Load-Deflection Behaviour of Simply Supported Rectangular Reinforced Concrete Slabs

Comportement charge-déformation de dalles rectangulaires en béton armé simplement appuyées sur leurs pourtours

Last-Durchbiegungs-Verhalten von einfach gelagerten Stahlbeton-Rechteckplatten

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SUMMARY
In this paper a method is presented to determine the complete load-deflection behaviour of simply-supported rectangular reinforced concrete slabs. The method is developed in two stages. In the first stage load-deflection behaviour upto Johansen's load is determined, while in the second stage load-deflection behaviour, which includes the effect of tensile membrane action is established. An experimental programme consisting of 12 simply-supported slabs was carried out to verify the results of the analysis. The load-deflection curves computed on the basis of the proposed method show satisfactory agreement with the test result.

RÉSUMÉ
Cette étude présente une méthode permettant de déterminer le comportement complet charge-déformation de dalles en béton armé simplement appuyées sur leurs pourtours. La méthode est subdivisée en deux parties. Dans la première, le comportement charge-déformation des dalles est étudiée pour des charges inférieures à la charge de Johansen. La deuxième partie traite du problème pour des charges supérieures, en tenant compte de l'effet de membrane correspondant. Un programme d'essai de 12 dalles a été exécuté pour vérifier les résultats de la méthode. Les flèches mesurées concordent de manière satisfaisante avec les déformations calculées sur la base de la méthode proposée.

ZUSAMMENFASSUNG
Die vorliegende Arbeit zeigt eine Methode, mittels der das vollständige Last-Durchbiegungs-Verhalten von einfach gelagerten Stahlbeton-Rechteckplatten erfasst werden kann. Die Methode basiert auf zwei Stufen. In der ersten Stufe wird das Last-Durchbiegungs-Verhalten der Platte bis zur Johansen'schen Traglast beschrieben, während die zweite Stufe sich auf höhere Lasten bezieht und auch Membrankräfte berücksichtigt. Ein Versuchsprogramm, in welchem 12 Rechteckplatten geprüft wurden, stützt die Berechnungsmethode. Die gerechneten Last-Durchbiegungs-Kurven passen sich ebenfalls gut an die Versuchsergebnisse an.

1. INTRODUCTION

In the case of simply supported reinforced concrete slabs, membrane forces are induced at finite deflections. These membrane forces are tensile in nature near the centre of the slab, while they are compressive near the edges. The compressive forces have a beneficial effect on the yield criterion, resulting in the increase in the load carrying capacity of the slab as the deflections increase. The effect of membrane action on the load carrying capacity of circular isotropic slabs was studied by WOOD [1], on the basis of rigid-plastic material behaviour. KEMP [2] gave the analysis for membrane action in square simply supported slabs. MORLEY [3] and SAWZUCK and WINNICK [4] gave methods which took into account the effect of membrane action on the load carrying capacities of simply supported isotropic slabs. Using an equilibrium approach, HAYES [5] presented the method for the determination of load-deflection relationship in orthotropic rectangular slabs. All these methods are based on rigid-plastic approach and hence, as shown in Fig. 1, the load-deflection behaviour predicted by them does not correspond to the behaviour of the actual slab. As seen from the figure the discrepancy between the analytical behaviour and the experimental behaviour is more pronounced in the vicinity of the yield line load. This is because the deflections and curvatures which occur prior to the yield line load are neglected in these analyses. However, the deflection at yield line load would be quite substantial in simply supported slabs and will affect their subsequent behaviour.

From the above discussion it is seen that most of the available methods of analysis of simply supported slabs for the effect of membrane action do not result in a correct picture regarding the load-deflection behaviour of these slabs. Hence in this paper, a method is presented to predict the complete load-deflection behaviour of simply supported slabs which gives a better representation of the same. Also tests have been done on 12 simply supported slabs and the results are compared with those of the proposed method.

2. PROPOSED METHOD

The analysis for the determination of load-deflection characteristics is carried out in two stages. In the first stage a semi-empirical method is given for the calculation of deflections upto Johansen's load. In the second stage the effect of membrane forces on the load carrying capacity is taken into consideration for the prediction of load-deflection behaviour beyond Johansen's load.

3. LOAD-DEFLECTION BEHAVIOUR UP TO JOHANSEN'S LOAD

In this stage the deflection upto Johansen's load are calculated using the results of classical theory of plates [6]. The cracking of concrete and the reduction in the modulus of elasticity of concrete under higher stresses are accounted for by suitably modifying the flexural rigidity of the slab. The method in general follows a method for the calculation of deflections of reinforced concrete beams as per CEB recommendations [7]. Fig.2 shows the typical stages in which the slab is likely to-behave as the load is increased from zero load to Johansen's load.

