

# Finite strip analysis of continuous folded plates

Autor(en): **Delcourt, C. / Cheung, Y.K.**

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## Finite Strip Analysis of Continuous Folded Plates

Analyse par bandes finies de toits plissés continus

Endliche Streifenelemente für die Berechnung von durchlaufenden Faltenwerken

**C. DELCOURT**

Ingénieur Physicien  
Dep. of Civ. Eng., University of Adelaide  
Adelaide, Australia

**Y. K. CHEUNG**

Professor, Ph. D., D. Sc., FICE, FIStructE.  
Dep. of Civ. Eng., University of Hong Kong  
Hong Kong

### SUMMARY

The finite strip method is generalized to study folded plates continuous over any number of spans and submitted to uniformly distributed loads or line loads. The end supports are either clamped, simply supported or free. The results obtained agree well with experimental and theoretical values available in the literature and a comparison of computational efforts required between finite element and finite strip methods demonstrates that the latter method has a very definite advantage over the former.

### RÉSUMÉ

La méthode par bandes finies est généralisée pour permettre l'étude de toits plissés continus, d'un nombre quelconque de travées, soumis à des charges uniformes ou linéaires. Les travées extrêmes sont soit encastées, soit simplement appuyées, soit libres. Les résultats concordent bien avec les valeurs théoriques et expérimentales données dans la littérature. La comparaison des efforts livrés par l'ordinateur dans les méthodes des éléments finis et des bandes finies montre que cette dernière est nettement plus avantageuse que la précédente.

### ZUSAMMENFASSUNG

Auf der Basis endlicher Streifenelemente wird eine Methode entwickelt für die Berechnung durchlaufender Faltenwerke beliebiger Felderzahl unter verteilten Lasten oder Linienlasten. Die Enden des Faltenwerks können eingespannt, frei drehbar gelagert oder frei sein. Die mit dieser Methode berechneten Ergebnisse stimmen gut mit Versuchsergebnissen und theoretischen Lösungen anderer Autoren überein. Vom Berechnungsaufwand her gesehen hat die vorgelegte Methode gegenüber Methoden endlicher Elemente eindeutige Vorteile.



## Introduction

Traditionally, the analysis of folded plate structures are based either on the "ordinary theory" or on the "elasticity theory" and most of the literature has been concerned with the analysis of single span structures. [1] More recently attention has been focused on continuous folded plates. Beaufait [2] used the ordinary theory and developed a computer solution for folded plate with arbitrary end conditions, although some of the assumptions appear to be questionable [3]. Pultar et al [4] presented a solution based on elasticity theory for continuous folded plates with simply supported ends, in which a force method is applied subsequently to the standard analysis in order to determine the redundant reactive forces at the intermediate supports.

The technique was also used by Scordelis et al [5] for the analysis of box girder structures with internal rigid diaphragms. Continuous folded plate structures were also analysed by Lee [6], using a finite difference technique.

The most versatile tool of analysis is obviously the finite element method. Rockey and Evans [7] were the first to study the behaviour of folded plate structures by using a rectangular element [8], and Lo and Scordelis [9] developed a finite segment method (which is a special type of finite element method based on ordinary theory) in an attempt to reduce the excessive number of degrees of freedom involved in a problem.

Recently, the finite strip method pioneered by Cheung [10] was applied to rectangular [11], curved [12,13] and skew [14] folded plate structures, all with simply supported ends. The object of the present paper is to extend the finite strip method to the analysis of continuous folded plate structures with arbitrary end conditions, but without resorting to the standard procedure of carrying out a subsequent flexibility analysis.

## Finite Strip Approach

It is now well known that a finite strip is a special finite element for which the boundary conditions of the structure, in the longitudinal direction, are *a priori* included in the approximation of the displacement field. In the present approach, such boundary conditions may include simply supported, clamped, or free edge conditions. However, as far as the loading is concerned the study is restricted to distributed loads or longitudinal line loads which are the dominant types of loadings for folded plate structures.

## Stiffness matrix and force vector

The procedure of formulating the stiffness matrix and the force vector for a flat shell strip has been already given elsewhere [10] and will not be repeated here. However, the approximation used for the displacement field will be given below. For a flat shell strip bounded by sides *i* and *j* (Fig. 1) and continuous over several spans the displacement field is:

$$u = \sum_{n=1}^N \left[ \left(1 - \frac{x}{b}\right) u_{in} + \left(\frac{x}{b}\right) u_{jn} \right] \sum_{s=1}^S Y_{sn}$$

$$v = \sum_{n=1}^N \left[ \left(1 - \frac{x}{b}\right) v_{in} + \left(\frac{x}{b}\right) v_{jn} \right] \sum_{s=1}^S Y'_{sn}$$

$$w = \sum_{n=1}^N \left[ \left(1 - 3\frac{x^2}{b^2} + 2\frac{x^3}{b^3}\right) w_{in} + \left(x - 2\frac{x^2}{b} + \frac{x^3}{b^2}\right) \theta_{in} + \left(3\frac{x^2}{b^2} - 2\frac{x^3}{b^3}\right) w_{jn} + \left(\frac{x^3}{b^2} - \frac{x^2}{b}\right) \theta_{jn} \right] \sum_{s=1}^S Y_{sn}$$

