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Influence of Ductility on Reliability of Reinforced Concrete Beams and Frames

Influence de la ductilité sur le niveau de sécurité des poutres et cadres en béton armé

Der Einfluss der Duktilität auf das Sicherheitsniveau von Balken und Rahmen aus Stahlbeton

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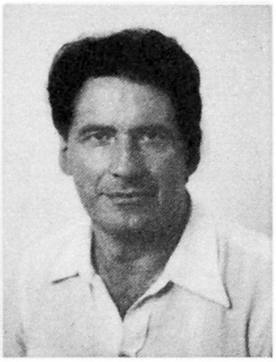

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SUMMARY

Reliability analyses, which are mainly used to determine and verify partial safety coefficients to be used in codes of practice; are usually based on linear elastic behaviour of structures. In this way the behaviour of the structure in the plastic field is not considered, and the influence of structural ductility on safety is ignored. In this paper the results of "reliability level two" parametric analyses are reported, which were performed on simple, although typical, beams and frames, to assess the influence of ductility on the level of safety of these structures, designed according to the partial safety coefficients given by the CEB Model Code.

RÉSUMÉ

Les calculs probabilistes des structures qui sont utilisés pour la détermination et la vérification des coefficients partiels de sécurité donnés par les codes sont fondés en général sur des analyses linéaires élastiques. De cette façon le comportement de la structure dans le domaine plastique n'est pas considéré et l'influence de la ductilité structurelle sur le niveau de sécurité est ignorée. L'article présente les résultats d'une étude probabiliste paramétrique de "niveau 2" concernant des poutres et cadres simples mais typiques, en vue de déterminer l'influence de la ductilité sur le niveau de sécurité de ces structures, dimensionnées selon les règles du Code Modèle CEB.

ZUSAMMENFASSUNG

Die Analysen des Sicherheitsniveaus, die hauptsächlich zur Bestimmung und Kontrolle der Teilsicherheitskoeffizienten benutzt werden, gründen sich vornehmlich auf das linearelastische Verhalten der Konstruktionen. Auf diese Weise betrachtet man das Verhalten der Konstruktion in der plastischen Phase nicht, und der Einfluss der Duktilität auf die Sicherheit wird vernachlässigt. In der vorliegenden Arbeit werden Parameteranalysen auf Stufe 2 am Beispiel einfacher und typischer Konstruktionen durchgeführt. Der Einfluss der Duktilität auf das Sicherheitsniveau gemäss der CEB Mustervorschrift wird untersucht.

1. INTRODUCTION

The Ultimate Limit State analysis of r.c. structures is performed in practice by using semi-probabilistic methods, as prescribed by the CEB Model Code |7|. In the same code suitable values of partial safety coefficient γ_f and γ_m are given, which were determined using more refined procedures such as the so called "level II" reliability methods.
If a reliability method of analysis is used, one should not only take care to choose proper distributions of random variables, a proper algorithm to determine the safety index |2||3||4|, but also to adopt a structural analysis procedure and a load history which realistically describes the "true" behaviour of the structure; also an ultimate limit state must be adopted which represents a true failure of the structure, whith possible extensive damage and loss on life, and not only a conventional limit state, which can lead to very different consequences, according to the "adaptability" of structure under consideration. As it is well known the behaviour of r.c. structures is non linear, due to geometrical effects (second order effects) and material behaviour (cracking of concrete in tension, non linear constitutive laws, plastic behaviour).
The non-linear behaviour of structures however was not considered in the determination of γ_f and γ_m coefficients of CEB Model Code, and therefore one may wonder if the same values of safety coefficients can be applied both to brittle and to ductile structures, despite the fact that the latters have a much greater reserve of strength, due to their capability of supporting additional loads in the plastic range, without local failures.
All this considered, a program was prepared and described in a previous paper |16| which can perform "level II" reliability analyses based on non linear behaviour of reinforced concrete plane frames.
This program was used to prepare a parametric study mainly intended to determine the influence of structural ductility on the level of safety, when the design is performed at "level I", using the partial γ coefficients specified by CEB Model Code and applying, in some cases, an arbitrary moment redistribution according to the ductility rule |15|.