OE in Fig.2 represents the elastic behaviour of the slab. The deflection at the centre of slab in this region is calculated on the basis of the plate theory as

$$\delta = \frac{\beta q L_1^4}{E_c I} \qquad (1)$$

where β is a constant for appropriate span ratio L_2/L_1 of the slab, q is the intensity of load, L_1 is the length of short span and I is the gross moment of inertia of the slab cross-section.

At point E of Fig. 2, the deflection is

$$\delta_1 = \frac{\beta q_{cr} L_1^4}{E_c I} \qquad (2)$$

where q_{cr} is the intensity of load on the slab corresponding to the stage when moment at the centre of the slab just reaches the cracking moment. Beyond the point E, as the slab would have cracked at certain locations, the load-deflection behaviour of the slab changes and is given by EF (Fig. 2). Hence the flexural rigidity in this portion is modified and is taken as $E_c I_{cr}$, where I_{cr} is the moment of inertia of the cracked transformed section. At point F, the yielding of steel at centre of the slab takes place at an intensity of load q_y, hence the deflection at F is,

$$\delta_2 = \delta_1 + \frac{\beta (q_y - q_{cr}) L_1^4}{E_c I_{cr}} \qquad (3)$$

In the portion FG , the yielding spreads and finally at G, the yield line mechanism forms. In this region FG, the effective flexural rigidity is taken as+ $0.5 E_c I_{cr}$. Therefore the total deflection at Johansen's load is

$$\delta_j = \delta_2 + \frac{\beta (q_j - q_y) L_1^4}{0.5 E_c I_{cr}} \qquad (4)$$

where q_j is the intensity of Johansen's load.

\+ The factor 0.5 in the effective flexural rigidity term, is chosen on the basis of preliminary calculations of the slabs of this investigation.

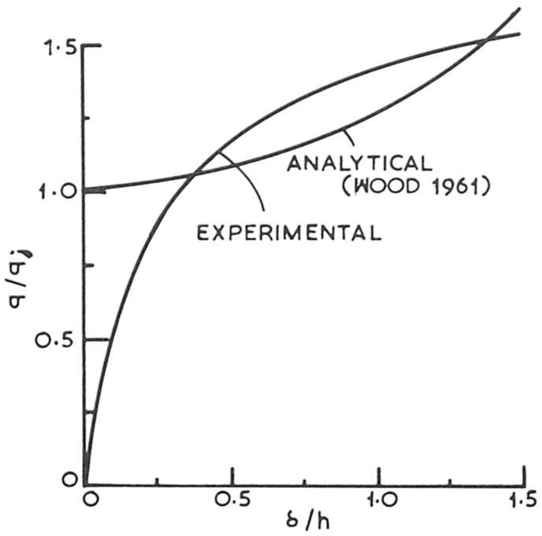

FIG. 1 - LOAD DEFLECTION BEHAVIOUR OF SIMPLY SUPPORTED SLAB

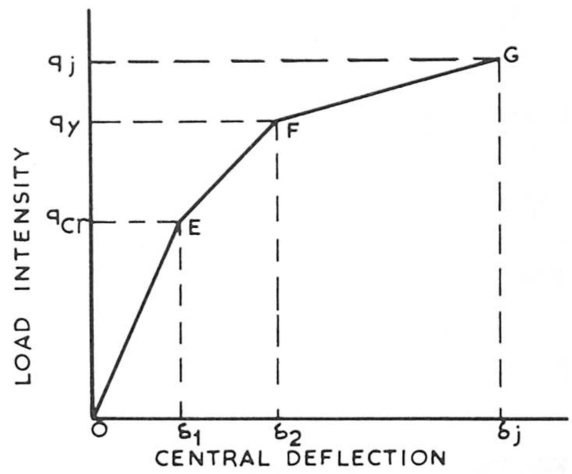

FIG. 2 - ASSUMED LOAD-DEFLECTION BEHAVIOUR FOR SIMPLY SUPPORTED SLABS UPTO JOHANSEN'S LOAD (PROPOSED METHOD)