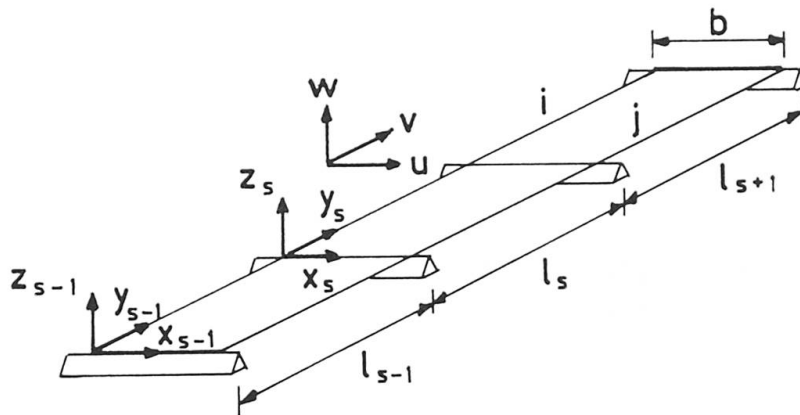


Fig. 1 A Typical continuous finite strip

where  $S$  and  $N$  are respectively the number of spans and the number of terms used in the series,  $Y'$  denotes the first derivative of  $Y$  with respect to  $y$ ;  $x$ ,  $y$ ,  $u$ ,  $v$ ,  $w$ ,  $\theta$  are the coordinates and displacements indicated in Fig. 1, and  $b$  is the width of the strip. By definition, the boundary conditions in the longitudinal direction must *a priori* be satisfied by the functions  $Y_{sn}$ , which are described hereafter. It should be noted that, in the present analysis, a simple end support or an intermediate support is a diaphragm which is infinitely stiff in its own plane but perfectly flexible normal to its plane.

#### Basic Functions for Multispan Finite Strip

The basic functions  $Y_{sn}$  are the eigenfunctions of a corresponding continuous beam which has the same characteristics as the structure to be considered in terms of end conditions, number and length of spans, relative rigidities, etc. The complete computation of these eigenfunctions can be found in ref. 15 and is not repeated herein in detail. However a resume of the procedure is given in the following.

The continuous beam eigenfunctions are computed by using a general stiffness approach. For a typical  $s^{\text{th}}$  span, the differential equation for vibration is given by

$$\frac{d^4 Y_s}{dy^4} - \frac{m_s \omega^2}{E_s I_s} Y_s = 0 \quad \dots\dots (1)$$

assuming harmonic motion as usual. In Eq. (1),  $Y_s$  is the amplitude of the mode with reference to a local set of coordinate system originated at the left end of  $s^{\text{th}}$  span.  $I_s$ ,  $m_s$ ,  $E_s$  are respectively the second moment of the area of the section, the mass per unit length and Young's modulus.



The general solution for this equation is

$$Y_{sn} = A_{sn} \sin G_s \mu_n y_s + B_{sn} \cos G_s \mu_n y_s + C_{sn} \sinh G_s \mu_n y_s + D_{sn} \cosh G_s \mu_n y_s \quad \dots\dots(2)$$

where  $\mu_n = \left[ \frac{m\omega_n^2}{EI} \right]^{1/4}$  and  $G_s = \left[ \frac{m_s}{EI_s} \right]^{1/4} \left[ \frac{m}{EI} \right]^{-1/4}$  denotes the common eigenvalue and the reciprocal of the relative wave propagation of  $s^{\text{th}}$  span. The unknowns,  $\mu_n, A_{sn}, B_{sn}, C_{sn}, D_{sn}$  are determined by expressing the boundary conditions at both end supports and at all intermediate supports.

#### Solution for $\mu_n$

By inserting the coordinates of the two end supports of the  $s^{\text{th}}$  span into Eq. (2), the following equations are derived:

$$\begin{bmatrix} Y_{sn}(0) \\ \theta_{sn}(0) \\ Y_{sn}(l_s) \\ \theta_{sn}(l_s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ -\beta & 0 & -\beta & 0 \\ s_1 & c_1 & s_2 & c_2 \\ -\beta c_1 & +\beta s_1 & -\beta c_2 & -\beta s_2 \end{bmatrix} \begin{bmatrix} A_{sn} \\ B_{sn} \\ C_{sn} \\ D_{sn} \end{bmatrix} \quad \dots\dots(3)$$

The two end moments for this particular span can be written as:

$$\begin{bmatrix} M_{sn}(0) \\ M_{sn}(l_s) \end{bmatrix} = EI_s \beta^2 \begin{bmatrix} 0 & -1 & 0 & 1 \\ s_1 & c_1 & -s_2 & -c_2 \end{bmatrix} \begin{bmatrix} A_{sn} \\ B_{sn} \\ C_{sn} \\ D_{sn} \end{bmatrix} \quad \dots\dots(4)$$

$$\begin{aligned} \text{In eq. (3 and (4)) } \theta_{sn} &= -\frac{dY_{sn}}{dy_s} & s_1 &= \sin \beta l_s & c_1 &= \cos \beta l_s \\ & & s_2 &= \sinh \beta l_s & c_2 &= \cosh \beta l_s \\ M_{sn} &= \frac{d^2 Y_{sn}}{dy_s^2} & \beta &= G_s \mu_n \end{aligned}$$

