Considerations to the design of prestressed concrete bridges

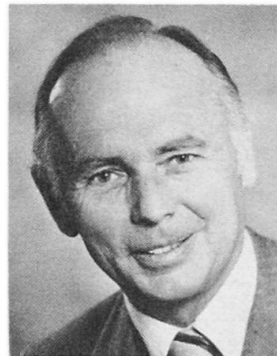# Considerations to the Design of Prestressed Concrete Bridges

Aspects du projet de ponts en béton précontraint

Zum Entwurf von vorgespannten Betonbrücken

Bruno THÜRLIMANN
Prof. Dr.
Swiss Federal Institute of Technology
Zurich, Switzerland

Bruno Thürlimann obtained a diploma in Civil Engineering from the Swiss Federal Institute of Technology in 1946 and a PhD degree from Lehigh University, Bethlehem, USA, in 1951. He did research work on the application of the theory of plasticity to the design of steel structures at Lehigh University. Since his return to Switzerland he extended this research to reinforced concrete, prestressed concrete and masonry structures. In 1977 he was elected president of IABSE.

SUMMARY

Regarding partial prestressing as a continuous transition between reinforced and fully prestressed concrete general guidelines for an appropriate choice of the degree of prestress are given. Using two continuous girder bridges as practical examples considerations on the selection of the cross sectional shape, the support system and the design of the columns are presented.

RÉSUMÉ

La précontrainte partielle peut être considérée comme une variation continue entre le béton armé et le béton totalement précontraint. Des directives permettent le choix judicieux du degré de précontrainte. A l'aide de deux cas pratiques de ponts continus, à caisson, l'auteur commente le choix de la section transversale du pont, du système d'appuis et du dimensionnement des piles.

ZUSAMMENFASSUNG

Ausgehend von der Auffassung, dass die partielle Vorspannung einen kontinuierlichen Übergang von Stahlbeton zum voll vorgespannten Beton erlaubt, werden generelle Gesichtspunkte für eine zutreffende Wahl des Vorspanngrades angeführt. Am Beispiel von zwei Balkenbrücken mit Durchlaufträgern werden Überlegungen zur Wahl der Querschnittsform, des Lagerungssystems und der Bemessung der Pfeiler gemacht.

The decisive progress in concrete bridge engineering over the past thirty years is mainly due to two developments:

1. The innovative idea of prestressing, and
2. The development of new construction equipment and erection techniques.

1. PRESTRESSING

In table 1 an estimate of the consumption of post-tensioning steel in some of the industrialized countries of the Western World for the year 1980 is presented. The total tonnage for the Western World amounts to about 250'000 tons. From Table 1 it appears that the cement as well as the prestressing steel consumption per capita vary considerably from country to country. Whereas the cement use is a reliable index for the construction market in a country the prestressing steel consumption reflects to a certain extent the state of development and application of the prestressing technique. Obviously, real possibilities for an increased use of prestressing still exist even in highly industrialized countries.

The essential features of prestressed concrete can be demonstrated with the following simple example of a tension member, Fig. 1 [1]. Such a member can also be regarded as the tension flange of a beam. The latter will exhibit essentially the same behaviour.

	Population $\times 10^6$	Cement kg/capita	Post T. Steel kg/capita
Australia	14.0	404	0.39
France	53.1	524	0.17
Germany (FRG)	61.4	528	0.57
Japan	114.3	707	0.26
Switzerland	6.3	667	1.00
United Kingdom	55.8	256	0.11
USA	216.8	300	0.22

Table 1: Cement and Post-Tensioning Steel Consumption in 1980.

The geometry as well as the material properties of the concrete, the ordinary steel and the prestressing steel are listed in Table 2. Taking the yield force of the total reinforcement constant

$$T_y = A_p \, \sigma_{py} + A_s \, \sigma_{sy} = \text{constant}$$

the four cases, listed in Fig. 1 are investigated.