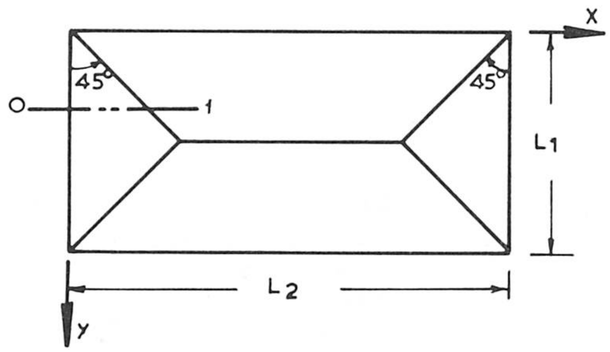

FIG. 3 - ASSUMED COLLAPSE MECHANISM AT YIELD LINE LOAD

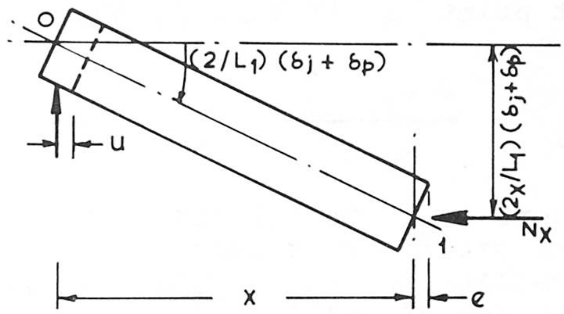

FIG. 4 - ENLARGED VIEW OF SECTION O-1

Table 1 : Details of simply supported slabs tested

Slab designation	Thickness of slab cms	Short span cms.	Long span cms.	Percentage of steel: Short span	Percentage of steel: Long span	Strength of Concrete: Cube strength N/mm^2	Strength of Concrete: Modulus of rupture N/mm^2	Intensity of yield line load N/mm^2
S_1	5.08	102	152	0.25	0.25	24.20	4.15	4.57
T_1	3.82		152	0.33	0.33	21.50	4.39	3.07
S_2	5.08		152	0.33	0.20	24.20	4.15	5.14
T_2	3.82		152	0.44	0.266	21.50	4.39	3.44
S_3	5.08		152	0.50	0.200	24.90	4.56	6.80
T_3	3.82		152	0.667	0.266	24.90	4.56	4.54
S_4	5.08		127	0.25	0.25	24.0	4.15	5.21
T_4	3.82		127	0.33	0.33	21.60	3.67	3.49
S_5	5.08		127	0.33	0.20	24.0	4.15	5.73
T_5	3.82		127	0.44	0.266	21.60	3.76	3.84
S_6	5.08		127	0.500	0.200	20.60	3.90	7.38
T_6	3.82		127	0.667	0.266	20.60	3.90	6.08

In the case of orthotropic slabs, the effective values of cracked transformed moment of inertia and the yield moment are taken as the average of the two values in two orthogonal directions.

The procedure mentioned above determines the load-deflection behaviour of simply supported slabs subjected to uniformly distributed load upto Johansen's load.

4. LOAD-DEFLECTION BEHAVIOUR BEYOND JOHANSEN'S LOAD

In this stage the load-deflection behaviour beyond Johansen's load is determined by a procedure which incorporates the effect of membrane action on the load carrying capacity. The procedure in general follows Kemp's approach [2] for square slabs, but has been modified as follows.

(a) The method has been generalised for rectangular orthotropic slabs, and (b) the effect of deflections prior to Johansen's load on the depth of neutral axis, membrane forces and their subsequent effect on the load carrying capacity is taken into consideration. The membrane forces are now determined using the geometrical relationship and the yield criterion.

4.1 Geometrical Relationship

Figure 3 shows a rectangular slab with short span L_1 and long span L_2, simply supported at the edges and with the assumed yield line pattern at Johaneen's load. To simplify the analysis the yield lines are assumed to make an angle of 45^o with the edges. Assuming that the membrane forces are induced in the slab only after the mechanism at Johansen's load has formed, a geometrical relationship between the extension of fibre and deflections is written. Considering a section 0.1 in x-direction (Fig. 3) extension at 1, can be written with reference to Fig. 4 as

$$e = [(x-u)^2 + \frac{4x^2}{L_1^2}(\delta_p + \delta_j)^2]^{\frac{1}{2}} - (x^2 + \frac{4x^2}{L_1^2}\delta_j^2)^{\frac{1}{2}} \qquad (5)$$

where u is the shift of the slab over the support and δ_p is the deflection at the centre of the slab after Johansen's load.

Following a procedure similar to that given in Ref. 2 and writing $e = 2\mu_x\delta_p/L_1$, we get

$$\mu_x = \mu_{ox} - \Delta(\frac{1}{2} - \frac{x}{L_1}) \qquad (6)$$

where μ_x is the height of neutral axis at any distance x, μ_{ox} is the height of neutral axis at $x = L_1/2$ and Δ is given by

$$\Delta = \frac{(\delta_p + \delta_j)^2 - \delta_j^2}{\delta_p}$$

A relation similar to that of equation (6) is written in y-direction as

$$\mu_y = \mu_{cy} - \Delta\left(\frac{1}{2} - \frac{y}{L_1}\right) \qquad (6a)$$