2. COMPUTER PROGRAM

The program under consideration was prepared by connectig together two existing programs:

a) Program SICA NL, which can perform incremental non linear analyses of r.c. plane frames, taking into account cracking, plastic behaviour, second order effects and creep; this program is described in ref.|1||17||18||19|. Plastic behaviour is considered using the plastic hinge approach.
b) Program FORM, which was developed in Munich |2| and subsequently modified for the current purpose; this program, given a set of uncorrelated random variables $\bar{x}$ and a limit state equation $g(\bar{x})=0$, permits the calculation of the reliability index β (which can be defined as the minimum distance of the coordinate origin from the failure surface in space of normalized random variables) and therefore of the failure probability.

The resulting program (program SIMO) permits first order reliability analyses of r.c. plane frames, based on non-linear behaviour.
The use of program raised some numerical problems which are extensively discussed in ref.|16|.

3. CHOICE OF LIMIT STATE FUNCTION

According to the criteria adopted by CEB Model Code a r.c. structure fails when in a critical section where a plastic hinge has previously formed a limiting value of plastic rotation is reached, correspondig to the section failure.
A r.c. frame behaves like a so called " series system", in the sense that when one element fails, all the structure is considered to fail. See ref.|12||20|.
This limit state can be reached in two basically different ways:

a) The limit plastic rotation is reached gradually and a "local" failure only takes place.

b) The limit plastic rotation is reached suddenly because a collapse mechanism or frame instability has developed and therefore an over-all failure occurs.
If the ultimate limit state is defined in this way, at least two possibilities exist in the definition of the limit state function:
1) The limit state function can be defined as

$$g(\underline{x}) = \theta(\underline{x}_{act}) - \theta_{lim}(\underline{x}_{res}) \tag{1}$$

where $\theta(\underline{x}_{act})$ is the current plastic rotation in the section, while $\theta_{lim}(\underline{x}_{res})$ is the rotation capacity.
This definition has the advantage that it does not impose limitations to the load history producing the structural failure.
2) The limit state function can also be defined as

$$g'(\underline{x}) = P_{act} - P_{res}(\underline{x}_{res})$$

P_{act} being the random acting load and P_{res} the load level corresponding to structural failure.
This formulation sharply reduces discontinuities in the failure surface and was therefore adopted in this study; however it implies that a type of load only proportionally increasing, can be considered critical from the point of view of collapse.

4. CHOSEN EXAMPLES - LIMITATIONS OF PARAMETRIC STUDY-DESIGN CRITERIA

4.1. Limitations introduced

Limitation had to be introduced in the parametric study, reducing the general validity of conclusions. These limitations can be described and classified as follows:
- Limitations concerning geometry of the structures:
Three very simple structural schemes were considered:
1) One span beam (see fig.1)
2) Two span beam (see fig.1)
3) Non sway, non slender simple frame (see fig.2).
These schemes permit to explore, in a very simplified way, some very important practical situations.
-Limitations concerning the choice of random variables:
The following variables were treated in a probabilistic way (in addition to the loads)
- Strength of concrete in compression f_c
- Strength of concrete in tension f_{ct}
- Strength of steel f_{ys}
-Cover of tensile reinforcement d'
The type of distribution which was adopted is in agreement with well established criteria and is summarized in table 1 of appendix 1.
Geometric variables (with the sole exception of cover d') were treated as deterministic and therefore the model uncertainties deriving from these variables have not been studied; moreover the distribution of materials strength is supposed to remain the same in every element of the considered examples.
- Limitations concerning loads:
Concentrated loads were applied to the examples (see Fig.2)
4) These loads were increased proportionally up to ultimate limit state.
Therefore one kind of load only was considered, with the exception of example 3, where also a deterministic load was applied to the column (in the first loading step), simply to simulate the influence of axial load on member ductility.
This simplification implies the following consequences:
-permanent and variable loads are assumed to have the same distribution and to increase contemporarily and proportionally, being defined by the same parameters.
-The ratio between loads on different spans does not change. This assumption corresponds, in semi-probabilistic design, to considering one loading condition only

(while design should be based in general on the envelope of action effects deriving from multiple loading conditions).
To account for possible limited random variations of the ratio between loads on different spans, for the second example, also the case was considered in which this ratio, in addition to the reference load, is assumed as random variable.
An extreme distribution of type I was assumed in every case, both for loads and ratios.

4.2 Design criteria

Reinforcement in the three examples was designed according to a linear elastic analysis, using the partial safety coefficients specified by CEB Model Code |7|:
γ_f = 1.5 for loads
γ_c = 1.5 for concrete strength
γ_s = 1.15 for steel strength
In the first example also a value of γ_f = 1.3 was considered.
Design was also performed for two cases, for the first two examples, with redistribution of moments according to the well known "ductility rule" which may be described by the following expression

$$\delta \geq 0.56 + 1.25\ x/d \qquad (2)$$

δ being the amount of redistribution permitted by the rule.