By inverting Eq. (3) and incorporating the result in Eq. (4), the moments can be written in terms of the deflections and the slopes. As all the intermediate supports are rigid supports, the condition

$$Y_{sn}(0) = Y_{sn}(l_s) = 0 \quad 1 < s < S$$

is used to establish, for each span (except the first and last spans) a relation between moments and slopes only:

$$\begin{bmatrix} M_{sn}(0) \\ M_{sn}(l_s) \end{bmatrix} = \begin{bmatrix} \alpha & \gamma \\ \gamma & \alpha \end{bmatrix} \begin{bmatrix} \theta_{sn}(0) \\ \theta_{sn}(l_s) \end{bmatrix} \quad \dots\dots(5a)$$



in which  $\alpha = \frac{EI}{S_s} \beta(s_1 c_2 - s_2 c_1)/(1 - c_1 c_2)$  .....(5b)

$\gamma = \frac{EI}{S_s} \beta(s_2 - s_1)/(1 - c_1 c_2)$  .....(5c)

By establishing the moment equilibrium (i.e.  $M_{s-1,n}(\ell_{s-1}) + M_{sn}(0) = 0$ ) and compatibility conditions (i.e.  $\theta_{s-1,n}(\ell_{s-1}) = \theta_{sn}(0)$ ) at all intermediate supports through Eq. (5a, b, c) the following set of equations can be obtained.

$$\begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix} = \begin{bmatrix} \gamma(\ell_1) & \alpha(\ell_1) + \alpha(\ell_2) & \gamma(\ell_2) & 0 & \cdot & 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma(\ell_2) & \alpha(\ell_2) + \alpha(\ell_3) & \gamma(\ell_3) & \cdot & 0 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \gamma(\ell_{s-1}) & \cdot & \cdot \\ 0 & 0 & 0 & 0 & 0 & \cdot & \gamma(\ell_{s-1}) & \alpha(\ell_{s-1}) + \alpha(\ell_s) & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta_{1n}(0) \\ \theta_{2n}(0) \\ \theta_{3n}(0) \\ \cdot \\ \cdot \\ \theta_{Sn}(0) \\ \theta_{Sn}(\ell_s) \\ Y_{1n}(0) \\ Y_{Sn}(\ell_s) \end{bmatrix} \dots\dots(6a)$$

The matrix in Eq. (6a) is overdeterminate, involving S-1 equations and S+3 unknowns; the boundary conditions at the end supports should be incorporated to make the matrix square. A non-trivial solution of  $\mu_n$  is obtained by equating the determinant of this matrix to zero. Eq. (6a) can be shortened as:

{o} = [K] {d} .....(6b)

and then the solutions will be given by

det [K] = 0 .....(6c)

[K] being a square matrix, including the boundary conditions, and {d} being the vector of displacements.

The coefficients of this determinental equation are transcendental functions of  $\mu_n$  and can be solved by the Newton Raphson method as suggested in ref.[15]. Unfortunately it was found that at times certain roots might be missing and in order to make sure that all the roots up to the desired number N can be computed, an adaptation of the "modified regula falsi" method [16, 17] is used herein, although it has the disadvantage of being somewhat time-consuming. Attention should be drawn to the fact that for the case of both ends clamped only, Eq. (6c) will miss out the roots corresponding to the situation in which the whole displacement vector {d} is equal to zero, i.e., the continuous structure degenerates into a series of single-span substructures with both ends clamped. The eigenvalues of such beams are of course common knowledge [10] and they can be inserted into the correct positions in the ascending array of  $\mu_n$  obtained from



Eq. (6c). The position of the missing roots can also be determined by using a method proposed by Wittrick and Williams [20], in which a "sign count" of the matrix  $[K]$  is made.

#### Solution for $A_{sn}$ , $B_{sn}$ , $C_{sn}$ , $D_{sn}$

If the ends are either supported or clamped all the supports are rigid supports; this leads to the condition  $Y_{sn}(0) = 0$  which implies that  $B_{sn} = -D_{sn}$ .

The moment equilibrium, the compatibility condition and the zero deflection conditions at each intermediate support provides 3S-3 equations for the remaining 3S constants. The second condition at the left end support (i.e.  $M_{1,n}(0) = 0$  if simply supported or  $\theta_{1n} = 0$  if clamped) provides one more equation, while the zero deflection at the last support gives another equation. Finally if  $A_{1n}$  is taken as a unit reference constant, the set of 3S equation can be solved. The remaining condition at the right end support is automatically satisfied because it has been incorporated once already in the solution of  $\mu_n$ . In the particular case of free ends, the process is exactly the same but 4 unknown must be kept for the first and last spans. The conditions of zero shear force and zero moment at the end will provide the necessary equations.

#### Convergence

In finite strip analysis, it is important to be able to predict the number of terms  $N$  required in the series so that results of reasonable accuracy can be obtained. It has been shown in ref. [11] that for single span folded plate structures, 5 non-zero Fourier series terms are sufficient to give accurate deflections, moments and membrane forces in a finite strip analysis. For continuous structures, the series will not converge as quickly and it is also difficult to fix a value  $N$  which is applicable to all problems involving different number of spans and span lengths.

