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Analysis of lateral buckling of continuous beams

Contribution à l'étude du déversement des poutres continues

Kippuntersuchung an Durchlaufträgern

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SUMMARY
The application of the transfer matrix and finite element methods to the inelastic lateral buckling of statically indeterminate steel I-beams is discussed. The predictions of both methods are compared with results of tests on both single span and continuous beams. The transfer matrix method is then used to investigate the effectiveness of different bracing arrangements at the internal support of a two-span continuous beam and the finite element method is used to study the influence of decreasing beam slenderness for the same structure.

RÉSUMÉ
Les auteurs discutent l'application de la méthode des matrices de transfert et de celle des éléments finis à l'étude du déversement des poutrelles en double-té hyperstatiques dans le domaine plastique. Ils comparent les résultats des deux méthodes à ceux d'essais entrepris sur des poutres simples et des poutres continues. On applique ensuite la méthode des matrices de transfert à l'examen de l'efficacité de diverses conditions d'entretoisement au droit de l'appui central d'une poutre continue sur deux travées et on utilise la méthode des éléments finis à l'étude de l'influence d'une diminution de l'élancement latéral pour le même système.

ZUSAMMENFASSUNG
Die Autoren besprechen die Anwendung der Methode der Übertragungsmatrizen und derjenigen der Finiten Elemente auf die Kippberechnung statisch unbestimmter I-Stahlträger im unelastischen Bereich. Die Resultate dieser zwei Methoden werden mit den Ergebnissen von Versuchen an einfachen Balken und an Durchlaufträgern verglichen. Das Verfahren der Übertragungsmatrizen wird dann bei der Untersuchung der Wirksamkeit verschiedener Bedingungen von Kipphalterungen an der Zwischenstütze von Zweifeldträgern verwendet, währenddem die Finiten Elemente eingesetzt werden, um den Einfluss einer Abnahme der Trägerschlankheit beim gleichen Tragsystem zu verfolgen.

1. INTRODUCTION

The design of an unbraced steel I-beam is usually based on the elastic in-plane moment distribution caused by the working loads, and on maximum permissible stresses [1, 3, 15] derived from the results of investigations of the elastic and inelastic lateral buckling of statically determinate beams. While this approach can logically be used [12, 16] for an indeterminate continuous beam which buckles elastically, its direct extension to the inelastic buckling of such a beam may not be valid if the final inelastic moment distribution at buckling differs significantly from the initial elastic distribution.

Recently [13], the effect of inelastic moment redistribution on the inelastic buckling of the two span continuous beam shown in Fig. 1 was investigated experimentally. The maximum test loads were compared with the experimental results obtained for simply supported beams of the same properties [6], and it was suggested that test results for statically determinate beams might not be wholly representative of the behaviour of indeterminate beams.

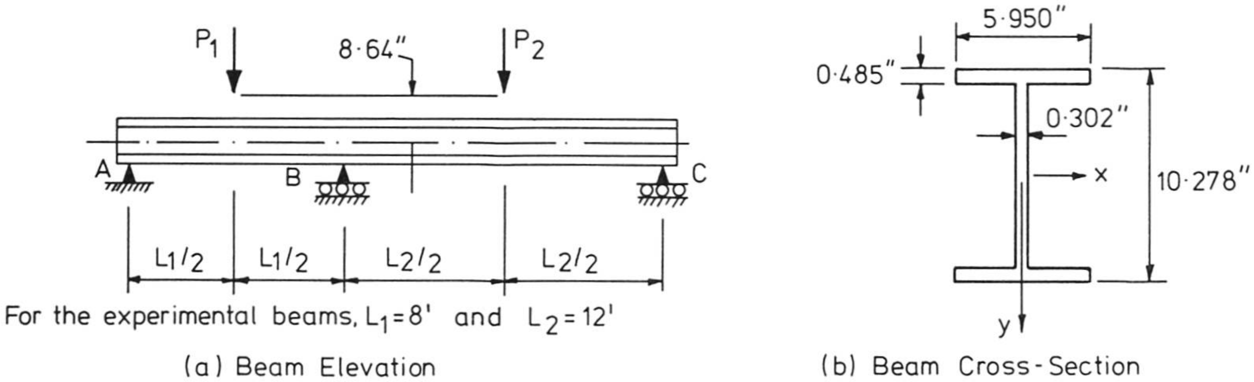


Fig. 1 Geometry of Two Span Continuous Beams
(1 in = 0.0254m, 1 ft = 0.3048m)

