

Table II. Fourier Coefficients for Membrane Stresses

	C_n	D_n	A_n	B_n
N_{ϕ_n}	$-\frac{n^2}{R} e^{n\phi}$	$-\frac{n}{2R^2} \phi e^{n\phi}$	$-\frac{EF}{aR} \frac{e^{a_n\phi}}{[\ldots][\ldots]} [n^2 (a_n^2 - b_n^2 - n^2) \cos b_n \phi + 2 a_n b_n n^2 \sin b_n \phi]$	$-\frac{EF}{aR} \frac{e^{a_n\phi}}{[\ldots][\ldots]} [n^2 (a_n^2 - b_n^2 - n^2) \sin b_n \phi - 2 a_n b_n \cos b_n \phi]$
N_{θ_n}	$\frac{n^2}{R^2} e^{n\phi}$	$\frac{1}{R^2} (e^{n\phi} + \frac{1}{2} n \phi e^{n\phi})$	$\frac{EF}{aR} \frac{e^{a_n\phi}}{[\ldots][\ldots]} [((a_n^2 + b_n^2)^2 - n^2 (a_n^2 - b_n^2)) \cos b_n \phi + 2 a_n b_n \sin b_n \phi]$	$\frac{EF}{aR} \frac{e^{a_n\phi}}{[\ldots][\ldots]} [((a_n^2 + b_n^2)^2 - n^2 (a_n^2 - b_n^2)) \sin b_n \phi - 2 a_n b_n n^2 \cos b_n \phi]$
$N_{\phi\theta_n}$	$n^2 e^{n\phi}$	$\frac{1}{2} (e^{n\phi} + n \phi e^{n\phi})$	$\frac{EF}{a} \frac{e^{a_n\phi}}{[\ldots][\ldots]} [(-b_n (a_n^2 - b_n^2 - n^2) + 2 a_n^2 b_n) \sin b_n \phi + (2 a_n b_n^2 + a_n (a_n^2 - b_n^2 - n^2)) \cos b_n \phi]$	$\frac{EF}{aR} \frac{e^{a_n\phi}}{[\ldots][\ldots]} [(b_n (a_n^2 - b_n^2 - n^2) - 2 a_n^2 b_n^2) \cos b_n \phi + (2 a_n b_n^2 + a_n (a_n^2 - b_n^2 - n^2)) \sin b_n \phi]$

$$[\ldots][\ldots] = [(a_n - n)^2 + b_n^2][(a_n - n)^2 - b_n^2]$$

We next integrate (32). By considering (36),

$$\psi_n'' + \cot\phi\,\psi_n' - n^2\psi_n = 0$$

and, under the same assumptions it reduces to $\psi_n'' - n^2\psi_n = 0$, with the solutions $e^{n\phi}$ and $e^{-n\phi}$. We must discard $e^{-n\phi}$ and retain only $e^{n\phi}$.

We introduce this result in (31) and consider again the expansion (36). Then, by taking account of the same simplifying assumptions,

$$\Phi_n'' - n^2\Phi_n = \frac{EF}{a}(A_n e^{a_n\phi}\cos b_n\phi + B_n e^{a_n\phi}\sin b_n\phi) + D_n e^{n\phi}.$$

The general solution of this differential equation is

$$\Phi_n = C_n e^{a_n\phi} + D_n\frac{1}{2n}\phi\, e^{n\phi} + \frac{EFR}{a}\frac{e^{a_n\phi}}{[(a_n-n)^2+b_n^2][(a_n+n^2+b_n^2]}$$
$$[A_n\{(a_n^2-b_n^2-n^2)\cos b_n\phi + 2a_n b_n\sin b_n\phi\} \quad (42)$$
$$+ B_n\{(a_n^2-b_n^2-n^2)\sin b_n\phi - 2a_n b_n\phi\}].$$

On account of the Fourier expansion (36), the stress resultants will have also the expansions below:

$$M_\phi = \sum_{n=1}^{\infty} M_{\phi_n}\cos n\theta; \quad M_\theta = \sum_{n=1}^{\infty} M_{\theta_n}\cos n\theta; \quad M_{\phi\theta} = \sum_{n=1}^{\infty} M_{\phi\theta_n}\sin n\theta. \quad (43)$$

$$N_\phi = \sum_{n=1}^{\infty} N_{\phi_n}\cos n\theta; \quad N_\theta = \sum_{n=1}^{\infty} N_{\theta_n}\cos n\theta; \quad N_{\phi\theta} = \sum_{n=1}^{\infty} N_{\phi\theta_n}\sin n\theta. \quad (44)$$