A simple but approximate method of predicting the value  $N$  is to examine the convergence characteristics of the distributed loads and take  $N$  as the number of terms required for approximating the loads over a continuous beam with the same number of spans and span length ratio. As the functions  $Y_{sn}$  are eigenfunctions obtained by solving Eq. (1), they possess the property of orthogonality

$$\int_0^{l_s} Y_{sm} Y_{sn} dy = 0 \quad \text{for } m \neq n$$

For this reason, any load  $q_s(y)$  can be resolved into the same eigenfunction series as

$$q_s(y) = \sum_N q_{sn} Y_{sn} \quad \dots\dots(7)$$

$$\text{in which } q_{sn} = \frac{\int_0^{\ell_s} q_s(y) Y_{sn} dy}{\int_0^{\ell_s} Y_{sn}^2 dy}$$

and the term by term convergence of the total integrated load can be examined. The above integration is carried out numerically.

An example, the special case of  $q(y) = 80 \text{ kg/m}$  on a two-span beam (either for equal spans or for unequal spans with  $\ell_1/\ell_2 = 0.8$ , which is characteristic of the examples treated later) is examined by using up to eleven terms of the series, while the convergence study for a single span beam (using Fourier series) with the same uniformly distributed loading is also made for comparison. In Fig. 2 (a,b,c), the approximate loads for specified values of  $N$  demonstrate clearly the faster convergence of the Fourier series. In Fig. 2d the percentage error of the total integrated load with respect to  $N$  is worked out for the three beams and it can be seen that for the unequal span continuous beam, the convergence becomes very slow for  $N > 10$ . It is therefore concluded that the small gain in accuracy obtained by increasing  $N$  even further cannot be justified, keeping in mind that the number of degrees of freedom per node is equal to  $4N$ . For the simply supported beam, convergence becomes very slow after five terms, and this is in accordance with the findings given in reference 11.

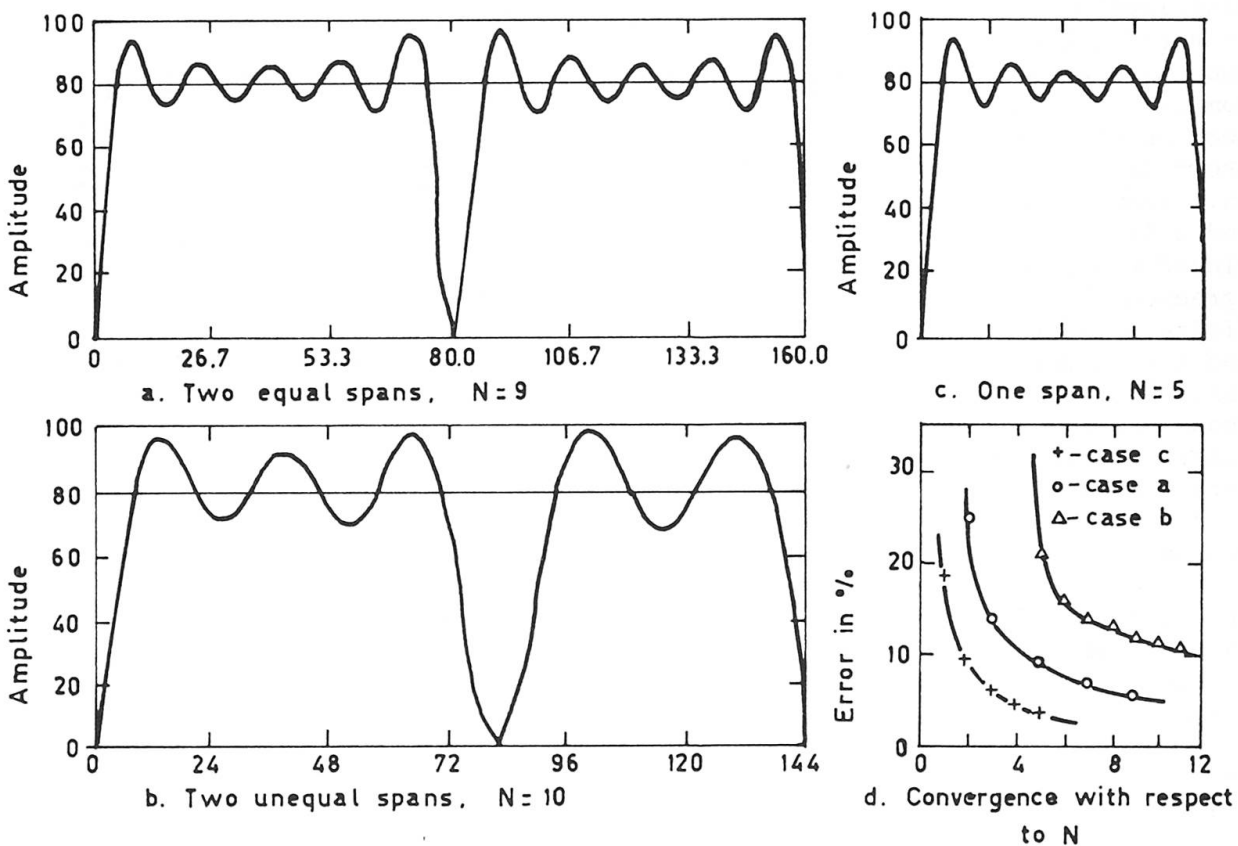


Fig. 2 Approximation of a uniform load of 80 kg/m on three different beams, with the series  $\sum_{N=1}^N Y_{sn}$





### Numerical Examples

The results for two-span continuous structures with simply supported ends and for a single-span structure with overhangs (treated as three-span structure in the finite strip analysis) are presented in this section.