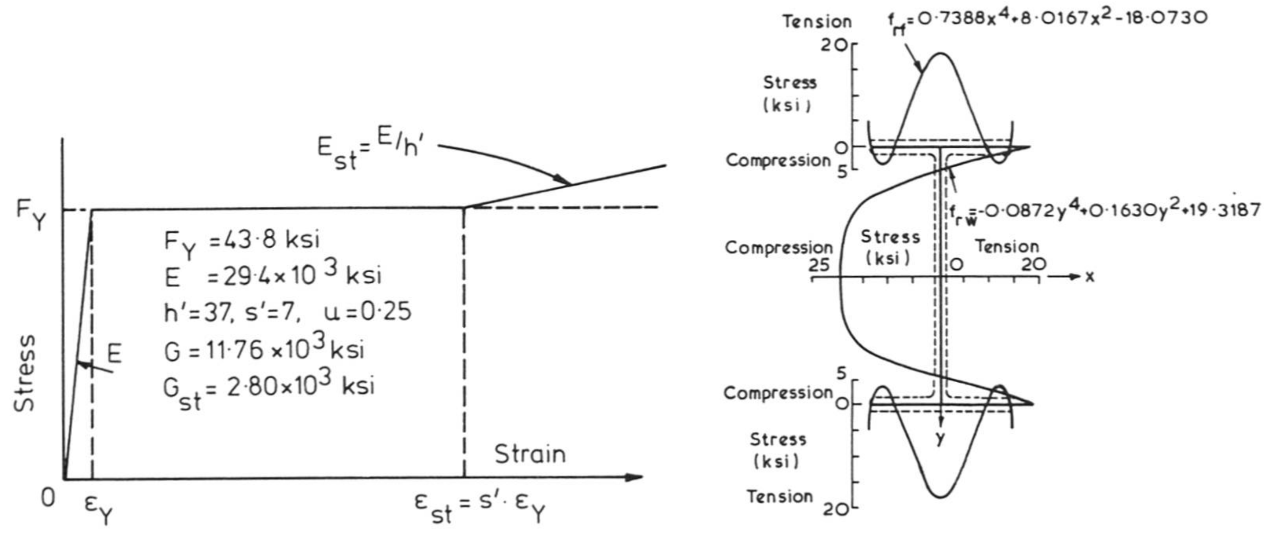
While the continuous beam tests [13] provided much valuable information, the test beams themselves were rather slender, and comparatively little in-plane moment redistribution occurred. The purpose of this paper is to investigate the inelastic buckling of continuous beams more fully, by extending the investigation to more stocky beams in which significant in-plane moment redistribution takes place before failure. This is done theoretically by using both the transfer matrix [18, 19] and the finite element [8, 14] methods of analysis. These methods are first used to predict the inelastic buckling loads of simply supported beams for which theoretical finite integral predictions [5, 6] and experimental test results [6] are available. The theoretical behaviour of the experimental continuous beams [13] is then analysed, the effects of different types of lateral bracing conditions at the internal support are studied, and finally, the buckling loads of stockier continuous beams are predicted.

2. THEORY

2.1 General

The geometry of the two span continuous beams investigated in this paper is shown in Fig. 1, and the material properties and residual stresses are shown in Fig. 2. The section properties which control inelastic lateral buckling are reduced by the spread of yielding through the section caused by the combined effects of the residual stresses and the in-plane bending moment.

The determination of these reductions has been discussed elsewhere [5, 6, 17].



(a) Material Properties (b) Residual Stresses

Fig. 2 Material Properties and Residual Stresses
(1ksi = 6.90MPa, 1 in = 0.0254m)

However, before this information can be used in a lateral buckling analysis, the distribution of the in-plane bending moment along the inelastic indeterminate beam must be determined. This can be done independently of the lateral buckling analysis by finite integrals [13] or by numerical integration [19]. Alternatively, this in-plane analysis can be done in conjunction with the lateral buckling analysis, since both are required at each different load level investigated for the inelastic indeterminate beam. A finite element method of doing this is discussed in a later sub-section.

Once the inelastic in-plane bending moment distribution has been calculated, the variations of the section properties along the beam can be determined, and the beam can then be analysed to find whether it is laterally stable or not. This inelastic lateral buckling analysis may be made in a number of different ways. In References 5 and 6, the finite integral method [4] was used for simply supported inelastic beams, and good agreement was obtained between the predicted inelastic buckling loads and the experimental maximum loads. However, this method has not yet been extended to inelastic indeterminate beams, and so the transfer matrix [19] and the finite element [8] methods were used to analyse the beams studied for this paper. For both of these methods, the continuous variations of the inelastic section properties along the beam length were approximated by values which remained constant within each small segment along the length of the beam, but which changed from one segment to the next. Thus the transfer matrix or finite element stiffness matrix for each segment was developed for these constant properties. The errors introduced by this assumption were reduced by decreasing the segment size. Details of the transfer matrix and finite element buckling analyses are given in the following sub-sections.

2.2 Transfer Matrix Method

In the transfer matrix method of analysing lateral buckling [19], the problem is treated as an initial value problem, and the solution proceeds from node to node along the beam. Thus the internal out-of-plane forces and deformations at the left and right ends of the beam divided into n segments, which are represented by the state vectors $\{V_n^R\}$ and $\{V_1^L\}$ respectively, are related by a linear equation of the form

$$\{V_n^R\} = \left[F_n\right] \left[T_{n-1}\right] \left[F_{n-1}\right] \ldots\ldots\ldots\ldots \left[F_2\right] \left[T_1\right] \left[F_1\right] \{V_1^L\} \tag{1}$$

In this equation $\left[F_n\right]$ is the field transfer matrix which relates the state vectors $\{V_n^R\}$ and $\{V_n^L\}$ at both ends of the beam segment n and $\left[T_{n-1}\right]$ is the point transfer matrix which relates the state vectors $\{V_{n-1}^R\}$ and $\{V_n^L\}$ for both sides of the node i. If the conditions at the left hand end of the beam are unknown so that $\{V_1^L\} = \left[R\right] \{A^L\}$, where $\left[R\right]$ is the left boundary matrix and $\{A^L\}$ is the vector of unknown variables, then the conditions at the right hand end of a beam of n segments can be expressed in the form $\left[R'\right] \{V_n^R\} = \{0\}$ in which $\left[R'\right]$ is the right boundary matrix. Substituting these conditions into Equation 1 leads to

$$\left[R'\right] \left[F_n\right] \left[T_{n-1}\right] \left[F_{n-1}\right] \ldots\ldots\ldots\ldots \left[F_2\right] \left[T_1\right] \left[F_1\right] \left[R\right] \{A^L\} = \{0\} \tag{2}$$

When restraints act at a nodal point and prevent some out-of-plane deformations, then some of the out-of-plane forces in the vector $\{A^L\}$ must be replaced by the reactions caused by the restraints [19].

