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7. Discussion of IABSE PROCEEDINGS

Théorie perfectionnée des poutres droites à parois minces (Improved Theory for Thin-Walled Straight Beams)

Charles Massonnet, P-55/82, published November 1982 in IABSE PERIODICA 4/1982

A discussion by W. H. Wittrick, Emeritus Professor, University of Birmingham, England

A recent paper by Massonnet (1) is concerned with a particular aspect of the problem of shear lag in thin-walled structural members ***of uniform*** cross-section subjected to a ***uniformly distributed transverse loading,*** so that the bending moment varies quadratically and the shearing force and torque vary linearly with the longitudinal co-ordinate z. He refers to a report by me (2), written in 1951, which was primarily concerned with shear lag in aircraft wing boxes ***of non-uniform cross-section*** subjected to a smoothly varying but ***non-uniformly distributed transverse loading,*** so that the bending moment and torque may have ***any*** smooth variation along the length.

Shear lag arises, of course, from the longitudinal warping of initially plane cross-sections due to shearing strains in the walls, and I pointed out in my 1951 paper (2) that it could be considered under two quite distinct headings, namely ***distributed shear lag*** and ***end effects.*** It is well known that if the distribution of shearing stress over any particular cross-section is specified, then the warping of that cross-section out of the plane can be determined, apart from rigid body displacements. In general, if the shearing force and torque vary with z in some smooth but arbitrary way, the difference in warping between neighbouring cross-sections due to engineers' theory shearing stresses is incompatible with the longitudinal strains due to engineers' theory bending stresses. A redistribution of stress is therefore required, in order to provide compatibility, and this is the origin of what I called distributed shear lag. A stress system that has been corrected so as to satisfy compatibility and which, like engineers' theory, still satisfies equilibrium requirements was referred to as a ***basic stress system.*** For any given distribution of load, there is an infinite number of basic stress systems; the particular one that provides the correct solution to any given problem is that which, in addition, satisfies the required conditions of freedom or constraint at the ends of the member. For example, suppose that the member is a cantilever with its tip free, and its root encastré; in general a basic stress system will satisfy neither the requirement of zero longitudinal stress at the tip nor zero warping at the root and there will be only one such system that does so. However, since every basic stress system is, by definition, in equilibrium with the transverse applied loads the difference between any two of them corresponds to the stresses that arise from a ***self-equilibrating*** system of external forces acting on each of the two end cross-sections. By Saint-Venant's Principle the stresses arising from such a self-equilibrating system decay with increasing distance from the end at which it is applied. Thus the whole problem can be split into two parts; (a) find ***any*** basic stress system, and (b) find the additional self-equilibrating systems of forces that must act at the two ends, and the stresses that they produce, in order to satisfy the end conditions. Each of these two parts of the problem results in a modification of engineers' theory stresses, though in the case of a tubular member with an encastré end it is the end effects that are likely to be the most significant.

The first point that I wish to make about Massonnet's paper is that his analysis is concerned only with the first of the two problems just described, namely distributed shear lag. However he makes the statement that his solution is unique, and quotes Kirchhoff's uniqueness theorem in justification. This is incorrect, since the uniqueness theorem requires for its validity not only that the requirements of equilibrium and compatibility throughout an elastic body be satisfied but also that the boundary conditions of specified tractions or specified displacements ***over the entire surface*** shall be satisfied. Since no end conditions were considered, it is clear that the uniqueness theorem does not apply, and his solution is merely one of an infinite number of solutions of the equations (5.1) and (5.2). The one that he has actually obtained is characterized by the fact that the distribution of ***shearing*** stress τ throughout the member is identical with that obtained from engineers' theory. As mentioned by Massonnet my 1951 paper (2) showed that such a basic stress system exists in the special case of a uniform member with linearly varying shearing force and torque. More generally, my paper showed that, if the bending moment and torque vary as polynomials in z, there exists a basic stress system in which σ contains a series of terms proportional to the bending moment and its even derivatives plus terms proportional to the odd derivatives of the torque whilst, conversely, τ contains a series of terms proportional to the torque and its even derivatives plus terms proportional to the odd derivatives of the bending moment. Two integral equations, one for σ and one for τ, were derived in the paper, which enable these terms, of progressively higher order, to be calculated for a thin tube of ***any*** uniform single-cell cross-section by repeated quadrature. As well as a thin wall the tube may also contain any number of concentrated "booms", i.e. longitudinal members that carry direct stress.