1. Reinforced Concrete, Ordinary Steel:

$$A_s = T_y/\sigma_{sy}$$

2. Reinforced Concrete, Prestressing Steel, No Prestress:

$$A_p = T_y/\sigma_{py}$$

3. Full Prestressing, Prestressing Steel, Post-Tensioned

$$A_p = T_y/\sigma_{py}$$

4. Partial Prestressing. Mixed Reinforcement:

Half of yield force provided by ordinary, other half by prestressing rein-forcement:

$$A_s = 1/2\ T_y/\sigma_{sy}$$

$$A_p = 1/2\ T_y/\sigma_{py}\ \ (Post\text{-}Tensioned)$$

Prestressing cable post-tensioned:

$$\sigma_p(P) = 1170\ N/mm^2$$

The degree of prestress is defined as the ratio

$$\lambda = \frac{A_p\ \sigma_{py}}{A_p\ \sigma_{py} + A_s\ \sigma_{sy}} = 1/2$$

Fig. 2 shows the result of the analysis. The tension stiffening effect of con-crete, i.e. the tensile resistance of the concrete between the cracks has been neglected. The figure shows in non-dimensional form the tensile force T versus the axial strain ε of the tensile member.

Concrete:	. Tensile Strength	σ_{ct} = 2.0 [N/mm²]
	. Cracking Strain	ε_{cr} = 0.05·10⁻³
	. Modulus of Elasticity	E_c = 40 [kN/mm²]
Steel:	. Yield Stress	σ_{sy} = 400 [N/mm²]
	. Yield Strain	ε_{sy} = 2·10⁻³
	. Modulus of Elasticity	E_s = 200 [kN/mm²]
Prestressing Steel:	. Yield Stress	σ_{py} = 1600 [N/mm²]
	. Yield Strain	ε_{py} = 8·10⁻³
	. Modulus of Elasticity	E_p = 200 [kN/mm²]

Table 2: Material Properties for Example of Fig. 1.

In case 1 - Reinforced Concrete with ordinary steel A_s - the initial stiffness of the member is considerably reduced at the cracking load T_{cr} to the stiffness of the steel reinforcement. The yield load is reached for $\varepsilon = \varepsilon_{sy} = 0.2$ %.

If prestressing steel A_p without prestress is used - case 2 - the cross sectional steel area A_p is only $1/4$ of A_s, the yield ratios being $\sigma_{sy}/\sigma_{py} = 1/4$. Accordingly, the fall-off in stiffness after cracking is four times larger and the yield force is reached at $\varepsilon_{py} = 0.8$ %. Obviously, high tensile steel cannot be used as a reinforcement without prestress. The tensile strain after cracking and hence the width of cracks would be excessive in comparison with ordinary reinforcement.

In case 3 - Full Prestressing - the reinforcement consisting of prestressing steel only is post-tensioned to a stress equal to about the difference between the yield stresses of the prestressing and the ordinary steel.

$$\sigma_p(P) \cong \sigma_{py} - \sigma_{sy} = 1600 - 400 = 1200 \text{ N/mm}^2$$

In Fig. 2 about 3/4 of the yield strain of the prestressing reinforcement are moved to the left of the zero point of the concrete strain. At $T \cong 0.75 \, T_y$ the concrete is decompressed, $\varepsilon_c = 0$, the corresponding prestressing steel strain is

$$\varepsilon_p = \varepsilon_p(P) + \varepsilon_p(D) \cong 0.6 \text{ %}$$

or equal to the post-tensioned strain $\varepsilon_p(P)$ and the strain $\varepsilon_p(D)$ caused by decompression. The latter is always a small, almost negligible quantity. After cracking near T_y the fall-off in stiffness occurs and the yield force is reached for a strain ε of the member nearly equal to $\varepsilon_{sy} = 0.2$ %.