The field and point transfer matrices in Eq. 2 are functions of the in-plane bending moment pattern and the section properties. In the inelastic range the section properties are reduced by yielding and vary with the bending moment. These properties are assumed to be constant along each segment, and are evaluated numerically by using the moment at the mid-point of the segment. In this way the coefficients in the matrices of Eq. 2 can be evaluated for any given load factor λ.

When the partially yielded beam buckles so that the vector $\{A^L\}$ is non-zero, then Eq. 2 can only be satisfied when the value of λ is such that its determinant is zero, i.e.

$$\left| \left[R'\right] \left[F_n\right] \left[T_{n-1}\right] \left[F_{n-1}\right] \ldots\ldots\ldots\ldots \left[F_2\right] \left[T_1\right] \left[F_1\right] \left[R\right] \right| = 0 \tag{3}$$

In the inelastic range, a trial and error approach must be used in which the left hand side of Eq. 3 is evaluated for a series of different values of λ.

2.3 Finite Element Method

The finite element method of solving lateral buckling problems [8, 14] uses the concept that the lateral stiffness of the structure vanishes at buckling, which can be expressed in determinental form by

$$\left| \left[K_{OP}\right] - \lambda_c \left[K_G\right] \right| = 0 \tag{4}$$

in which $\left[K_{OP}\right]$ is the matrix which respresents the lateral resistance of the structure, and $\left[K_G\right]$ is the matrix which, when multiplied by the load factor λ, represents the destabilising effects of the applied in-plane loads. The critical load at which the structure buckles is defined by the lowest load factor λ_c which satisfies Eq. 4.

In the inelastic buckling of steel beams, the matrix $[K_{OP}]$ depends on the load factor λ, since it contains terms which depend on the reductions in the section properties caused by yielding. Because of this, a trial and error approach is required, in which a trial value of λ is assumed so that $[K_{OP}]$ can be evaluated, and the determinant in Eq. 4 is calculated. This will not normally be zero, and so a second trial value of λ must be selected and the determinant recalculated. An iterative procedure is then used to find the value of λ_c which satisfies Eq. 4. This method has been used for a wide range of statically determinate inelastic beams [7-11].

In statically indeterminate inelastic beams, the in-plane bending moment distribution caused by a particular load factor λ cannot be determined by statics alone, and so a further analysis must be made before the reduced matrix $[K_{OP}]$ can be determined. This may be done by an incremental in-plane analysis which solves iteratively the stiffness equation.

$$\Delta\lambda \{P\} = [K_{IP}] \{\Delta\delta\} \tag{5}$$

In this equation, $\Delta\lambda$ defines the increment in the load set $\{P\}$, $\{\Delta\delta\}$ is the incremental set of in-plane deflections, and $[K_{IP}]$ is the current in-plane stiffness matrix which contains terms which depend on the magnitude of the load factor λ. The control of this iterative solution is based on a comparison of the moment distribution used to form the in-plane stiffness matrix $[K_{IP}]$ with that determined from the solutions for the deflection increments $\{\Delta\delta\}$.

Because of the trial and error process used to determine the lateral buckling load factor λ_c, it is convenient to check for stability at the end of each iterative in-plane analysis. Thus at each increment $\Delta\lambda$, the in-plane moment distribution is determined iteratively, and then the determinant in Eq. 4 is evaluated. The value of the critical load factor is approached from below by making successive increases $\Delta\lambda$, the magnitudes of which may be decreased as the determinant of Eq. 4 approaches zero.

3. COMPARISON WITH EXPERIMENT

3.1 Simply Supported Beams

Theoretical solutions for the elastic and inelastic critical moments M_E and M_c of simply supported single span steel beams [6] have been obtained by each of the analytical methods, and these solutions are given in Table 1. The cross-section and the material properties of these beams, including the residual stresses, are the same as those given in Fig. 1 and 2, while a full discussion of the effects of progressive yielding on the rigidities which control lateral buckling has been given in Ref. 6.

The theoretical solutions in Table 1 are in close agreement. The maximum test moments M_m (see Ref. 6) are also given in Table 1, and these are generally somewhat less than the corresponding theoretical inelastic critical moments M_c. This is caused by the small residual geometrical imperfections which could not be eliminated from the test beams, and which are not accounted for by the buckling theories. It may be concluded that each analytical method is satisfactory, as it has led to predictions which are in close agreement with those of the other theories and in reasonable agreement with the test results.

Quantity	Units	Values		
Span L	ft.	8	10	12
M_Y	kip in.	1353	1353	1353
M_P	kip in.	1576	1576	1576
M_m	kip in.	1267	1248	1174
Finite Integral M_E	kip in.	2161	1596	1278
Transfer Matrix M_E	kip in.	2075	1515	1203
Finite Element M_E	kip in.	2169	1616	1320
Finite Integral M_c	kip in.	1366	1311	1252
Transfer Matrix M_c	kip in.	1344	1317	1170
Finite Element M_c	kip in.	1367	1350	1270

Table 1 Comparison of Results for Simply Supported Beams
(1 kip in. = 0.113 kNm, 1 ft. = 0.3048m)

3.2 Continuous Beams

The transfer matrix method and the finite element method have also been used to obtain the theoretical elastic and inelastic buckling loads P_E and P_c for the two span continuous beams shown in Fig. 1 (see Ref. 13), and these are compared in Table 2 with the maximum test loads. The elastic buckling loads P_E predicted by the two theoretical methods are in close agreement with those obtained previously by the finite integral method [13].