The second point that I wish to make is concerned with the equation of compatibility — equation (3.7) or (5.2) — that Massonnet uses, namely

$$\frac{1}{E}\frac{\partial^2\sigma}{\partial s^2} = \frac{1}{G}\frac{\partial^2\tau}{\partial z\partial s}$$

I believe that this equation is correct only if the boundary of the cross-section consists of a series of straight sides. The reason is that if the boundary is curved the co-ordinate s is one of a triad of moving axes and it is necessary to use the strain-displacement relations for a shell, rather than those of plane stress, in deriving the compatibility equation. When this is done an additional term equal to

$$\frac{\partial^2}{\partial z^2}\left(\frac{u_n}{r}\right)$$

appears on the right hand side of the above equation, where u_n is the displacement normal to the wall surface and r the radius of curvature of the boundary of the cross-section.

Moreover, in deriving equation (5.10), Massonnet states that

$$\frac{\partial^2}{\partial s^2}\left(\frac{M_x y}{I}\right) = 0$$

where the term in parentheses is the engineers' theory bending stress. But, this also is correct only if the boundary is straight, in which case y varies linearly with s. Thus the theory on which the paper is based is, I believe, valid only for structures formed from ***flat*** walls. The analysis of my 1951 paper (2) was not restricted in this way.

My remaining comments are concerned with the solution which is given in §6.2 for the uniformly loaded Tee-section. It is stated at the beginning that the effects of shear lag are negligible in the web, which is assumed to be "massive", and thereafter the correction $\Delta\sigma$ is considered only for the flange. The constants of integration are then determinded from (i) symmetry, and (ii) equation (6.10), which says that the total longitudinal force in the flange due to the $\Delta\sigma$ stresses is zero. I do not think that the neglect of the web in this way can be justified, since it is obvious that, for compatibility of longitudinal strains, the $\Delta\sigma$ stress must be the same in both the web and the flange at their junction. This must surely imply that the resultant force and bending moment arising from the $\Delta\sigma$ stresses in the web are just as significant as those in the flange. Finally, if account is taken of the web, the variation of $\Delta\sigma$ across the width of the flange, although symmetrical about the centre, has a discontinuity of slope $\partial(\Delta\sigma)/\partial s$ on the centreline, and does not have zero slope as suggested by equation (6.11).

1. MASSONNET, Ch.: Improved Theory for Thin-walled Straight Beams. IABSE Proceedings, P-55/82, November 1982, pp. 81-95.

2. WITTRICK, W. H.: On the Problem of Shear Lag in Non-Uniform Cylindrical Tubes. Report SM164 of the Aeronautical Research Laboratories, Melbourne, 1951.

Reply of the author

The author agress generally with the considerations of professor Wittrick. He feels that it should be emphasized that his paper [1] and Wittrick's paper of 1951 [2] have not the same aim. The problem solved by Wittrick, namely that of shear lag in non-uniform cylindrical tubes subjected to non-uniform transverse loading, leads to two coupled integral equations that the author has found too cumbersome for practical use. In the "computer age", where we live, such problems would be solved by using a computer program.

The author's aim was very different from that of Wittrick : it was to show that for prismatic beams, the classical theory of bending-torsion of thin-walled members could be extended to the case of quadratically varying bending moments and linearly varying torques. The only correction is the introduction of a distribution of normal stresses $\Delta\sigma$ called "distributed shear lag".

For this reduced problem, the theory presented is much simple than Wittrick's equation. Be said in passing, it is rather unfortunate that because of the limitation imposed to the paper, the author has been obliged to leave the practical application of the theory to the distribution of shear lag in plate and box girders to another paper [3].

However, professor Wittrick is right in his two critics:

1) The compatibility equation on which the theory is developed is only correct for thin-walled profiles with ***straight walls.*** This should have been explicitely said in the paper.
2) The distributed shear lag computed in the paper assumes some relaxed conditions at the ends of the beam, namely $N(\Delta\sigma) = 0$ and $M_x(\Delta\sigma) = 0$ [equations (5.14.) and (5.15.)].

   The simplicity of the stress distribution yields obviously from the neglect of the end conditions which are discussed in Wittrick's paper but lead to very cumbersome computations. Anyway, the author agrees with professor Wittrick that Kirchhoff's unicity theorem cannot be invoked (see bottom of page 85) because the boundary conditions have not been specified here on the end surfaces of the beam.

Additional reference.

[3] R. Maquoi et Ch. Massonnet: Une évaluation simple de la largeur efficace due au traînage de cisaillement. Construction Métallique N° 2-1982, pp. 17 à 24.

Professor Ch. Massonnet