4.2 Yield Criterion

The yield criterion as given by KEMP [2] is

$$\frac{M}{M_p} = 1 + \gamma\frac{N}{T} - \eta\left(\frac{N}{T}\right)^2 \qquad (7)$$

where

$$\gamma = \frac{\left(\frac{1}{2}\frac{h}{d} - \frac{2k_2T}{k_1k_3f_{cu}d}\right)}{\left(1 - \frac{k_2T}{k_1k_3f_{cu}d}\right)} \quad ; \quad \eta = \frac{\frac{k_2T}{k_1k_3f_{cu}d}}{\left(1 - \frac{k_2T}{k_1k_3f_{cu}d}\right)}$$

If f is the yield function, then the height of neutral axis μ is expressed as

$$\mu = \frac{-\frac{\partial f}{\partial N}}{\frac{\partial f}{\partial M}} = \frac{\frac{\gamma}{T} - 2\eta\frac{N}{T^2}}{1/M_p} \qquad (8)$$

Equation (8) gives the value of axial force in x-direction, with proper subscripts as,

$$N_x = \frac{T_x\gamma_x}{2\eta_x} - \frac{\mu_xT_x^2}{2\eta_xM_{px}} \qquad (9)$$

Substituting the value of μ_x from eq.(6) in eq.(9), we have

$$N_x = A + Bx \qquad (10)$$

where $$A = \frac{T_x\gamma_x}{2\eta_x} - \frac{T_x^2\mu_{ox}}{2\eta_xM_{px}} + \frac{T_x^2\Delta}{4\eta_xM_{px}} \qquad (10a)$$

and $$B = -\frac{T_x^2\Delta}{2\eta_xM_{px}L_1} \qquad (10b)$$

Similarly in y-direction with proper change of subscripts, N_y is given by

$$N_y = C + Dy \qquad (11)$$

where $$C = \frac{T_y \gamma_y}{2\eta_y} - \frac{T_y^2 - \mu_{oy}}{2\eta_y M_{py}} + \frac{T_y^2 \Delta}{4\eta_y M_{py}} \qquad (11a)$$

$$D = -\frac{T_y^2 \Delta}{2\eta_y M_{py} L_1} \qquad (11b)$$

The values of N_x and N_y depend on the values of μ_{ox} and μ_{oy} and they are calculated by considering the equilibrium of forces for the rigid portions of the slab between the yield lines.

4.3 Calculation of μ_{ox} and μ_{oy}

Considering in-plane equilibrium of the portion ABC we have (Fig. 5)

$$2\int_0^{L_1/2} (A + B.x)\,dx = 0 \qquad (12)$$

which gives $$\mu_{ox} = \frac{\gamma_x M_{px}}{T_x} + \frac{\Delta}{4} \qquad (13)$$

The in-plane equilibrium equation for the portion ABDE (Fig.5) is

$$2\int_0^{L_1/2} (C + D.y)\,dy + 2\int_{L_1/2}^{L_2/2} \left(C + \frac{DL_1}{2}\right) dy = 0 \qquad (14)$$

which gives $$\mu_{oy} = \frac{\gamma_y M_{py}}{T_y} + \frac{\Delta}{4}\,\frac{L_1}{L_2} \qquad (15)$$

Knowing the values of μ_{ox} and μ_{oy} the axial forces at any section are determined from eqs.(10) and (11) and the corresponding moments from the yield criterion (eq.(7)).

4.4 Determination of the Load

From equations (6) and (6a) it is seen that the heights of neutral axes μ_x and μ_y are maximum at $x = y = L_1/2$ and are equal to μ_{ox} and μ_{oy}. Hence as the deflection of slab increases μ_{ox} or μ_{oy} will attain a value of h/2. At this condition the slab will be cracked throughout the depth, and the moment will be reduced to a minimum. Hence comparing the relative magnitudes of μ_{ox} and μ_{oy} following three cases are considered:

Case 1 : $\mu_{ox} \leq h/2$ and $\mu_{oy} < h/2$

Case 2 : $\mu_{ox} > h/2$ and $\mu_{oy} \leq h/2$

Case 3 : $\mu_{ox} > h/2$ and $\mu_{oy} > h/2$

4.4.1 Case 1 : $\mu_{ox} \leq h/2$ and $\mu_{oy} < h/2$

The intensity of load on the slab is calculated separately for the triangular portion ABC and the trapezium ABDE. As the derivations involved are quite lengthy only the final expressions are given in this paper. The detailed derivations can be found in Ref. 8.