Quantity	Units	Values							
Load Ratio P_1/P_2	-	146	4.95	2.47	1.56	1.54	1.00	0.60	0.01
P_{1m}	kip	58.4	61.4	65.9	61.4	59.4	44.4	25.4	0.4
P_{2m}	kip	0.4	12.4	26.7	39.4	38.6	44.4	42.4	39.4
Finite Integral P_{1E}	kip	121.3	119.1	105.0	76.7	75.8	50.4	30.4	0.5
Finite Integral P_{2E}	kip	0.8	24.1	42.5	49.2	49.3	50.4	50.6	50.4
Transfer Matrix P_{1E}	kip	122.2	120.3	106.6	77.7	76.9	51.2	30.7	0.5
Transfer Matrix P_{2E}	kip	0.8	24.3	43.2	49.9	50.0	51.2	51.3	50.9
Finite Element P_{1E}	kip	123.0	120.0	107.3	78.5	77.4	51.6	31.1	0.5
Finite Element P_{2E}	kip	0.8	24.2	43.5	50.2	50.3	51.6	51.8	51.5
Transfer Matrix P_{1c}	kip	67.7	73.3	79.7	70.0	69.2	46.3	27.8	0.5
Transfer Matrix P_{2c}	kip	0.5	14.8	32.3	44.9	45.0	46.3	46.5	46.0
Finite Element P_{1c}	kip	70.1	73.7	80.1	75.9	75.1	49.9	29.8	0.5
Finite Element P_{2c}	kip	0.5	14.9	32.4	48.7	48.8	49.9	49.5	47.2

Table 2 Comparison of Results for Continuous Beams
(1 kip = 4.45 kN)

However, the predicted inelastic buckling loads P_c are noticeably and consistently higher than the corresponding experimental results. The test beams were comparatively slender and the maximum moments at failure were only 60 per cent approximately of the fully plastic moment M_P. These comparatively low moments cause little yielding, and the rigidities controlling lateral buckling are very close to their elastic values (see Fig. 9 of Ref. 6). The limiting elastic buckling loads P_E are substantially higher than the maximum test loads P_m, and it is therefore surprising that the maximum test loads were so low and not closer to the predicted inelastic buckling loads P_c. It seems unlikely that these discrepancies are due to a breakdown of either theoretical method of analysis, since these have given predictions which are in close agreement with each other, while their use for the simply supported beams has led to satisfactory agreement between theory and experiment. It is, perhaps, possible that both theories do not account for some facet of continuous beam behaviour which does not occur in simply supported beams. Further and more detailed experimental investigations need to be made before these discrepancies can be resolved.

4. EFFECT OF BRACING CONDITIONS AT THE INTERNAL SUPPORT

The transfer matrix method has been used to investigate the effects of changes in the conditions of lateral restraint at the internal support on the elastic and inelastic buckling behaviour of the two span continuous beams shown in Fig, 1. Five different conditions were used; completely free, lateral deflection prevented $u_B = 0$, twist prevented $\phi_B = 0$, lateral deflection and twist prevented $u_B = \phi_B = 0$, and completely fixed $u_B = \phi_B = u_B' = \phi_B' = 0$. The elastic buckling loads were determined for an initially stress free beam. The complete set of results is presented in Figs. 3 and 4.