4.3 Data for first example

Dimensions of the beam are reported on fig. 1
A total of 12 design cases were considered.
The different designs correspond to increasing values of design load; in this way gradually less ductile structures are obtained; two cases have been considered in which the moments deriving from linear elastic analysis have been redistributed according to the ductility rule.

4.4 Data for second example

Dimensions of the frame are reported on fig.3.
A total of 7 basic design cases were considered.
The various cases were obtained using the procedure of the first example.

4.5 Data for third example (non sway frame)

Dimensions of the frame are reported on fig. 2.
A total of 12 basic design cases were considered.
The different designs correspond to different values (assumed as deterministic) of the axial load P' applied to the column; the load applied to the midspan of the beam has the same mean value and distribution in all the cases: reinforcement in the beam is as well the same in all the cases and has been increased with reference to the value resulting from linear elastic analysis by 30 and 50%.

5. OBTAINED RESULTS

On Fig. 3 and 4 values of safety indexes β are given against ductility factors in the critical section for the first beam example.
As may be expected a decrease of β with decreasing ductility is obtained.
On fig. 5 values of β are given for the second beam example.
On fig. 5 the values of β were also plotted which were obtained using a computation procedure aimed at separating the influence of the increase of tensile steel ratio alone on safety, from the influence of ductility.
Therefore in these cases the rotation capacity has been considered the same, ignoring the influence of the increase of $\xi = x/d$ on this capacity.
On Fig.6 the values of β are given which were obtained by assuming, as