The forces and moments acting on the yield lines of the slab are shown in Fig. 6. Considering the portion ABC and taking moments of all the forces on the portion ABC about the edge AC and equating them to zero, we get

$$\frac{q_1 L_1^3}{24} = 2\,M_{px} L_1 \left[0.5 + \frac{\gamma_x}{\eta_x}\left(0.5A + \frac{BL_1}{8}\right) - \frac{\eta_x}{T_x^2}\left(0.5A^2 + \frac{B^2L_1^2}{24} + \frac{ABL_1}{4}\right)\right] - (\delta_p + \delta_j)(0.5\,AL_1 + BL_1^2/6) \qquad (16)$$

where q_1 is the intensity of load on the portion ABC.
For portion ABDE (Fig. 6) we have

$$\frac{q_2 L_1^3}{24} + q_2 \frac{(L_2 - L_1)}{8} L_1^2 = 2\,M_{py} L_1 \left[0.5 + \frac{\gamma_y}{T_y}\left(0.5C + \frac{DL_1}{8}\right) - \frac{\eta_y}{T_y^2}\left(0.5C^2 + \frac{D^2L_1^2}{24} + \frac{CDL_1}{4}\right)\right] + M_{py}(L_2 - L_1)\left[1 + \frac{\gamma_y}{T_y}\left(C + \frac{DL_1}{2}\right) - \frac{\eta_y}{T_y^2}\left(C^2 + \frac{D^2L_1^2}{4} + CDL_1\right)\right] - (\delta_p + \delta_j)\left(\frac{CL_1}{2} + \frac{DL_1^2}{6}\right) - (\delta_p + \delta_j)\left(C + \frac{DL_1}{2}\right)(L_2 - L_1) \qquad (17)$$

The two values of q_1 and q_2 obtained from equations 16 and 17 would be different due to 45° yield line approximation involved in the assumed mechanism. This results in the introduction of nodal forces at the points B and D.

Accounting for this nodal force the true intensity of load q can be obtained as

$$q = \frac{q_1 + q_2\left(\dfrac{3L_2 - 2L_1}{L_1}\right)}{2 + 3\,\dfrac{(L_2 - L_1)}{L_1}} \qquad (18)$$

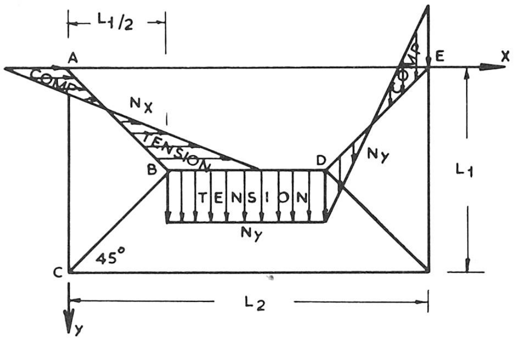

FIG. 5-VARIATIONS OF FORCES IN X AND Y DIRECTIONS

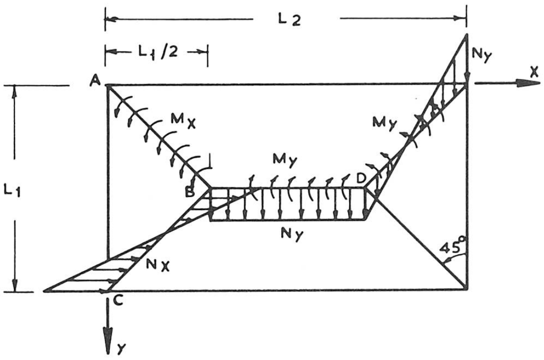

FIG. 6-FORCES AND MOMENTS ON YIELD LINES

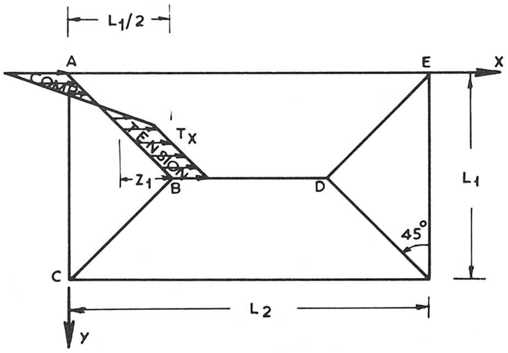

FIG. 7-SPREAD OF TENSION ZONE IN X-DIRECTION

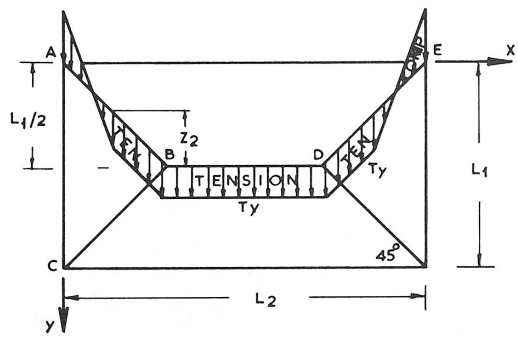

FIG. 8-SPREAD OF TENSION ZONE IN Y-DIRECTION

The equations 16 and 17 are valid so long as μ_{ox} and μ_{oy} are less than h/2 . With increase in deflections, μ_{ox} reaches h/2 first, this happens when (from eqn. 13)

$$\Delta = 4 \left[\frac{h}{2} - \frac{\gamma_x M_{px}}{T_x} \right]$$

After this value of Δ is reached, the slab at B or D (Fig. 5) will crack throughout the depth and the net axial force acting on the section will be tensile and equal to T_x. Hence for such a situation the analysis is modified as follows.