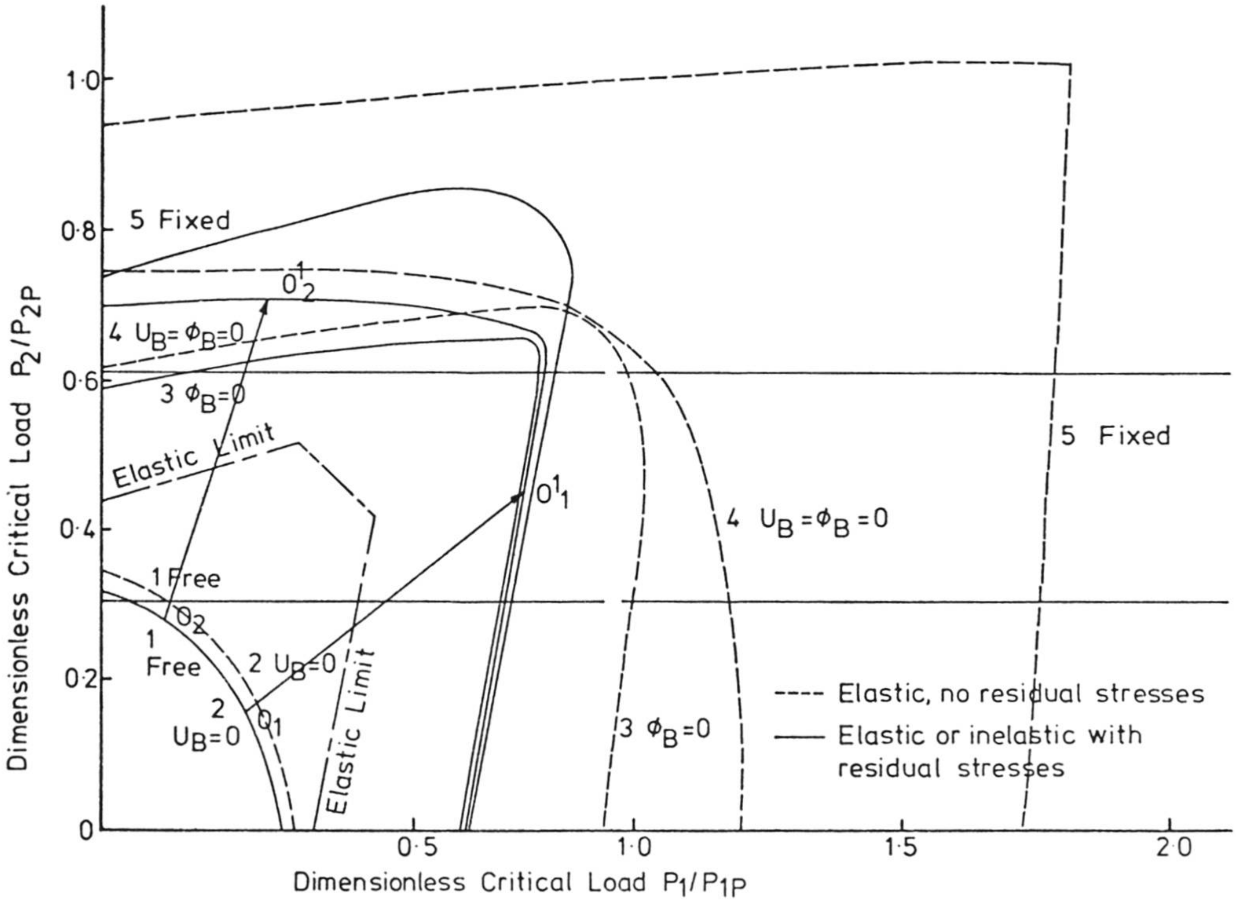


Fig. 3 Effect of Lateral Bracing Conditions at the Internal Support on the Buckling of Two Span Continuous Beams.

The predicted elastic and inelastic critical loads P_{1c}, P_{2c} are shown in the non-dimensional interaction diagram of Fig. 3 by the dashed and solid curves respectively. Also shown is the curve corresponding to the attainment of first yield in a beam with residual stresses. The results show that prevention of lateral deflection alone produces almost no increase in the resistance to lateral buckling, the critical loads being virtually identical to those for the case where no lateral restraint is provided at the internal support. Since buckling is always elastic in these two cases, the differences between the solid and dashed curves are due to the effects of residual stresses on elastic buckling. The increases in the buckling resistance caused by the three other bracing conditions are shown by the positions of the respective pairs of curves in Fig. 3. These increases are greatest for elastic buckling, and the increased elastic buckling loads exceed the plastic collapse loads in several cases. The effects of the changes in the bracing conditions on inelastic buckling are less than for elastic buckling, particularly when the shorter span is the less heavily loaded.

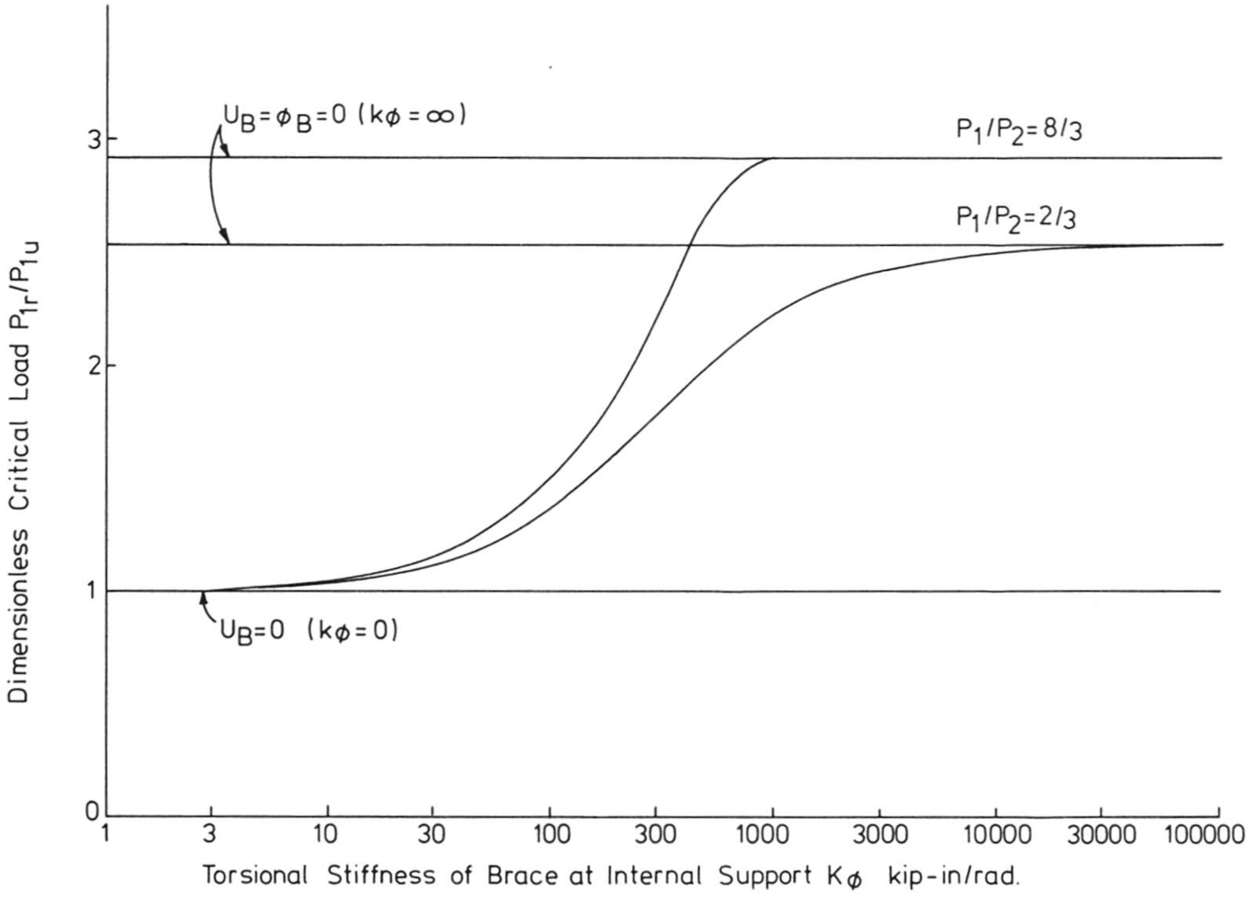


Fig. 4 Effect of Torsional Stiffness of Internal Brace on Buckling of Two Span Continuous Beam (1 kip - in/rad. = 0.113 kNm/rad.)