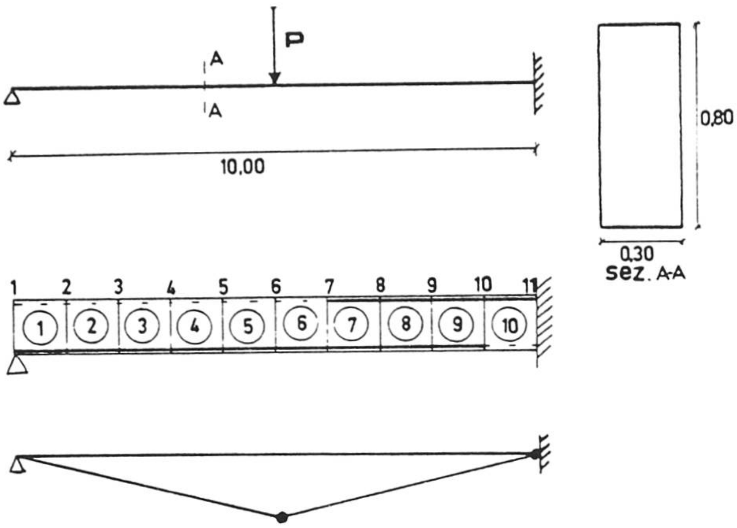

Fig. 1a - Example 1

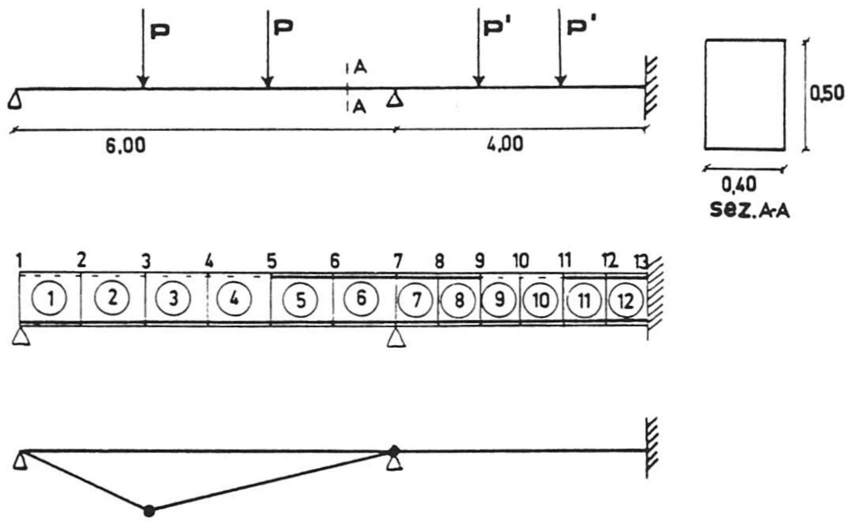

Fig. 1b - Example 2

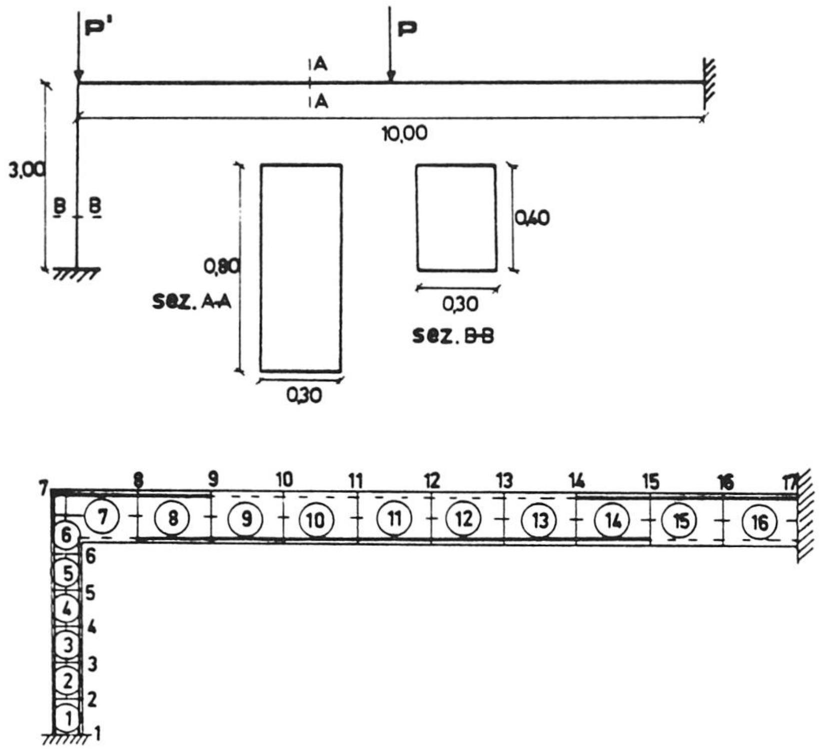

Fig. 2 - Example 3

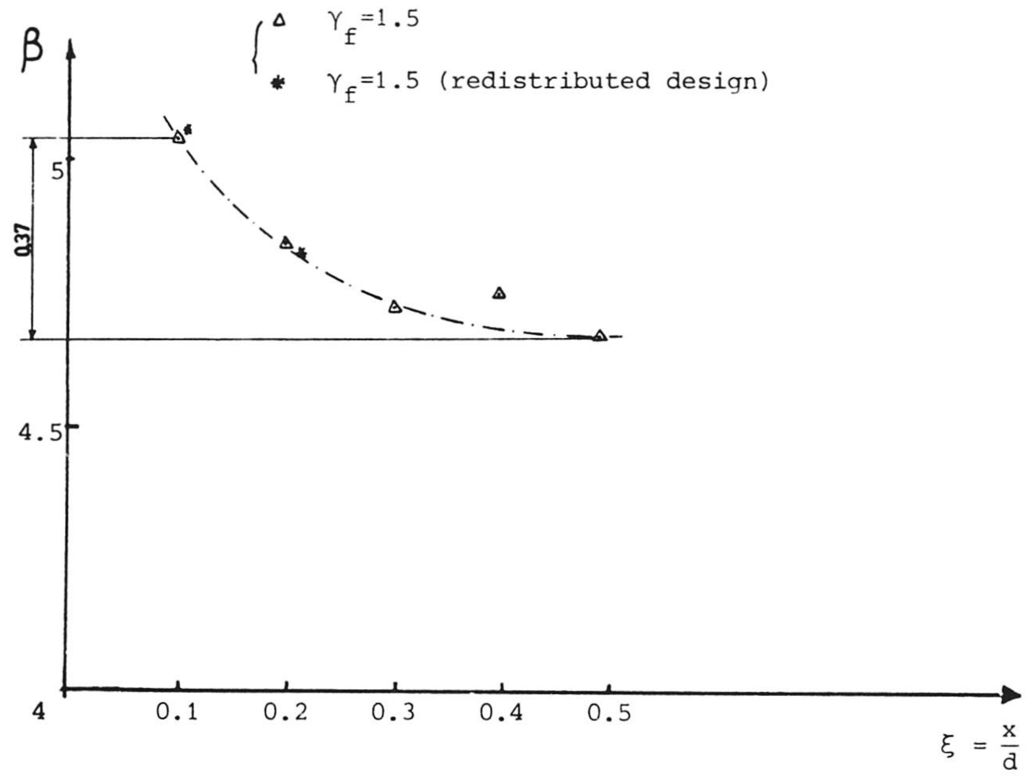

Fig. 3 - Example 1; Values of $\beta(\gamma_f=1.5)$

an additional random variable, the ratio of loads on adjacent spans, with a distribution of the same type of the loads (extreme of type I) and a mean value of 1.
On Fig.7 values of β are given for the third example(1).
Two series of design cases were considered. In the first series the needed reinforcement in the beam was increased by 30%, while in the second it was increased by 50%.
This criterion of design was adopted to obtain, for the cases where the column is brittle, a "local" failure in the column itself, without formation of failure mechanism, which invariably develops in a "normal" design.

6. CONSIDERATIONS ON RESULTS

Ductility can be influenced, once the distribution of material strengths have been defined, mainly by two factors: reinforcement ratios and axial loads on columns.
The first factor has been considered in the first two examples, where a series of cases was studied in which the design loads, and therefore tensile reinforcement, were gradually increased.
The second factor has been considered in the third case, where the column ductility was gradually decreased by applying an increasing axial load, while the action on the beams was not modified.
Design of reinforcement was performed within the framework of the semi-probabilistic partial coefficients method, using the γ coefficients prescribed by CEB Model Code to test the validity of these coefficients when a non-linear model is adopted. The case of design with arbitrary redistribution of moments according to the "ductility rule" was also considered for some cases concerning the first two examples.
This stated, the conclusions which may be drawn from this study can be summarized as follows:
- <u>Influence of structure ductility on safety</u>

The maximum variation of β in the first two examples considered, are indicated on Figs. 3, 4, 5, 6 and are summarized on Table 1.

Example	$\Delta\beta_{max}$
1	0.37
2	0.26

Table 1 - Maximum variations of safety level (γ_f = 1.5)

As it is possible to notice, variations in reinforcement ratio (and corresponding variations in structural ductility) produce variations of safety levels which are much below the admissible level, which can be taken as ± 0.5 with reference to the safety index β. On the other hand the probabilities of failure are always sufficiently low.
It may be concluded that γ_f and γ_m coefficients used in design are adequate even after consideration of non linear behaviour. This fact can be explained as follows:
- Application of partial coefficient γ_c = 1.5 to concrete strength f_{ck} has the consequence that the design ductility factor ξ of critical sections is much higher than the one resulting from the adopted distribution in level II calculations.

In other words the "true" ductility is much higher than the "fictitious" ducti-

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(1) It must be noticed that, while for the first two examples values of ξ were considered with reference to the design values of material strengths, in the third case, where the behaviour of an extremely brittle element was investigated, the values of ξ were computed with reference to the mean values of resistances.