4.4.2 <u>Case 2</u> $\mu_{ox} > h/2$ and $\mu_{oy} \leq h/2$

Assuming the slab has fully cracked upto distance Z_1 in x-direction, and considering the in-plane equilibrium of portion ABC (Fig. 7) Z_1 can be determined from the following relation

$$\left(\frac{L_1}{2} - Z_1 \right)^2 = \frac{2\eta_x M_{px} L_1^2}{T_2} \qquad (19)$$

and the load q_1 as

$$\frac{q_1 L_1^3}{24} = 2 M_{px} \left(\frac{L_1}{2} - Z_1\right) \left[1 + \frac{\gamma_x}{\eta_x} \left\{A + \frac{B}{2} \left(\frac{L_1}{2} - Z_1\right)\right\} - \frac{\eta_x}{T_x^2} \left\{ A^2 + (B^2/2)\left(\frac{L_1}{2} - Z_1\right)^2 + AB \left(\frac{L_1}{2} - Z_1\right)\right\}\right] + 2 M_{px} (1 - \gamma_x - \eta_x) Z_1$$

$$- 4 \frac{(\delta_p + \delta_j)}{L_1} \left(\frac{L_1}{2} - Z_1\right)^2 \left[\frac{A}{2} + \frac{B}{3} \left(\frac{L_1}{2} - Z_1\right)\right]$$

$$+ 2 \frac{T_x}{L_1} (\delta_p + \delta_j)(L_1 Z_1 - Z_1^2) \qquad (20)$$

The intensity of load q_2 is again given by eqn.(17) and knowing q_1 and q_2, q is calculated from eqn.(18).

At a deflection

$$\Delta = \frac{4L_2}{L_1} \left(\frac{h}{2} - \frac{\gamma_y M_{py}}{T_y}\right)$$

μ_{oy} also reaches h/2 , the slab cracks throughout the depth along BD (Fig. 7) and the analysis is further modified as follows.

4.4.3 <u>Case 3</u> $\mu_{ox} > h/2$ and $\mu_{oy} > h/2$

For portion ABDE (Fig. 8) assuming Z_2 to be the spread of pure tension zone in y-direction, Z_2 is obtained from

$$\left(\frac{L_1}{2} - Z_2\right)^2 = \frac{2\eta_y M_{py} L_1 L_2}{T_y \Delta} \quad \text{and } q_2 \text{ as}$$

$$\frac{q_2 L_1^3}{24} + \frac{q_2 (L_2 - L_1) L_1^2}{8} = 2 M_{py} \left(\frac{L_1}{2} - Z_2\right)\left[1 + \frac{\gamma_y}{T_y}\left\{C + \frac{D}{2}\left(\frac{L_1}{2} - Z_2\right)\right\} - \frac{\eta_y}{T_y^2}\left\{C^2 + \frac{D^2}{3}\left(\frac{L_1}{2} - Z_2\right)^2 + C.D\left(\frac{L_1}{2} - Z_2\right)\right\}\right]$$

$$+ 2 M_{py} (1 - \gamma_y - \eta_y) Z_2 + M_{py} (1 - \gamma_y - \eta_y)(L_2 - L_1)$$

$$- 4 \frac{(\delta_p + \delta_j)}{L_1} T_y \left(\frac{L_1}{2} - Z_2\right)^2 \left[\frac{C}{2} + \frac{D}{3}\left(\frac{L_1}{2} - Z_2\right)\right]$$

$$+ 2 \frac{(\delta_p + \delta_j)}{L_1} T_y (L_1 Z_2 - Z_2^2) + (\delta_p + \delta_j) T_y (L_2 - L_1) \qquad (21)$$

As μ_{ox} is also greater than $h/2$, the intensity of load q_1 is given by eqn.(20), and knowing q_1 and q_2, q is obtained from eqn. 18.

Depending on the values of δ_j, the deflection at Johansen's load, it is likely that for values slightly larger than δ_j, μ_{ox} will be greater than $h/2$, in such a case, the analysis givenin case 1 does not arise.

If for deflections slightly greater than δ_j both μ_{ox} and μ_{oy} are greater than $h/2$, the analysis starts directly with case 3.

3. EXPERIMENTAL PROGRAMME

The experimental programme consisted of casting and testing under uniformly distributed load twelve slabs. These covered two ratios of L_1/h (= 20 and 26.7), two aspect ratios L_2/L_1 (= 1.5 and 1.25) and coefficients of orthotropy ranging from 1.0 to 2.88. The details of slabs are given in Table 1. For all slabs steel used was of 4 mm dia. with a yield strength f_y = 555 N/mm^2 and ultimate strength f_u = 645 N/mm^2.