As case 4 - Partial Prestressing - with the degree of prestress $\lambda = 1/2$ is illustrated. Again the prestressing reinforcement is prestressed to its full stress $\sigma_p(P) \cong 1200$ N/mm^2. Cracking occurs at about $0.5 \, T_y$. After a fall-off in stiffness the yield force is reached at $\varepsilon = \varepsilon_{sy} = 0.2$ %.

The example shows clearly that:

1. Fully prestressed concrete and reinforced concrete are the two extremes of the whole field of partially prestressed concrete, $1 \leqslant \lambda \leqslant 0$.

2. If prestressing steel is used it should be stressed to a high value, i.e. $\sigma_p(P) \cong \sigma_{py} - \sigma_{sy}$. In this way the yield force of any reinforcement - full prestress, $\lambda = 1$, partial prestress with mixed reinforcement $1 < \lambda < 0$, and ordinary reinforcement, $\lambda = 0$, will be reached for a tensile strain of the ordinary reinforcement and the concrete of $\varepsilon_s = \varepsilon_c = \varepsilon_{sy} = 0.2$ %.

3. The cracking force depends on the tensile strength of the concrete and the degree of prestress. It can be set at any desired limit by varying the degree of prestress λ.

4. Partial prestress exhibits a deformation behaviour which lies between full prestress, $\lambda = 1$, and reinforced concrete, $\lambda = 0$. With a reasonable disposi-

tion of the ordinary steel a cracking behaviour can be obtained which is
superior to that of reinforced concrete.

5. At ultimate load reinforced concrete, prestressed concrete and partially
   prestressed concrete reach their limit at strains in the concrete and the
   ordinary steel which are equal for all cases. Hence, a unified design approach
   for all cases is warranted.

Obviously, partial prestressing allows a much broader use of prestressing. It
gives the designer the freedom to select any combination of ordinary and
prestressing reinforcements appropriate to the technical and economical require-
ments of a particular design [2]. The following points may be useful as a guide-
line to arrive at an adequate selection:

1. In designing a concrete structure the use as well as the degree of prestress-
   ing λ should not be decided at the outset.

2. Generally, a design is governed by the ultimate limit state. Besides the con-
   crete dimensions it determines the required yield forces of the bending,
   shear, torsion, etc. reinforcements.

3. The decision to split up the yield forces into prestressing reinforcement
   and ordinary reinforcement is governed by the following criteria:

   3.1 Deflection Control:
       Partial prestressing allows any desired deflection control irrespective
       of the dead to live load ratio.

   3.2 Use of High Tensile Steel:
       Prestressing Steel, having about a four times higher yield strength than
       ordinary reinforcement, reduces the reinforcement volume correspondingly.

   3.3 Improvement of the State of Stress:
       Properly placed prestressing cables reduce the shear and torsional forces
       acting on the concrete. Local conditions at points of high load transfers
       can be favorably influenced.

   3.4 Improvement of Structural Details:
       The reduced steel areas, concentrated in a few cables, allow a better
       detailing of sections. Splicing of reinforcing bars in tower structures,
       reactor containement vessels, bridges, can be avoided by the use of
       prestressing cables. In the tight situations requiring
       large anchorage forces such as cross girders of bridges, transfer gir-
       ders and highly concentrated loads, the anchors of cables allow often
       clearer and more compact details.

   3.5 Improved Corrosion Protection:
       Special conditions may require an improved crack control or even the
       elimination of cracking. Taking as an example a bridge, an additional
       partial prestress in the transverse direction will often lead to better
       results than a full prestress in the longitudinal direction only. Partial
       prestressing with a reasonable disposition of the ordinary reinforcement
       to cover secondary stresses due to temperature and/or volumetric changes
       leads often to the soundest solution.
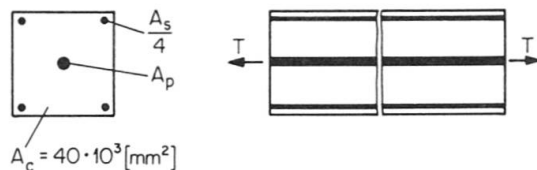
Constant Yield Load T_y: $T_y = A_p \cdot \sigma_{py} + A_s \cdot \sigma_{sy} = 800$ [kN]