The amplitudes of the stress resultants are now determined by simple differentiations by means of (1) and (26), by accounting for (27). On account of the simplifying assumption, we are allowed to write instead

$$N_\phi \cong \frac{1}{R^2}\Phi^{\cdot\cdot}; \quad N_\theta \cong \frac{1}{R^2}\Phi''; \quad N_{\phi\theta} = -\frac{1}{R^2}\Phi'^{\cdot};$$
$$k_\phi \cong -\frac{1}{R^2}w''; \quad k_\theta = -\frac{1}{R^2}w^{\cdot\cdot}; \quad k_{\phi\theta} = \frac{2}{R^2}w'^{\cdot}. \quad (45)$$

The results are summarized in Tables I and II. The tables should be self-explaining, all Fourier coefficients being given as

$$\textit{Fourier coefficient} = \textit{stiffness} \times [f_n(\phi)A_n + g_n(\phi)B_n + \ldots].$$

For example,

$$M_{\phi_n} = -\frac{3(3+\mu)}{8aR^2}\left\{e^{a_n\phi}\left[\left(a_n^2-b_n^2-\frac{1-\mu}{3+\mu}n^2\right)\cos b_n\phi - 2a_n b_n\sin b_n\phi\right]A_n\right.$$
$$\left. + e^{a_n\phi}\left[\left(a_n^2-b_n^2-\frac{1-\mu}{3+\mu}n^2\right)\sin b_n\phi + 2a_n b_n\cos b_n\phi\right]\right\}.$$

The generalized shear forces Q_ϕ^* and the shear forces Q_ϕ and Q_θ are also given.