The relationship between the torsional stiffness k_ϕ of the bracing at the internal support and the corresponding inelastic critical load P_{1r} (non-dimensionalised by dividing by the critical load P_{1u} for no torsional restraint) is shown in Fig. 4. The two values of the load ratio P_1/P_2 of 8/3 and 2/3 have been considered, and these load ratios correspond to the lines O_1O_1' and O_2O_2' in Fig. 3. The effect of torsional restraint is generally greater when the shorter span is the more heavily loaded.

5. EFFECT OF BEAM SLENDERNESS

The finite element method has been used to investigate the effects of beam slenderness on the inelastic buckling behaviour of the two span continuous beams shown in Fig. 1. For this investigation, the value of the span ratio L_2/L_1 was held constant at 1.5 while the span length L_1 was decreased from 8.0 ft. (2.44m) to 6.0 ft, 4.8 ft, 4.0 ft, and 3.2 ft (1.83m, 1.46m, 1.22m, and 0.98 m). Nine different values of the load ratio P_1/P_2 which vary between 0 and ∞ were considered. The results of this investigation are shown in Fig. 5, 6 and 7.

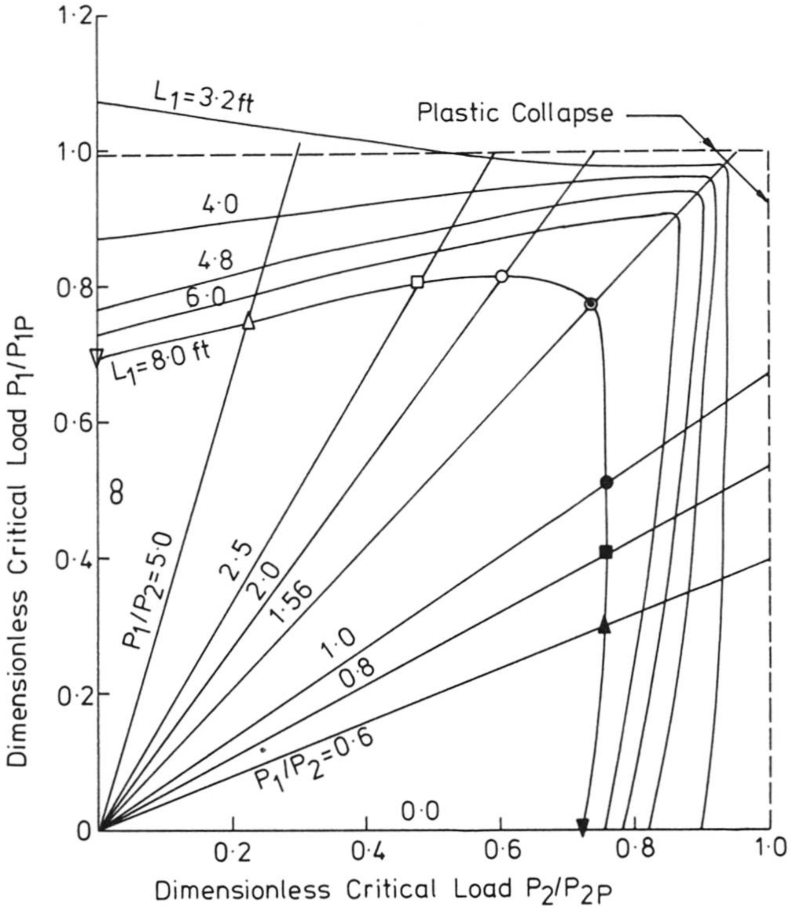


Fig. 5 Buckling Loads of Two Span Continuous Beams
(1 ft = 0.3048m)

The predicted inelastic critical loads P_{1c}, P_{2c} are shown in the non-dimensional interaction diagram of Fig. 5. It can be seen that as the beam slenderness decreases, the inelastic buckling curve generally approaches the fully plastic collapse boundaries. The exception to this takes place at high values of P_1/P_2, for which significant strain-hardening occurs in the yielded region at the centre of the span L_1. This causes the plastic collapse load P_{1P} for this span to be exceeded for very stocky beams. Significant strain-hardening also occurs for low values of P_1/P_2, and there is some indication of the consequences of this in Fig. 5. Extensive strain-hardening under these high and low load ratios P_1/P_2 is associated with the redistribution of the in-plane bending moments which takes place before inelastic buckling occurs. This redistribution is favourable with respect to the resistance to lateral buckling, as the increased yielding which it causes occurs at the interior support, where the consequent decreases in the beam rigidities have comparatively small effects on lateral buckling. Thus the resistance to lateral buckling tends to increase more rapidly with decreasing slenderness than it does for intermediate values of P_1/P_2 for which there is little in-plane moment redistribution. The degree of in-plane moment redistribution is more clearly shown in the upper diagram of Fig. 6 by the departure from linearity of the lines of constant load ratio P_1/P_2. These are especially marked for the extreme cases of $P_1/P_2 = 0$ and ∞.