The first example consists of a continuous folded plates structure initially studied by Beaufait [1] using the ordinary theory and subsequently by Scordelis and Lo [3,9], using both the elasticity theory and a finite segment ordinary method. The dimensions and loadings of the structures, which has two unequal spans, are shown in Fig. 3. The modulus of elasticity is taken as  $25.25 \times 10^6 \text{ kN/m}^2$  and Poisson's ratio as zero. The number of terms used in the series will be equal to 10 unless otherwise specified. Two different meshes using 9 and 18 strips for half of the section have been used in the analysis. Little difference can be detected in the results except for transverse bending moment; for this reason, results for the two meshes are only given here for the transverse bending moment.

The longitudinal variations of the vertical deflections, the longitudinal stresses and the transverse bending moments at B and D are listed in Table 1, 2, and 3 respectively, while the transverse distribution of longitudinal stresses and transverse bending moments for mid-section of span 1 is shown in Fig. 4a and 4b. From Tables 1, 2 and 3 it can be concluded that the agreement between the finite strip results and the values from ref.[9] (in particular those due to the elasticity method) is

very good and the only point worth commenting concerns the transverse bending moments, in which a marked discrepancy exists for the case of the ordinary theory because of its one-way slab assumption.

For completeness, the transverse distribution of shear stresses  $\tau_{xy}$  at the left end support and at the intermediate support is shown in Fig. 5. These curves were obtained by extrapolating the values of  $\tau_{xy}$  between the supports. Finally, in order to study the convergence of the results, the values of the vertical deflection at joint A and the longitudinal stress and transverse moment at joint B (mid-section of span 1) are given in Table 4 for  $N = 7, 8, 9$  and 10. It can be observed that for  $N > 9$ , the convergence becomes very slow, as predicted by the test proposed previously.

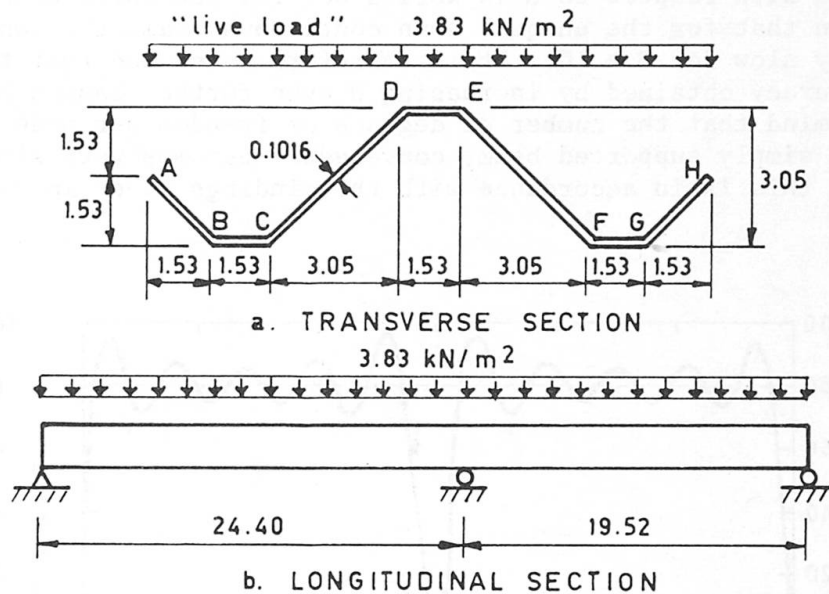


Fig.3 Dimensions (in m) and loading for Example 1



Table 1  
Comparison of vertical deflections, in metres times  $10^4$

x in metres	finite strip method 9 strips		finite segment ordinary theory		elasticity theory	
	B	D	B	D	B	D
3.66	42.40	17.39	47.28	17.69	42.70	18.00
8.54	76.25	31.72	84.49	32.64	77.47	32.64
10.98	80.52	33.55	88.45	34.77	81.74	34.47
13.42	75.34	31.42	82.96	32.94	76.86	32.64
18.30	44.23	18.91	48.50	20.13	45.45	19.83
30.50	15.56	3.36	20.13	3.97	16.47	3.97
35.38	28.37	7.32	35.38	7.63	29.59	7.93
37.82	27.15	7.32	33.55	7.63	27.76	7.93

Table 2  
Comparison of longitudinal stresses ( $\text{kN/m}^2$ )

x in metres	finite strip method 9 strips		finite segment ordinary theory		elasticity theory	
	B	D	B	D	B	D
3.66	1311	-1242	1477	-1201	1311	-1256
10.98	1932	-2001	2056	-2015	1960	-2056
13.42	1691	-1704	1801	-1753	1739	-1787
18.30	476	-400	518	-469	518	-483
23.18	-2360	1932	-2491	1691	-2256	1739
24.40	-3450	2691	-3636	2346	-3436	2843
25.62	-2415	2001	-2539	1801	-2312	1849
30.50	207	69	297	83	255	55
37.82	1256	-897	1470	-842	1263	-918