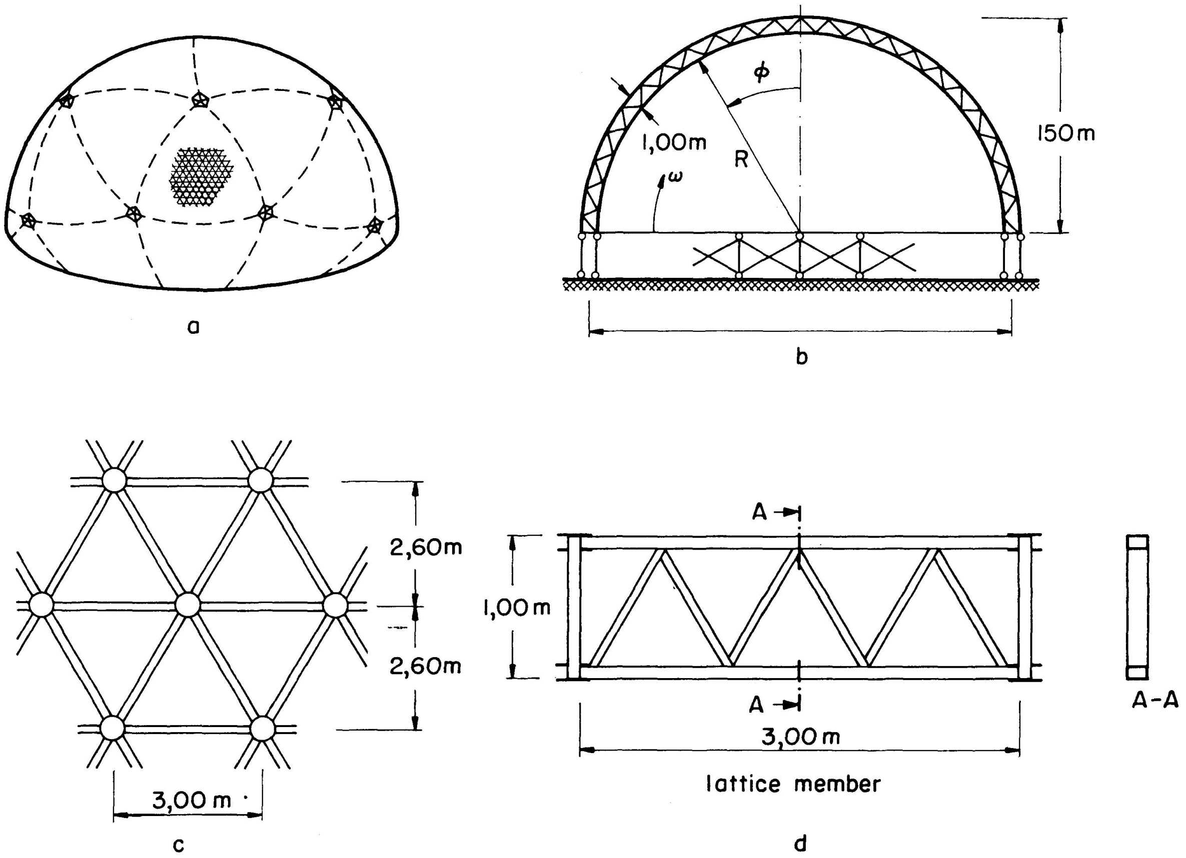

Fig. 4.

5. Numerical Applications

For a numerical illustration of the above theory, we shall report some concrete results obtained in the design of Manaus Dome, a geodesic dome with 300 m in diameter, in high tensile steel.

The main characteristics of Manaus Dome are sketched in Fig. 4, and it is a double-layer shell, each lattice member being a truss with a 1 m depth and 3 m length (see Fig. 4c, d). The dome is supported along the fundation in such a way, that the tangential displacements (normally denoted by u and v) and the meridian rotation vanish. The support is also not able to resist a horizontal thrust, so that $Q_\phi^* = 0$ at $\phi = \pi/2$. The characteristics of the lattice in the edge zone are

$$a = 260\,\mathrm{cm};\; F = 89\,\mathrm{cm}^2;\; J = 1.80 \times 10^5\,\mathrm{cm}^4 \text{ and } R = 150\,\mathrm{m}.$$

5.1. Axissymmetric Edge Disturbances

With the above values (15) gives

$$k = 12.5.$$

The moment of restraint at the foundation will be determined for dead weight, which was variable from 124 kg/m² to 182 kg/m². A calculation of edge displacements was performed by means of the membrane theory with an equivalent membrane thickness determined from

$$\frac{1}{Eh} = \frac{a}{EF} \quad \text{[See (5)]}, \quad \text{i. e.,} \ h = \frac{F}{a} = 0.34 \ \text{cm},$$

The results were

$$\Delta_r^{(g)} = \frac{1.61 \times 10^6}{E}; \quad \chi^{(g)} = -\frac{1.87 \times 10^4}{E}.$$

The elasticity modulus must be inserted in t/m².

With these results and $k = 12{,}5$ above, $\phi_0 = \pi/2$, $\mu = 0$, $H = M = 1$, formulas (21) and (23) yield

$$\chi_{(H=1)} = -\frac{2ak^2}{EF} = -\frac{9.1 \times 10^4}{E}; \quad \Delta_{r(H=1)} = \frac{2aRk}{EF} = \frac{110 \times 10^4}{E};$$

$$\chi_{(M=1)} = -\frac{8}{3}\frac{aR}{kJ(3+\mu)E} = -\frac{1.54 \times 10^4}{E}; \quad \Delta_{r(M=1)} = \frac{4}{3}\frac{aR^2}{k^2J(3+\mu)E} = \frac{9.1 \times 10^4}{E}.$$

With these results, any boundary conditions may be accounted for.

The condition for a vanishing meridian rotation, with no thrust is obviously

$$\chi_{(M=1)}M + \chi^{(g)} = 0 \quad \text{so that} \quad -\frac{1.54 \times 10^4}{E}M - \frac{1.87 \times 10^4}{E} = 0$$

and therefore

$$M = -1.21 \ \text{tm/m}.$$

The corresponding hoop stress at the edge is found from the last formula of (22) and it is

$$N_{\theta(\omega=0)} = \frac{4}{3}\frac{MRF}{k^2J(3+\mu)} = -2.55 \ \text{t/m}.$$

5.2. *Non-symmetrical Edge Disturbances*

We report the results of the calculation for the first and second harmonics of the wind loading.

The wind pressure was determined from a wind tunel test and the result of the calculations for the first and the second harmonics of the meridian rotation were

$$\chi_1^{(w)} = -\frac{5530}{E} \quad \text{and} \quad \chi_2^{(w)} = -\frac{1880}{E},$$

where E must be inserted in t/m². With the present numerical values (40) yields

$$\mu_{R_e} = 315; \quad p_n = 17.8; \quad \omega_n = \frac{\pi}{4}$$

so that $$a_n \simeq b_n = 17.8 \times 0.707 = 12.6$$

from (39). As a_n and b_n are responsible for the damping of the perturbations, we see that this result is very close to the corresponding parameter $k = 12.5$ for the axissymmetric edge disturbances. This conclusion is a further concrete demonstration of the equivalence of Geckler's and the shallow shell theory.