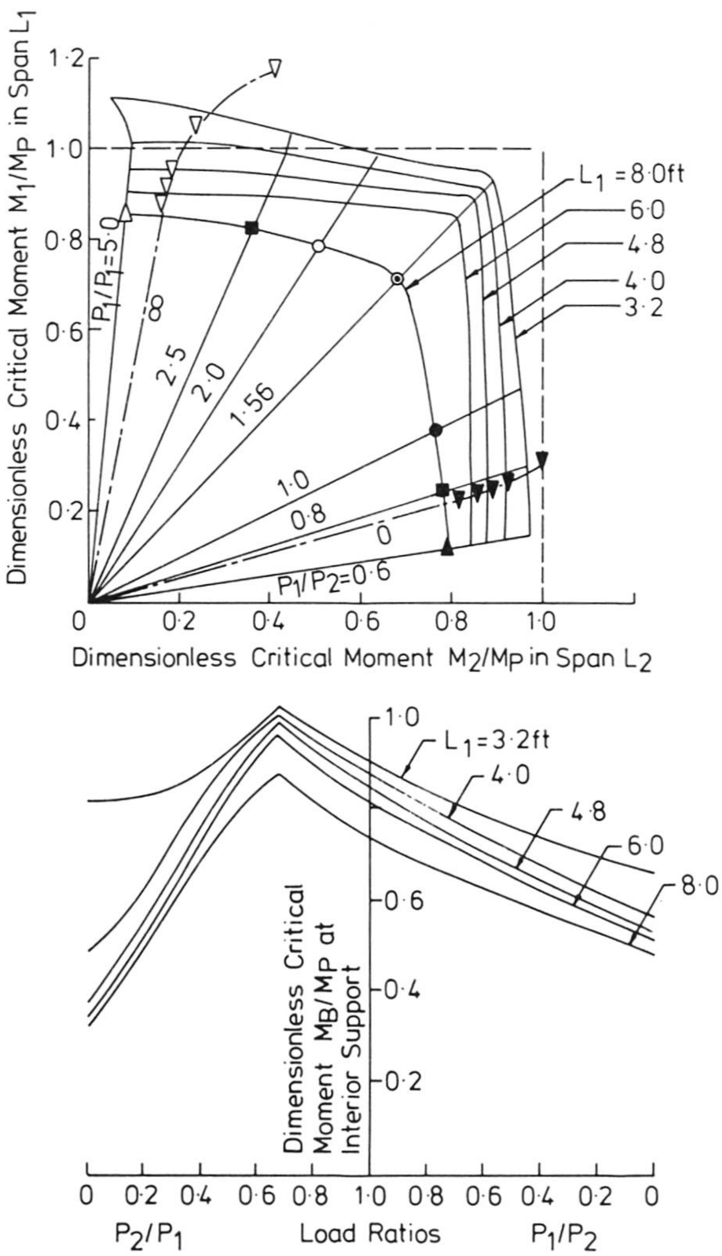


Fig. 6 Moments at Buckling in Two Span Continuous Beams
(1 ft = 0.3048 m)

The variations of the dimensionless critical load P_c/P_Y with a modified slenderness parameter $\sqrt{P_Y/P_E}$ are shown in Fig. 7. All of the plotted points, with the exception of those for $P_1/P_2 = 1.56$, are very closely grouped. Moreover, for modified slendernesses in excess of 0.6 approximately, this group is quite close to the curve for simply supported single span beams with central concentrated loads. This is because the resistance to lateral buckling of all of these beams is dominated by the effects of yielding near the load points. However, with decreasing slenderness, the plotted points for the continuous beams rise more quickly than the curve for the single span beams because of the more extensive strain-hardening which leads eventually to favourable in-plane moment redistribution in the continuous beams.