Table 3  
Comparison of transverse moments (kg-m/m)

x in metres	finite strip method 9 strips		finite strip method 18 strips		finite segment ordinary theory		elasticity theory	
	B	D	B	D	B	D	B	D
1.22	-149	-159	-149	-168	-453	-225	-202	-150
3.66	-334	-183	-362	-195	-453	-178	-336	-195
10.98	-378	-154	-396	-163	-453	-128	-397	-169
18.30	-381	-199	-399	-208	-453	-190	-398	-205
23.18	- 63	- 72	- 63	- 91	-453	-251	-132	-105
25.62	- 59	- 68	- 59	- 91	-453	-245	-130	-101
30.50	-381	-190	-394	-199	-453	-168	-391	-187
37.82	-358	-172	-376	-181	-453	-135	-372	-171
42.70	-104	-113	-104	-127	-453	-221	-201	-146

Table 4  
Influence of "n" on the deflections,  
stresses and moments (example 1)

value at midspan 1 \ n	7	8	9	10
w (metres) at A	$149 \times 10^{-4}$	$151 \times 10^{-4}$	$153 \times 10^{-4}$	$153 \times 10^{-4}$
$\sigma_x$ (kN/m <sup>2</sup> ) at B	30.37	37.60	46.46	46.27
$\sigma_y$ (kN/m <sup>2</sup> ) at B	1787	1811	1839	1839
$M_x$ (kg-m/m) at B	-339	-361	-390	-390
$M_y$ (kg-m/m) at B	19.88	20.43	21.47	21.52

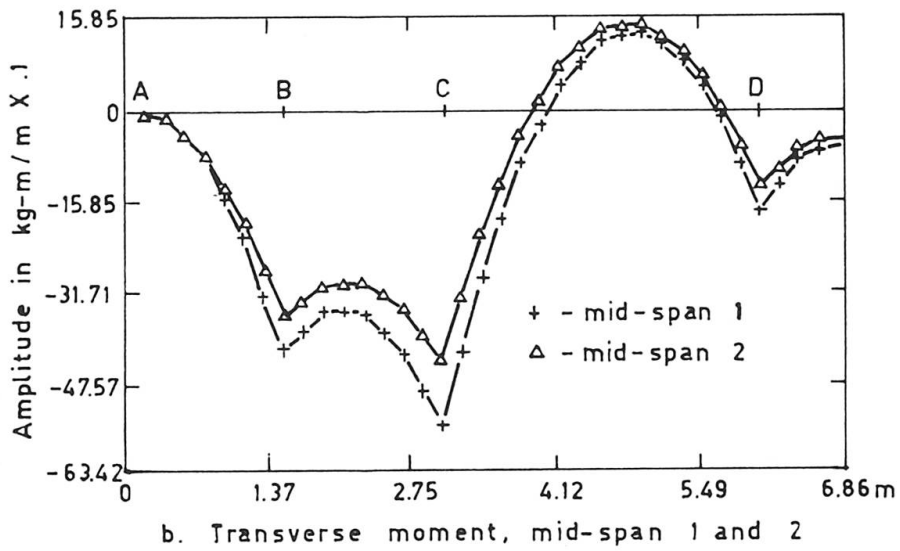
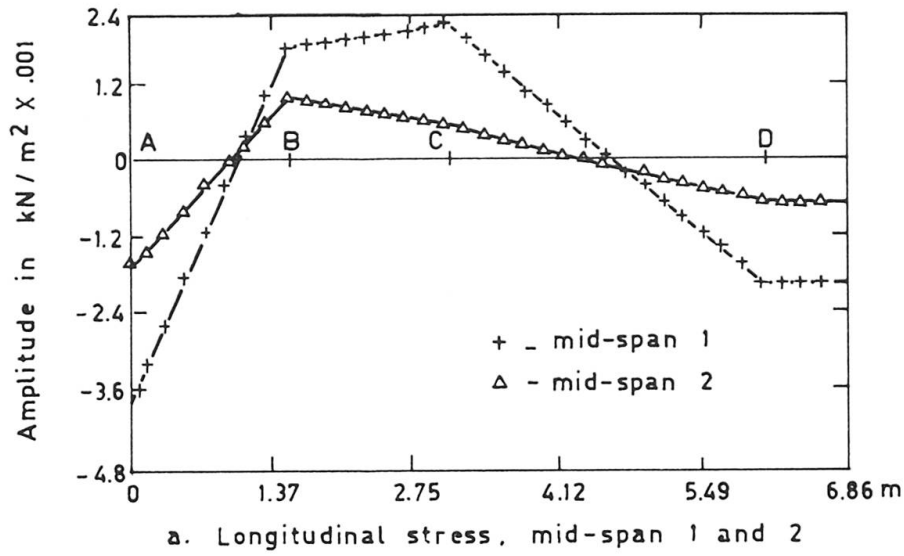


Fig.4 Example 1. Transverse distribution of longitudinal stress and transverse moment