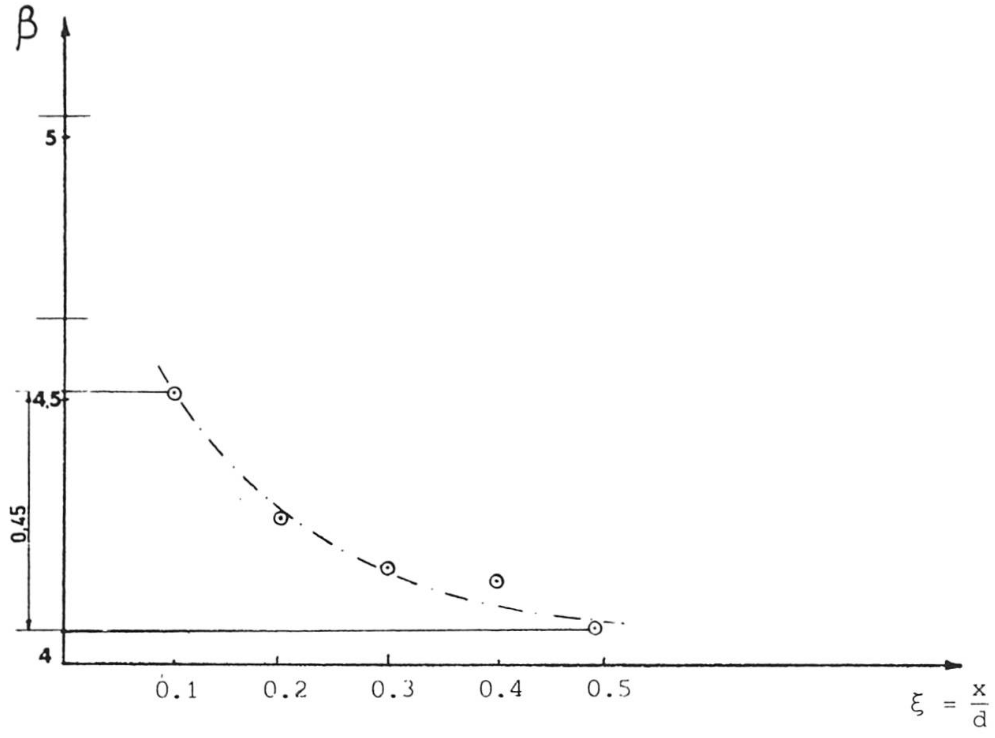

Fig. 4 - Example 1; Values of $\beta(\gamma_f=1.3)$

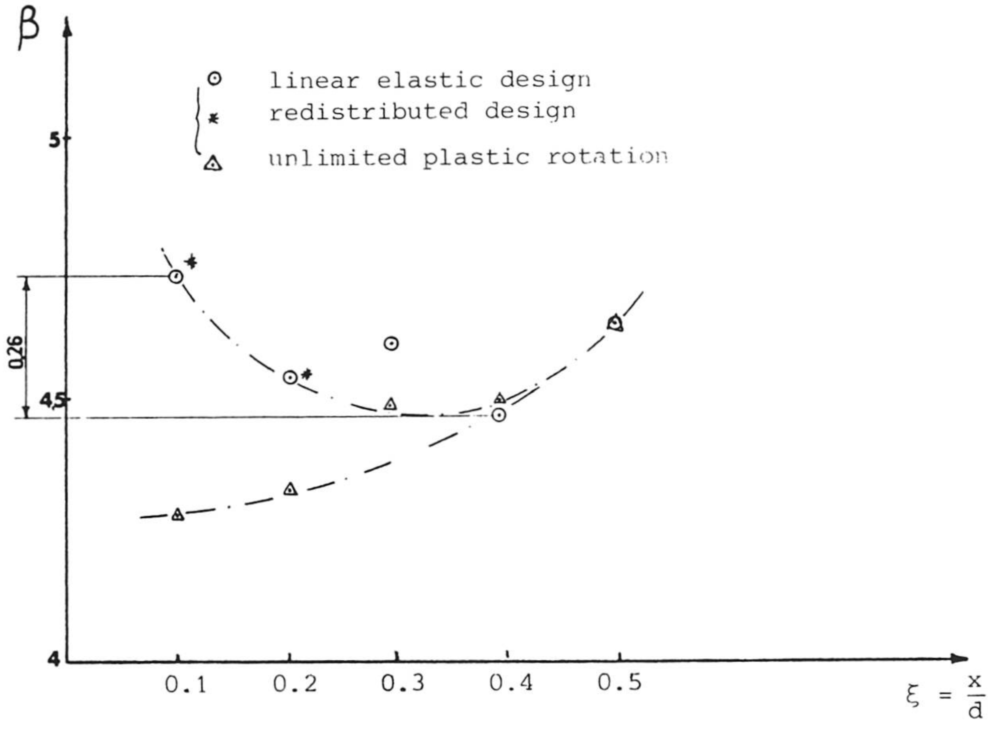

Fig. 5 - Results for example 2

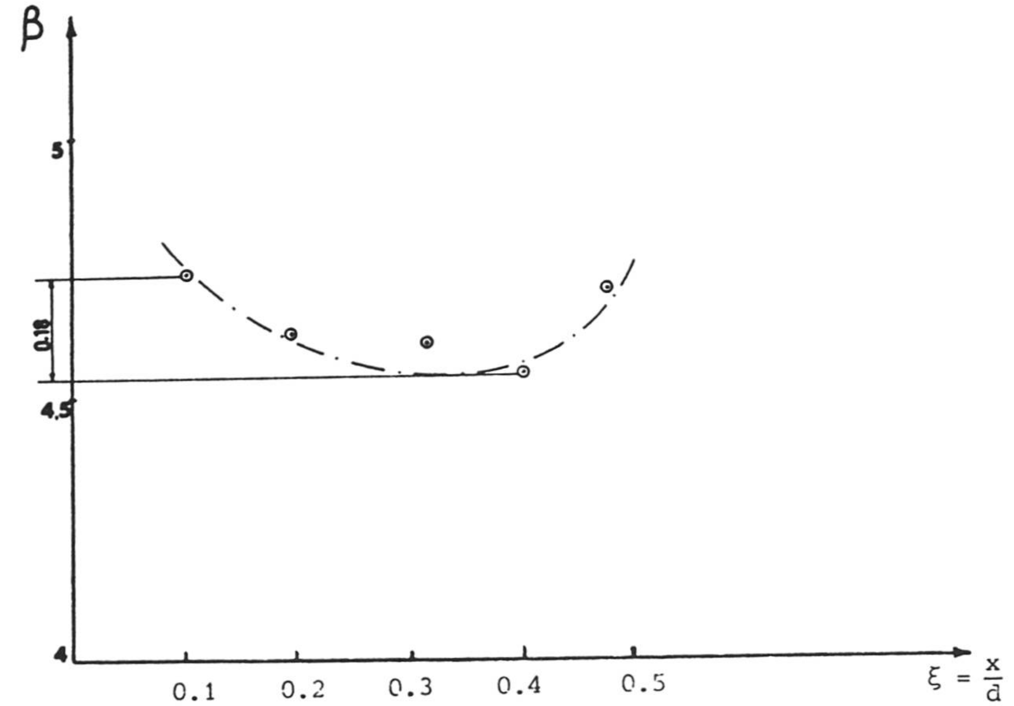

Fig. 6 - Results for example 2 (with P/P' as random variable)

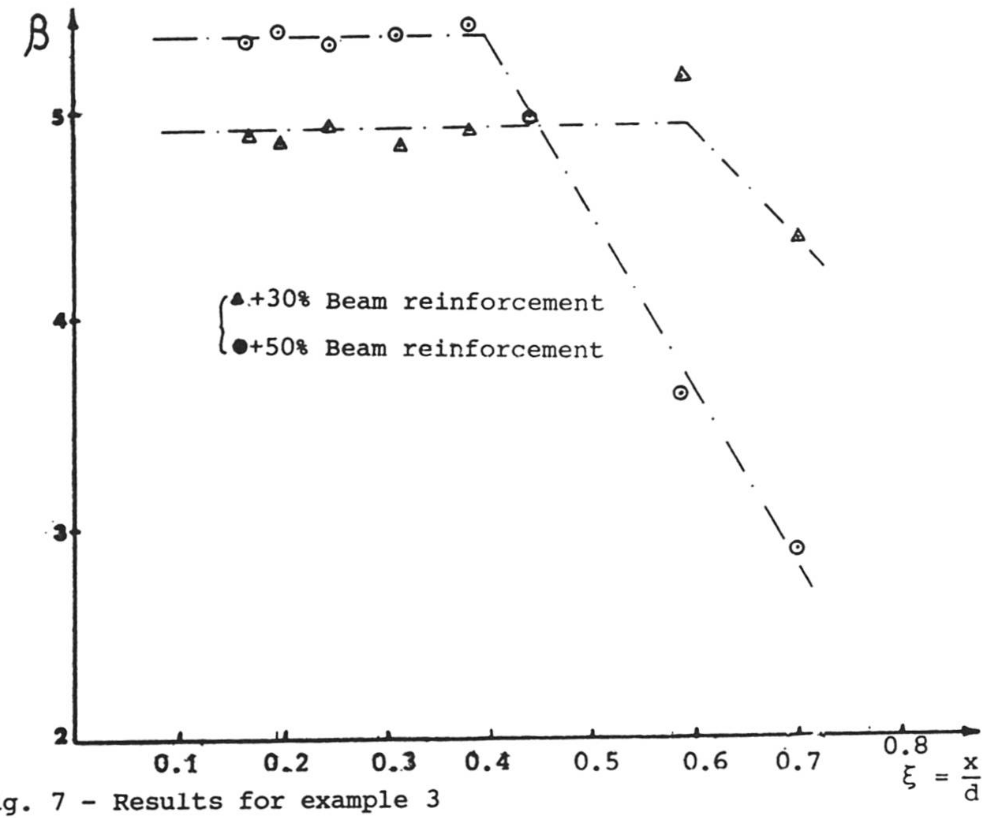

Fig. 7 - Results for example 3

lity based on design values of strength.
It is therefore not easy, at least for beams in simple flexure, using the γ_c coefficient given by Model Code, to design sections which are so brittle to produce a sizable reduction of safety index β.
Much smaller seems to be the influence on $\Delta\beta$ of γ_f coefficients: in the first example two different values of γ_f (1.5 and 1.3) were considered in design, and the maximum variation of β which was obtained in the two cases is not much different, as results from Table 2 on which these variations are reported.