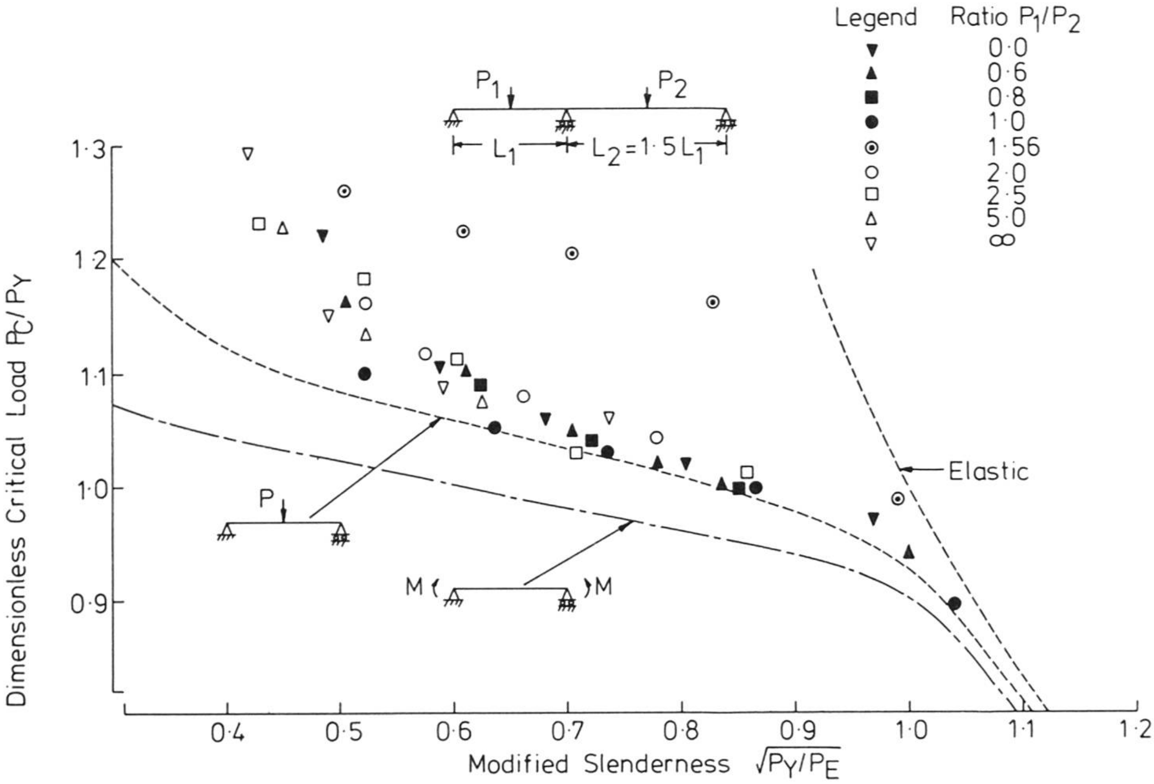


Fig. 7 Effect of Beam Slenderness on Buckling of Two Span Continuous Beams

The predicted values of P_c/P_Y for $P_1/P_2 = 1.56$ are, however, significantly higher than those for other load ratios, as can be seen in Fig. 7. In this particular case, the point of maximum moment is located at the interior support instead of under one of the loads, and so the most extensive yielding occurs at a point of lateral support, instead of at the centre of an unbraced span. Previous investigations [10] of single span beams have shown that yielding at braced points has comparatively little effect on the resistance to lateral buckling, and that the inelastic buckling load does not fall significantly below the elastic buckling load until after substantial reductions have occurred in the section rigidities at the braced points. A similar effect occurs in the continuous beams with $P_1/P_2 = 1.56$, and the inelastic buckling loads are therefore much closer to the elastic buckling loads.

6. CONCLUSIONS

The application of both the transfer matrix method and the finite element method to problems of the inelastic lateral buckling of beams have been discussed. For statically determinate beams both methods have been shown to yield results that are in good agreement with a previous series of tests on as-rolled steel I-beams. The extension of both methods to deal with the inelastic buckling of statically indeterminate beams has been described and results have been presented for a series of two span continuous beams. These theoretical results are consistently higher than the experimental test results. The reason for this is not clear, and it is concluded that further experimental investigations should be made.

The transfer matrix method has been used to investigate the effects of variations in the conditions of lateral restraint at the internal support on the elastic and inelastic buckling behaviour of two span continuous beams. Prevention of twist was found to be the most effective method of bracing, as prevention of lateral deflection alone produced very little increase in the resistance to lateral buckling. For inelastic buckling, the more highly restrained conditions such as complete lateral fixity produced only comparatively small increases in resistance.

The finite element method has been used to investigate the effects of decreased slenderness on the inelastic buckling of two span continuous beams. The results obtained show that until the beams are sufficiently stocky for the effects of strain-hardening and the eventual redistribution of the in-plane moments to become important, their inelastic buckling loads are close to those for single span beams with similar patterns of in-plane moment.

ACKNOWLEDGMENT

This paper was prepared in the Department of Civil and Structural Engineering of the University of Sheffield whilst N. S. Trahair was there on study leave.

NOTATION

$\{A\}$	Vector of unknown variables required in transfer matrix method
E	Young's modulus
E_{st}	Strain-hardening modulus
f_{rf}	Residual stress in flange
f_{rw}	Residual stress in web
$[F_n]$	Field transfer matrix for beam segment n
F_Y	Static yield stress of the material
G	Shear modulus of elasticity
G_{st}	Shear modulus for strain-hardened material
h'	Ratio of Young's modulus to strain-hardening modulus = E/E_{st}
i	Node number
k	Lateral stiffness of restraint at internal support
k_ϕ	Torsional stiffness of restraint at internal support
$[K_G]$	Geometric stiffness matrix accounting for destabilising effects
$[K_{IP}]$	Stiffness matrix for beam's in-plane resistance
$[K_{OP}]$	Stiffness matrix for beam's out-of-plane resistance
L_1, L_2	Spans of two span continuous beam
M_c	Inelastic critical moment
M_E	Elastic critical moment
M_m	Maximum test moment
M_P	Fully plastic moment
M_Y	Moment at nominal first yield
n	Segment number
$\{P\}$	Vector of in-plane beam loads
P_1, P_2	Beam loads
P_{1c}, P_{2c}	Inelastic buckling loads
P_{1E}, P_{2E}	Elastic buckling loads
P_{1P}, P_{2P}	Plastic collapse loads
P_{1r}	Buckling load with torsional restraint at internal support
P_{1u}	Buckling load with no torsional restraint at internal support
P_Y	Nominal first yield load
$[R], [R']$	Boundary matrices

s'	Ratio of strain-hardening strain to the yield strain $= \varepsilon_{st}/\varepsilon_Y$	x, y	Major and minor principal axis
$\left[T_n\right]$	Point transfer matrix for beam segment n	$\{\delta\}$	Vector of in-plane beam displacements
u_B	Lateral deflection at internal support	ε_{st}	Strain at onset of strain hardening
$\left[V_n^L\right]$, $\left[V_n^R\right]$	State vectors, at left and right ends of beam segment n	ε_Y	Yield strain $= F_Y/E$
		λ	Load factor
		λ_c	Load factor corresponding to critical buckling load
		ϕ_B	Twist at internal support
		μ	Poisson's ratio

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