6. DISCUSSION AND COMPARISON OF RESULTS

The proposed analysis has been used to calculate load deflection relationship for all the twelve slabs tested in this investigation. The typical computed and experimental load-deflection curves are compared in Fig. 9. The points E, F, G corresponding

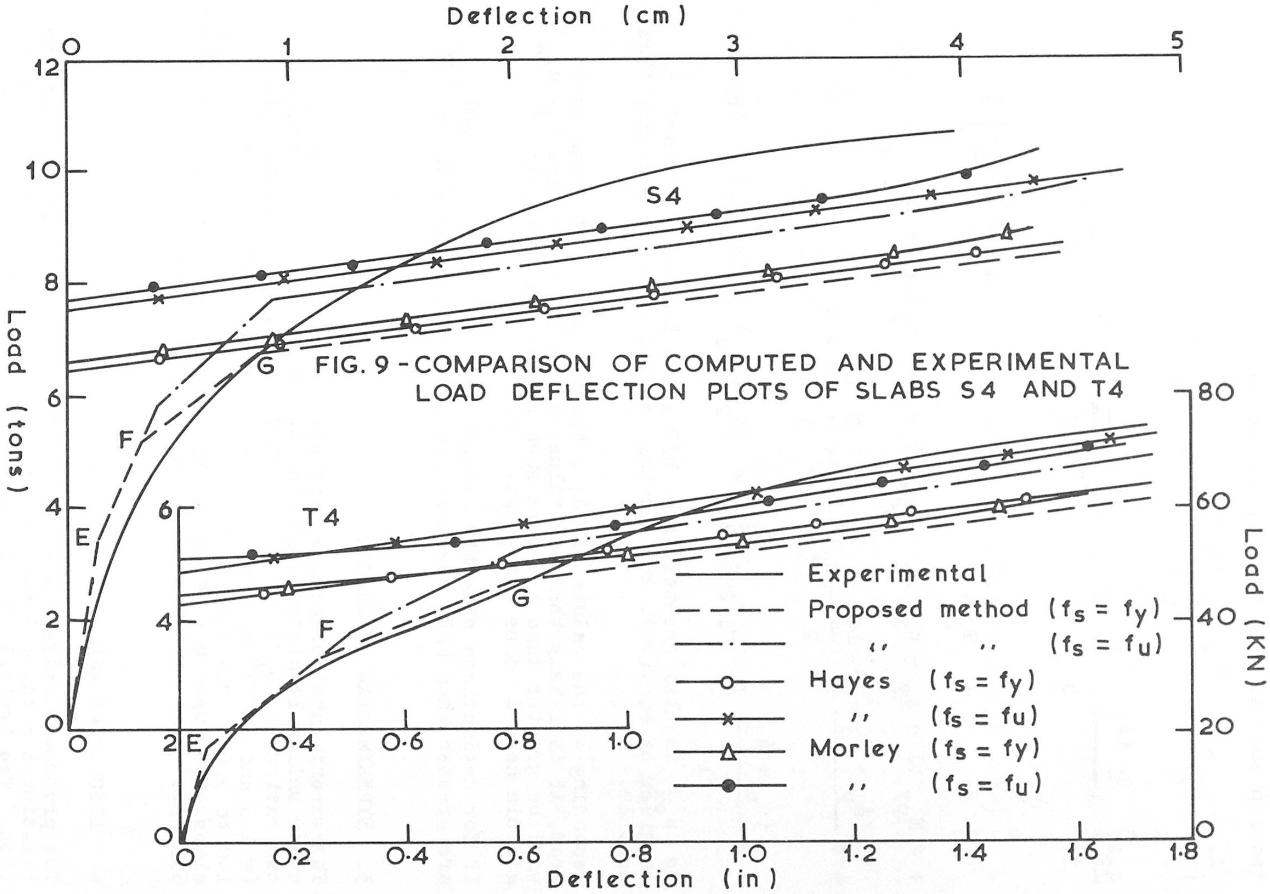

FIG. 9 - COMPARISON OF COMPUTED AND EXPERIMENTAL LOAD DEFLECTION PLOTS OF SLABS S4 AND T4

Table 2 : Comparison of deflections at the working load and at Johansen's load

Sl. No.	Slab designation	Deflections at working load			Deflections at Johansen's load		
		δ_{exp} mm	δ_{cal} mm	$\frac{\delta_{exp}}{\delta_{cal}}$	δ_{jexp} mm	δ_{jcal} mm	$\frac{\delta_{jexp}}{\delta_{jcal}}$
1.	S_1	4.20	3.30	1.269	12.7	10.70	1.190
2.	T_1	4.60	5.60	0.818	10.40	13.80	0.641
3.	S_2	5.75	4.10	1.435	12.00	10.10	1.191
4.	T_2	6.35	6.75	0.943	11.80	15.50	0.764
5.	S_3	5.10	5.60	0.909	9.90	13.40	0.736
6.	T_3	6.60	8.65	0.788	11.00	17.60	0.622
7.	S_4	3.81	2.80	1.360	8.64	7.62	1.130
8.	T_4	5.73	5.73	1.000	10.30	9.70	1.067
9.	S_5	3.82	3.18	1.200	8.40	7.90	1.064
10.	T_5	4.95	5.85	0.848	10.10	12.70	0.797
11.	S_6	5.85	4.70	1.243	9.90	9.35	1.057
12.	T_6	7.10	7.90	0.903	11.00	14.20	0.773
		Average		1.060	Average		0.920
	Coefficient of variation			0.213	Coefficient of variation		0.230

to q_{cr}, q_y and q_j respectively are also shown on these curves. In the calculations for computing ultimate moment of resistance of slab section, usually the value of yield strength of steel is used. However because of the considerable deflections suffered by the slab, snapping of steel was noticed in some test specimens, therefore it is possible that the steel has attained stresses close to the ultimate strength. Hence calculations have also been made using the value of ultimate strength of steel, and the resulting plots are also superposed for comparison with experimental plots in Fig. 9. These figures also show the load-deflection relationship obtained from the analysis due to MORLEY [3] and HAYES [5]. In determining the load-deflection relationship according to their method again two values of steel strengths ($f_s = f_y$ and $f_s = f_u$) are used.

Figure 9 shows that the use of $f_s = f_y$ has underestimated the effect of membrane action on the load carrying capacity of slabs. The use of $f_s = f_u$ has shown better agreement with experimental curves. It is also noted from these figures, that the results of the proposed analysis for membrane action compare favourably with the analyses due to MORLEY [3] and HAYES [5] while using $f_s = f_y$ or $f_s = f_u$.