γ_f	$\Delta\beta_{max}$
1.5	0.37
1.3	0.45

Table 2 - Maximum variations of β for different values of γ_f

- Variations of β corresponding to variations in reinforcement ratio are produced by two effects of opposite sign:
I-Reduction of rotation capacity with increasing ξ; this reduction obviously produces a decrease of β.
This effect is very sharp for low values of ξ, but decreases rapidly as ξ increases, following the trend of the $\theta_{lim}(\xi)$ function.
II-Increase of β produced by an increase in reinforcement ratio. As it has already been observed by Giuffrè-Pinto |13|, on the basis of "Level II" computations based on a R-S type limit state function and on linear elastic analysis, thus excluding the influence of ductility, the β coefficient increases as reinforcement ratio increases.
This phenomenon has been verified for the second example, by excluding the influence of ductility using the method, which has been described in Chapter 4.
The increase of safety level seems to be based on the fact that this level is raised when the number of random variables influencing the resistance R increases.
In the cases under consideration the influence of concrete resistance f_c on the value of the ultimate moment M_{uR} increases when increasing the reinforcement ratio.
As a conclusion the two factors affecting safety seem to interfere in the following way:
- For low values of ξ the first factor, dependent on rotation capacity, is prevalent and therefore the curve of β decreases.
- By increasing ξ the influence of the first factor is gradually reduced until the second factor becomes prevalent and the curve of β starts to increase.
The presence of the second factor seems thus to reduce the variation of β, when the ductility is changed by modifying the reinforcement ratio.
Different considerations must be made about example 4, in which column ductility is gradually reduced by increasing a deterministic axial load.
In this cases the ultimate limit state can be reached in two ways, in the sense already specified in Chapter 3:
a) Failure occurs because a failure mechanism forms in the beam.
b) A "local" failure occurs, because the limit rotation is reached in the upper critical section of the column.
Case a) is by far the most frequent and always occurs when the beam reinforcement is designed according to a linear elastic analysis, even for high column brittleness.
In all these cases, column ductility does not influence the safety level β, which remains about the same. If, on the other hand, beam reinforcement is overdimensioned, when the column is brittle, a "local" failure in the column does really occur.
In these cases,by increasing column brittleness, a sharp reduction of β is obtained as can be seen on Fig.7.
These situations can be dangerous, although they arise in very particular situa-

tions, in which axial load on column is very high, while the beam design has not been correctly executed.

- Influence of application of "ductility rule" on safety level.

As already stated, to verify the validity of this rule, which is given by the CEB Model Code, and which is extensively justified in Ref.|15|, reinforcement distributions based on this rule have been considered for examples 1 and 2. It is interesting to notice that the β values which were obtained for "redistributed" designs are in practice the same as in the "non redistributed" cases. It may be therefore concluded that application of the rule has no influence, for cases of this type, on safety level.

7. FINAL CONCLUSIONS

Within the limits of the performed reliability tests, specified in chapter 4, the following final conclusions can be drawn, which are relevant from the point of view of the practical design:

- Design of r.c. continuous beams at Ultimate Limit State, according to a linear analysis (with or without redistribution of moments according to the ductility rule), using the partial safety coefficients specified by CEB Model Code, seems to be sufficiently safe even in the case of abnormal brittleness of the beams, determined by a high tensile reinforcement ratio.
- The same design can lead to unsafe structures in the case of frames with brittle columns and overreinforced beams.

In fact, in these situations, load failures can occur in columns, to which sharp reductions of the safety indexes correspond.

These situations are therefore in any case to be avoided in design.

8. ACKNOWLEDGEMENTS

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APPENDIX 1

RANDOM VARIABLE	CHARACTERISTIC VALUE	MEAN VALUE	STANDARD DEVIATION	VARIATION COEFFICIENT	TYPE OF DISTRIBUTION
COVER d	0.012	0.02	0.005	0.25	NORMAL
STEEL STRENGTH f_{ys}	4100	4300	120	0.03	NORMAL
CONCRETE STRENGTH (compression) f_c	400	485	48.5	0.1	LOGNORMAL
CONCRETE STRENGTH (tension) f_{ct}	44	34	8.5	0.25	LOGNORMAL
LOAD P	VARIABLE	VARIABLE	VARIABLE	0.2	EXTREME TYPE

TABLE 1 - Distribution of random variables (cover in m, strength in MPa x 10)