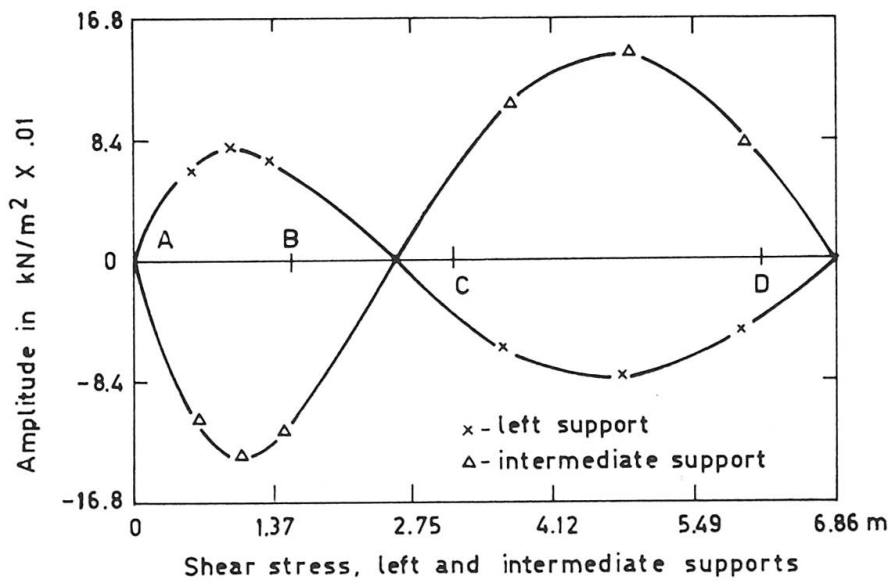


Fig.5 Example 1. Transverse distribution of shear stress



In order to assess the efficiency of the finite strip analysis, two finite element solutions using  $9 \times 9$  and  $9 \times 18$  meshes of flat shell elements [19] for half of the structure are carried out, and a comparison of number of degrees of freedom and CPU time used are presented in Table 5. It should be mentioned that while the deflections differ by only 1% (maximum) between the three sets of results, the difference in the stresses and moments amounts to 15% (maximum) between the coarse mesh ( $9 \times 9$ ) and finite strip results, and 5% (maximum) between the fine mesh ( $9 \times 18$ ) and finite strip results. Thus it can be concluded that for comparable accuracies, the saving in CPU time by using the finite strip solution is very significant.

Table 5  
Comparison between finite element and  
finite strip results (example 1)

	number of degrees of freedom	number of points where displacements are given	number of points where stresses are given	CPU time in sec. (*)
finite strip n = 10	400	180	270	120
finite element mesh $9 \times 9$	1320	100	81	342
finite element mesh $9 \times 18$	2544	190	162	1206

(\*) The computer used is a CDC-6400

The second example corresponds to model 3 from Beaufait's paper [18], in which experimental as well as theoretical (ordinary theory) results are available. The dimensions and the loadings are shown in Fig. 6, and the modulus of elasticity is taken as  $73.14 \times 10^6$  kN/m<sup>2</sup> and Poisson's ratio as 0.33. Because the two spans are equal, only 9 terms of the series are used for the analysis. Twenty finite strips are used for half of the structure because of symmetry.

The transverse distribution of the longitudinal stress and the transverse bending moment at mid-span is shown in Fig. 7(a,b), and the values of transverse bending moment and the longitudinal stress at point H are given at in Table 6 for several different cross-section locations. In general, the finite strip results are much closer to the experimental values when compared with Beaufait's theoretical results.

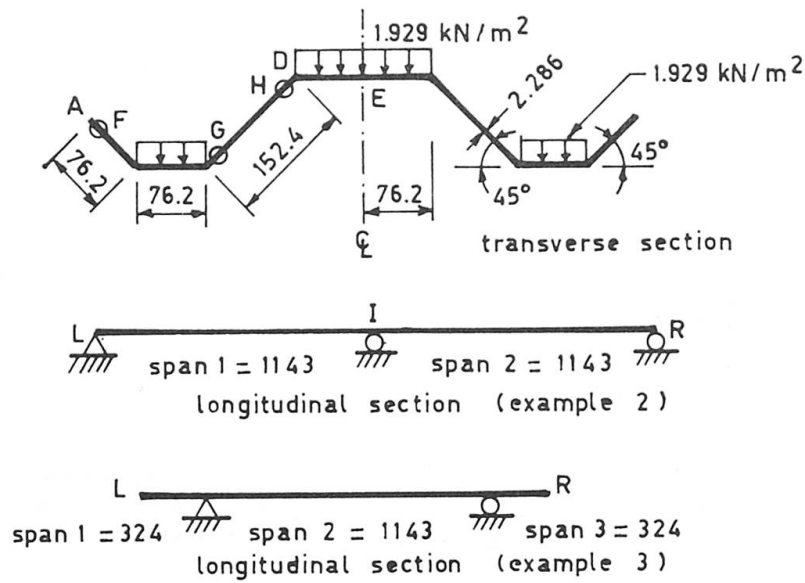


Fig. 6. Dimensions (in mm) and loading for Examples 2 and 3

Table 6  
Longitudinal Stresses and Transverse Moments at point H (example 2), at different cross-sections

Section	$\sigma_y$ (kN/m <sup>2</sup> )			$M_x$ (kg-m/m)		
	Beaufait experiment	Beaufait theory	finite strip method	Beaufait experiment	Beaufait theory	finite strip method
Mid-Span 1	- 656	- 704	- 718	-0.207	-0.205	-0.210
Mid-Span 2	- 738	- 704	- 718	-0.192	-0.205	-0.210
52.4 mm from centre support	-1642	-1428	-1559	-0.116	-0.218	-0.102
155.6 mm from centre support	490	538	504	-0.210	-0.215	-0.216



The third example corresponds to model 2 from the same paper by Beaufait, and here the versatility of the finite strip method over that of the elasticity method is amply demonstrated, since the latter method cannot be applied to the present example which has fairly long overhangs. The dimensions and loadings are shown in Fig. 6, and the material properties are the same as those given for example 2. Ten finite strips are used to represent half of the structure, and eight terms of the series (determined from the load convergence test) are used in the analysis. In addition to the finite strip solution, a finite element analysis using 70 flat shell elements [19] for a quarter of the structure was also carried out. All results are presented in Table 7, and once again the finite strip results agree very well with the experimental and numerical values. The discrepancy of the longitudinal stresses at the support was attributed by Beaufait to local effects caused by the supporting diaphragm.