The meridian rotation for bending effects is obtained from (41), as the derivative w_n' for $\phi = \pi/2$. By taking instead $a_n = b_n = 12$ in order to simplify computations, we find

$$w_1' = A_1 a_1 e^{a_1(\pi/2)} + B_1 b_1 e^{a_1(\pi/2)}; \quad w_2' = A_2 a_2 e^{a_2(\pi/2)} + B_2 b_2 e^{a_2(\pi/2)}.$$

The conditions of vanishing meridian rotation are then

$$w_1' + \chi_1^{(w)} = 0; \quad w_2' + \chi_2^{(w)} = 0$$

from which
$$A_1 + B_1 = \frac{1}{12\, e^{6\pi}} \frac{5530}{E},$$
$$A_2 + B_2 = \frac{1}{12\, e^{6\pi}} \frac{1880}{E}. \tag{a}$$

The other boundary condition of vanishing generalized normal shear Q_ϕ^* at $\phi = \pi/2$ is obtained from the last line of Table I ($a_n = b_n$), which now reduces to

$$\left(-2 a_n b_n^2 - n^2 a_n + \frac{4}{3} n^2 a_n\right) A_n + \left(2 a_n^2 b_n - n^2 b_n + \frac{4}{3} n^2 b_n\right) B_n = 0.$$

By inserting the numerical values we see that $A_n \simeq B_n$, so that (a) reduces to

$$A_1 \simeq B_1 = \frac{1}{2 \times 12\, e^{6\pi}} \frac{5530}{E}; \quad A_2 = B_2 = \frac{1}{2 \times 12\, e^{6\pi}} \frac{1880}{E}.$$

The corresponding bending moments are determined from the first line of Table I, with $\phi = \pi/2$.

We would find

$$M_{\phi_1} = -\frac{3(3+\mu) E J}{8 a R^2} e^{6\pi} \left(-\frac{1}{3} A_1 + 2 a_1 b_1 B_1\right)$$

and
$$M_{\phi_2} = -\frac{3(3+\mu) E J}{8 a R^2} e^{6\pi} \left(-\frac{1}{3} A_2 + 2 a_2 b_2 B_2\right).$$

By inserting the numerical values

$$M\phi_1 = -2.26 \times 10^{-3}\, \mathrm{tm/m}; \quad M\phi_2 \doteq -0.76 \times 10^{-3}\, \mathrm{tm/m}$$

which are negligible.

6. Concluding Remark

An important difference between lattice shells and isotropic shells is the essential deviation between membrane and bending stiffness. Particularly, in the case of double-layer lattice shells, the equivalent bending thickness of the shell is a multiple of the corresponding membrane thickness.

If we estimate the edge effects in a lattice shell by means of the equivalent isotropic shell, by using a meridian stiffness obtained from the bending stiffness, the forces will be highly overestimated.

The edge displacements are more directly connected with the hoop stresses, which depend directly on the membrane stiffness.

The appropriate combination of both effects yields the correct results.

7. References

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Summary

The problem of edge disturbances for lattice shells with triangular meshes is investigated approximately considering both the rotationally symmetric and the asymmetric edge stresses.

The behaviour of the lattice is accounted for by means of a continuous analogue model. Numerical results are reported from the design of the Manaus Dome, with 300 m in diameter.

Résumé

Le problème des perturbations aux coupoles en treillis à subdivision triangulaire est étudié par voie approximative, en tenant compte des sollicitations rotationnelles symétriques et des sollicitations asymétriques.

Le comportement du treillis est simulé par un modèle continu. On donne des résultats numériques basés sur le projet de construction pour la coupole de Manaus mesurant 300 m en diamètre.

Zusammenfassung

Das Randstörungsproblem für Gitterkuppeln mit dreieckiger Unterteilung wird in angenäherter Weise untersucht, wobei sowohl drehsymmetrische als auch unsymmetrische Randbeanspruchungen berücksichtigt werden.

Das Verhalten des Gitters wird mit einem kontinuierlichen Modell nachgebildet. Es werden numerische Resultate aus dem Entwurf der Manaus-Kuppel mit 300 m Durchmesser mitgeteilt.

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