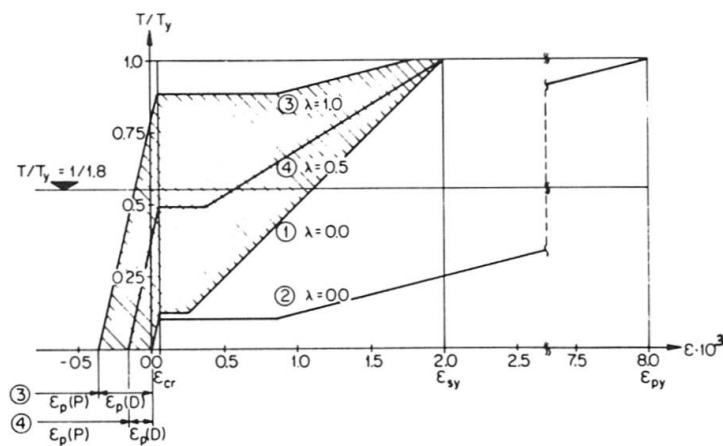| Case	A_s [mm²]	A_p [mm²]	Prestress A_p $\sigma_p(P)$ [N/mm²]	λ	Legend
①	2000	0	0	0	Reinforced Concrete
②	0	500	0	0	Reinforced Concrete
③	0	500	1170	1.0	Full Prestress
④	1000	250	1170	0.5	Partial Prestress

Fig. 1: Tension Member, Data
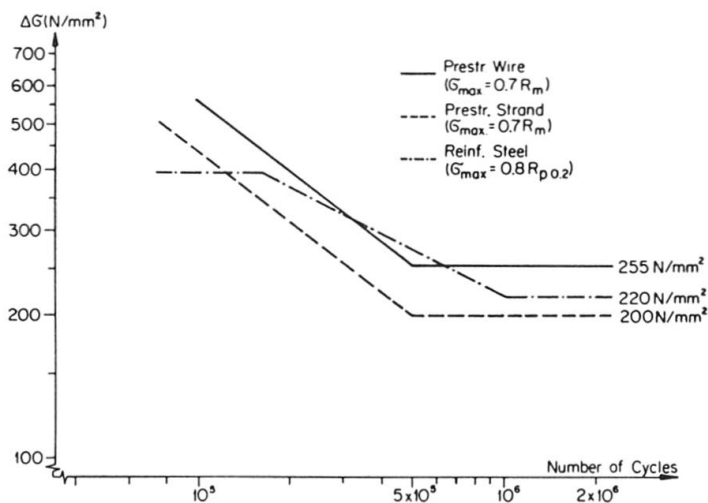

Fig. 2: Tension Member, Load vs. Elongation

Fig. 3: Fatigue Strength of Reinforcing and
        Prestressing Steels

3.6 Fatigue Consideration:

By limiting the stress variations $\Delta\sigma$ in the reinforcement and the cables
a prescribed fatigue limit can be obtained. If the stress variation is
kept below the endurance limit - safe limit $\Delta\sigma \leqslant 150$ N/mm^2 - fatigue
can be completely eliminated (Fig. 3).

4. The replacement of ordinary steel reinforcement by prestressing cables re-
sults often in direct cost savings. Stress-wise the ratio of the yield
stresses of prestressing wires and ordinary reinforcement is

$$\frac{\sigma_{py}}{\sigma_{sy}} \cong \frac{1600}{400} \cong 4$$

On the other hand, the ratio per unit weight of prestressing cables (including
placing, grouting and finishing) and ordinary reinforcement (placed, ready
to pour) has diminished to

$$\frac{\text{Price Prestressing Steel}}{\text{Price Ordinary Reinforcement}} = 3 \text{ to } 4$$

Obviously, this ratio may vary considerably with regional economic conditions,
size and type of structures, length and size of cables, etc.