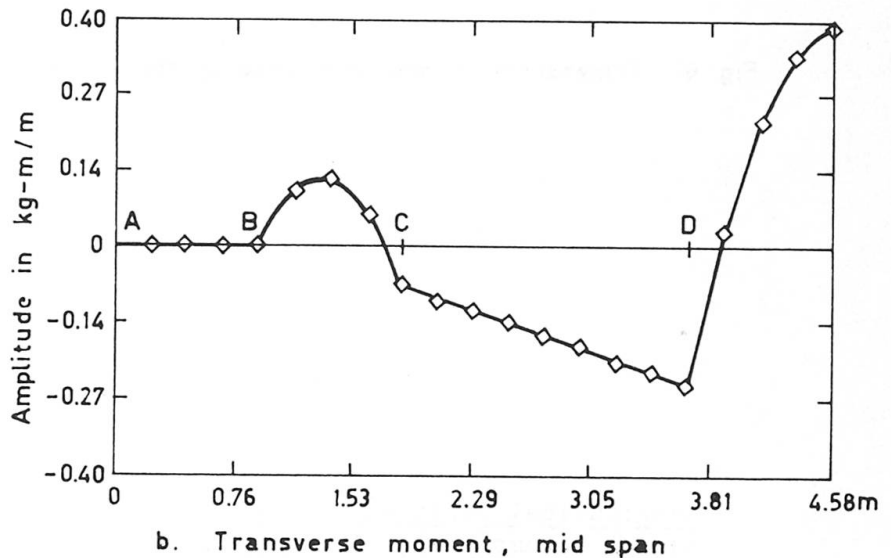
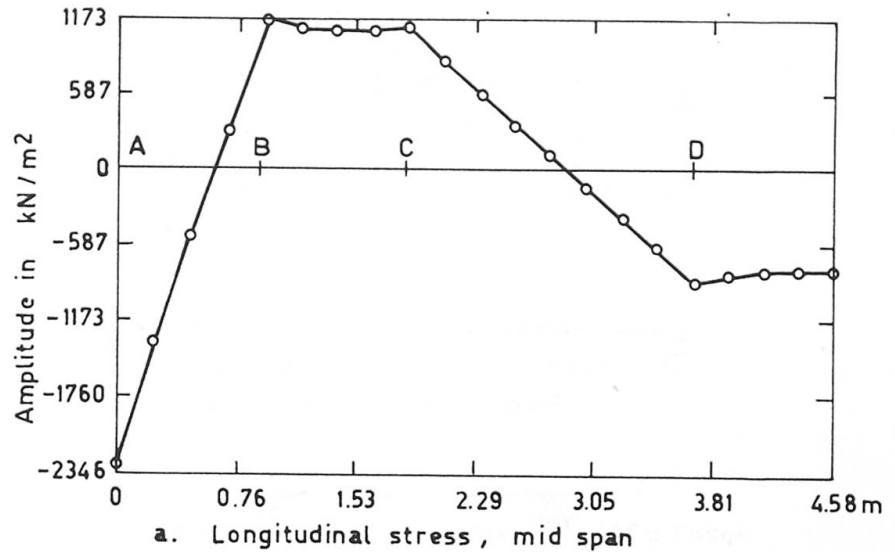


Fig. 7 Example 2. Transverse distribution of longitudinal stress and transverse moment

### Conclusion

The finite strip method has been generalized to study multi-span structures with any type of boundary conditions. The number of terms necessary for the finite strip analysis is dependent on the considered structure, and can be determined by a simple test of load convergence. Three numerical examples are presented and the results are compared with numerical and experimental results available in the literature. The finite strip method produces values which agree very well with the experimental

results. Finally, a comparison between finite element method and finite strip method is presented. The conclusion is that for such continuous structures the finite strip method is much more economical than the finite element method.

Table 7(a)  
Longitudinal stresses (kN/m<sup>2</sup>) (Example 3), at  
different cross-sections

		Beaufait theoretical	Beaufait experimental	Finite strip	Finite element
Mid-span	point F	-2525	-2539	-2567	-2456
	point G	1408	1477	1490	1497
	point H	- 973	-987, -904 <sup>(*)</sup>	- 932	- 994
**	point G	490	490	469	483
At support	point G	- 276	166	- 311	- 345
	point H	373	407	380	414

(\*) different values measured at symmetric points

\*\* 155.6 from support in main span

Table 7(b)  
Transverse moments (kg-m/m) Example 3 at point G

	Beaufait theoretical	Beaufait experimental	Finite strip	Finite element
Section mid-span	-0.0589	-0.0580, -0.0874 <sup>*</sup>	-0.0680	-0.0702
Section 155.6 mm from support main span	-0.0145	-0.0072	-0.0177	-0.0181

\* different values measured at symmetric points

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