The comparison of the deflections calculated using the proposed analysis with experimental deflections at working loads are shown in Table 2. In this comparison the working load is taken as (yield line load)/1.5. The load factor 1.5 is taken as the average of the load factors for dead and live loads specified in CP 110 (1972) [9]. The average ratio of experimental deflection to calculated deflection is 1.060, and a coefficient of variation of 0.213. In Table 2, the deflections of Johansen's load are also compared. The average ratio of experimental deflection to calculated deflection at Johansen's load is 0.92 with a coefficiend of variation of 0.230.

7. CONCLUSIONS

(i) The proposed method predicts the complete load-deflection behaviour of simply supported slabs, which is in satisfactory agreement with the test results, if $f_s = f_u$ is used, while the use of $f_s = f_y$, results in an underestimation of the load carrying capacity.

(ii) The deflections under working load predicted by the proposed method agree satisfactorily with experimental deflections. The average ratio of experimental to calculated deflection being 1.060, with a coefficient of variation of 0.213. The average ratio of deflections at Johansen's load is 0.92 with a coefficient of variation of 0.23.

(iii) The results of the proposed method in the second stage which accounts for the membrane action compare favourably with the results of methods available in literature which are based on rigid plastic approach.

8. ACKNOWLEDGEMENT

Authors thank Prof.K.T.S.Iyengar for his constant encouragement during this investigation.

NOTATIONS

A,B,C,D	deflection dependent constants
d	effective depth upto centre of tension steel
e	extension of central fibre
E_c	modulus of elasticity of concrete
f	yield function
f_{cu}	cube strength of concrete
f_s	stress in steel
f_y	yield strength of steel
f_u	ultimate strength of steel
h	total depth of slab
I	gross moment of inertia of slab cross-section
I_{cr}	cracked transformed moment of inertia
k_1, k_2, k_3	stress block parameters
L_1	short span length
L_2	long span length
M	bending moment
M_p	plastic moment
M_{px}, M_{py}	plastic moment capacity in x and y directions respectively
N	axial force
N_x,N_y	axial forces in x and y directions
q	intensity of distributed load
q_{cr}	intensity of load corresponding to cracking at the centre of the slab
q_y	intensity of load at yielding of steel at the centre of slab
q_j	intensity of Johansen's load
T_x, T_y	tensile forces in x and y directions
u	shift of the slab over the support
β	constant depending on aspect ratio
δ	deflection
δ_j	deflection at Johansen's load
δ_p	deflection beyond Johansen's load
Δ	function of δ_j and δ_p
η, γ	constants in yield criterion
μ_x, μ_y	heights of neutral axes in x and y directions
μ_{ox}, μ_{oy}	heights of neutral axes at $x = y = L_1/2$ in x and y directions

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