5. In general, if prestressing reinforcement is used it should be substantial
in order to achieve economical advantages. As a first assumption, balancing
of the moments due to dead and permanent loads by prestressing provides
a good start. Additional considerations as indicated above may then modify
this choice somewhat.

It is with such an attitude and orientation that in the future the use of
prestressing in reinforced concrete bridges and structures should be considered.

2. CONSTRUCTION TECHNIQUES

Concrete bridges have usually been cast in place on fixed falsework in timber
and/or steel. In order to substantially reduce the relatively high cost of the
falsework in comparison with the total cost of a bridge, new erection techniques
have been developed and successfully applied. Today, it is increasingly recognized
that the construction method has definite and decisive influences on the concep-
tion, the statical system and the design of a bridge [3].

3. ILLUSTRATIVE EXAMPLE: GATEWAY BRIDGE

As an illustrative example of a widely used bridge type the Gateway Bridge in
Brisbane, Australia, will be presented. It will offer the possibility to add
remarks on the general conception of concrete bridges.

The Gateway Bridge, presently under construction, will be the longest span
girder bridge with a main span of 260 m. Figs. 4 and 5 show the central part
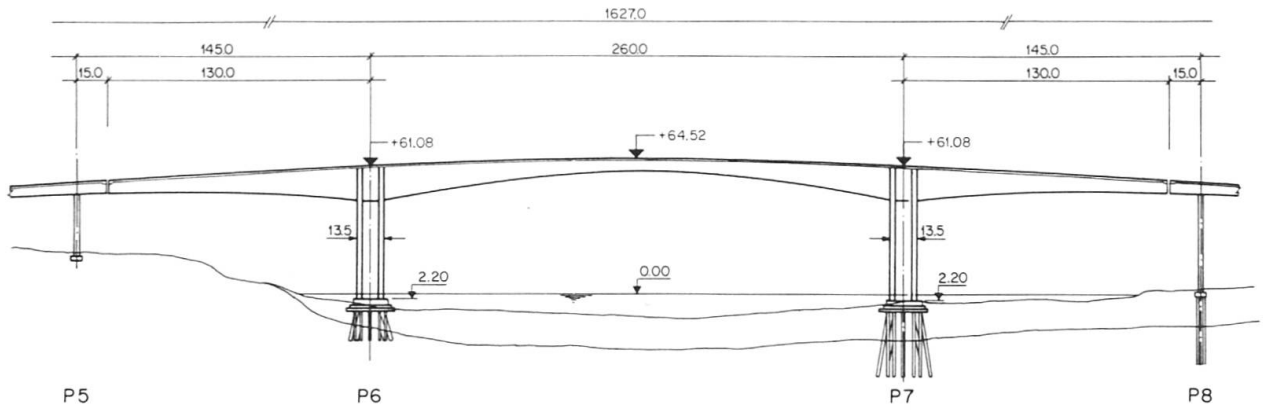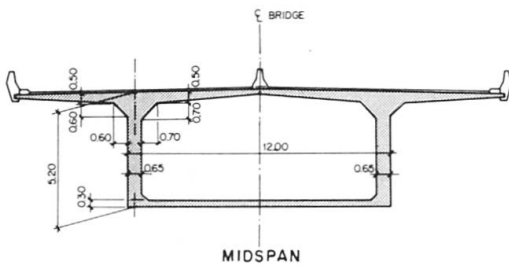
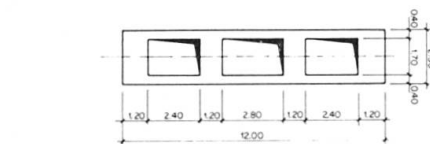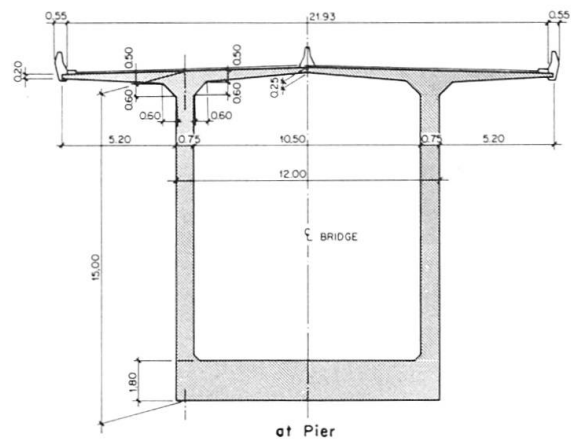Fig. 4: Gateway Bridge (Brisbane, Australia)
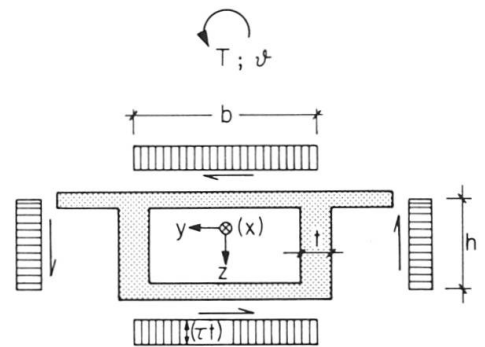
Fig. 5: Gateway, Cross Sections

Fig. 6: Gateway, Pier Section

Circulatory Torsion : $T = GK \dfrac{d\vartheta}{dx}$

Shear Flow : $(\tau t) = \dfrac{T}{2A_0}$

$A_0 = b \cdot h$

Fig. 7: Box Section

with a jointless length of 520 m and the cross sections over the piers and at
midspan. The total bridge width of 22 m is formed with a single box section.
Cross girders are only placed over the supports. The piers are resting on pile
foundations and are rigidly connected to the superstructure which in turn is
joined to the access viaducts with hinges. This floating support requires an
appropriate stiffness of the piers in order to assure the horizontal stability
of the system and to accommodate the horizontal movements of the superstructure
due to prestressing, shrinkage, creep and temperature movements. These require-
ments are achieved by piers formed of twin columns, each with a depth of 2.50 m
(Fig. 6).

The following features of this design will now be discussed in a more general
way:

1. Shape of Cross Section

2. Floating Support

3. Column Design.

4. CROSS SECTIONAL SHAPE

The tendency is toward functional, efficient and simple cross sections. Fig. 5
shows a typical example. The bridge slab over the entire width forms the top
flange of a single cell box section. Statically, such a section, Fig. 7, is
also very effective in resisting torsional moments due to eccentric loads or a
curved girder axis. The circulatory torsion resistance is dominant over the
warping resistance such that the latter is usually neglected [4].

The question of transverse bending and of the necessity of diaphrams to main-
tain the cross sectional shape is treated in Fig. 8. Considering an eccentrical-
ly applied line load $\bar{F}$, a piece of unit length $\Delta x = 1$ is taken as a free body.
The transverse forces acting are besides $\bar{F}$ the forces resulting from the shear
stresses τ at the section x and $(\tau + d\tau/dx)$ at the section $(x + \Delta x)$. As the
τ's are cancelling each other the resulting specific shear forces are due to
$d\tau/dx$ caused by the specific shear $\bar{V} = dV/dx$ and the specific torsion $\bar{T} = d\bar{T}/dx$.
For the case of Fig. 8 they are shown in (A). Together with $\bar{F}$ they form an
equilibrium system. The rest forces to be taken by transverse bending of the
cross section considered as a frame are given in (B) and the resulting trans-
verse bending moments in (C). The values in parenthesis are for the case of
equal stiffness of the top and bottom slab and the two webs respectively. Ob-
viously, if the loading $\bar{F}$ is zero the specific shear $\bar{V}$ and the specific torsion
$\bar{T}$ are also zero and hence no transverse bending will occur. On the other hand,
if a concentrated load P is applied eccentrically, then the jump in V and T at
the section is equal to P and P times the eccentricity. They in turn lead to
a jump in the resultant shear forces in the web and flange plates of the sec-
tion. For large concentrated loads P the resulting transverse bending moments
may exceed the transverse bending strength of the cross section. In this case
a cross girder is necessary to properly introduce the concentrated load, stiffen
the cross section and maintain its shape. Instead of a complete diaphram one
or two tension - compression diagonals could also be used as it is usually done
to stiffen box sections of steel bridges. Surprisingly enough such diagonals
are practically not used for concrete box sections despite of the fact

Sp. Shear: $\bar{V} = \bar{F}$
Sp. Torsion: $\bar{T} = \bar{F}b/2$

(A) Sp. Shear Forces (Shear + Torsion)

(B) Rest Forces (Eq. System)

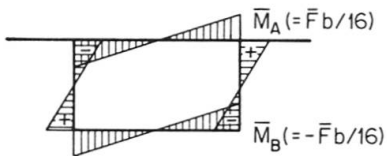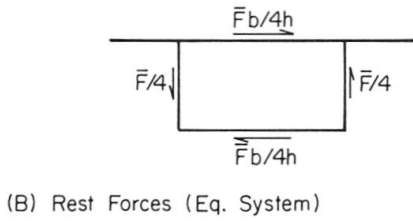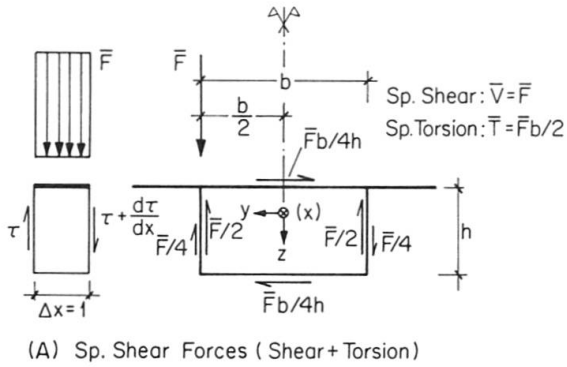$\bar{M}_A (= \bar{F}b/16)$

$\bar{M}_B (= -\bar{F}b/16)$

(C) Transv. Bending

Fig. 8: Box Section,
Transverse Bending

Warping Torsion : $T = -EI_\omega \dfrac{d^3\vartheta}{dx^3}$

Shear Stress : $\tau = \dfrac{T S_\omega}{I_\omega t}$

Normal Stress : $\sigma = \dfrac{M_\omega \omega}{I_\omega}$

Fig. 9: Open Section

(A) Temp. Movements

Stop

N_B N_C H N_D N_E

(B) System Stability
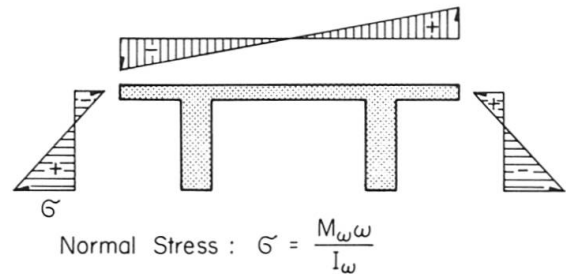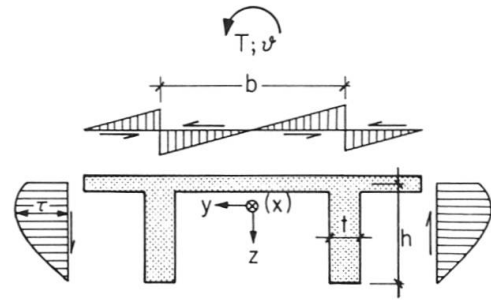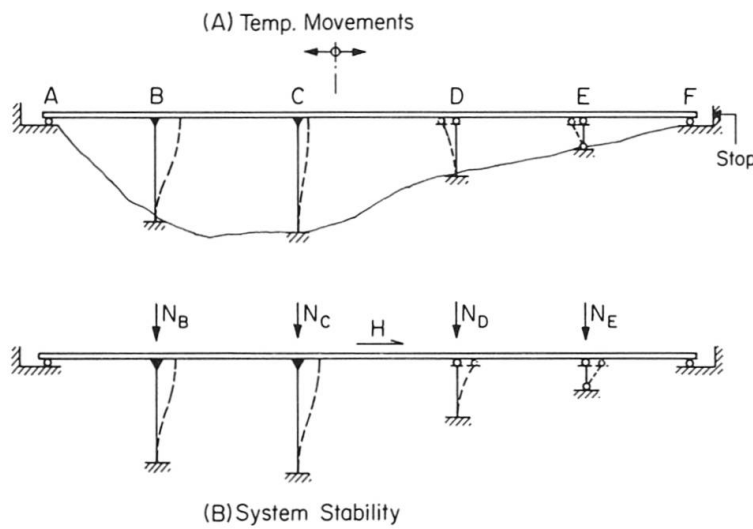
Fig. 10: Floating Support

that they would produce considerable weight and cost savings.

In the case of the Brisbane bridge it has been found that cross girders were only required for the sections over the piers and at the hinged ends.

For shorter span bridges of up to 60 m with a straight axis an open cross section, Fig. 9, may be fully adequate. Torsional effects are essentially resisted by warping torsion producing not only shear but also normal stresses.

5. SUPPORT SYSTEM

Horizontal and vertical movements in girder bridges may require joints in the bridge deck and superstructure as well as hinges and special bearings at the abutments and at intermediate supports. On the other hand, it is a common knowledge that the best joint is no joint and the best bearing is no bearing. For, the most frequent and most costly maintenance problems in bridges are caused or initiated by these two elements. It is hence one of the foremost tasks of a bridge designer to choose an adequate, simple support system with a minimum of joints and bearings.

A floating support system is illustrated in Fig.10. The continuous girder is floating on the columns without a fixed support at the abutments. In general, prestressing, creep, shrinkage and temperature variations will produce longitudinal changes of about -40 ± 25 mm per 100 m length. The center of movement for such displacements falls somewhere near the middle depending on the relative stiffnesses of the columns, Fig. 10 (A). Whereas the flexibility of the long, fixed columns B and C is sufficient, higher flexibilities for the shorter columns D and E are achieved by placing hinges or sliding bearings at the ends. On the other hand, the horizontal stability of the system has to be assured, Fig.10 (B), requiring sufficient stiffness against a horizontal load H. In order to tolerate a relatively low safety margin for horizontal stability and to provide additional safety for exceptional catastrophic loads such as earthquakes, horizontal stops at the abutments should be provided.

Using such a support concept jointless continuous girder bridges of over 500 m length have been successfully built (Fig. 4 and Fig. 16).

6. PIER DESIGN

It has just been pointed out that a detailed knowledge of the stiffness, strength and stability of reinforced concrete columns is necessary to design an appropriate support system. However, most design specifications do not treat this particular problem in an adequate manner and lead in general to unnecessary restrictions. Hence, it becomes necessary to develop a design starting from the basic material properties of concrete and steel. For a pier of the Brisbane bridge with twin shafts Fig. 11 gives the moment-curvature relationship for axial loads $P_1 = 175$ MN and $P_2 = 67$ MN for the leg 6S and the leg 6N of pier P6 respectively. The influence of creep with $\varphi = 1.5$ (creep = 1.5 times elastic strain) is also shown. Column deflection curves can then be derived. Fig. 12 shows the horizontal force H as a function of the horizontal displacement u of the pier top. In Fig. 13 the influence of an additional wind load w
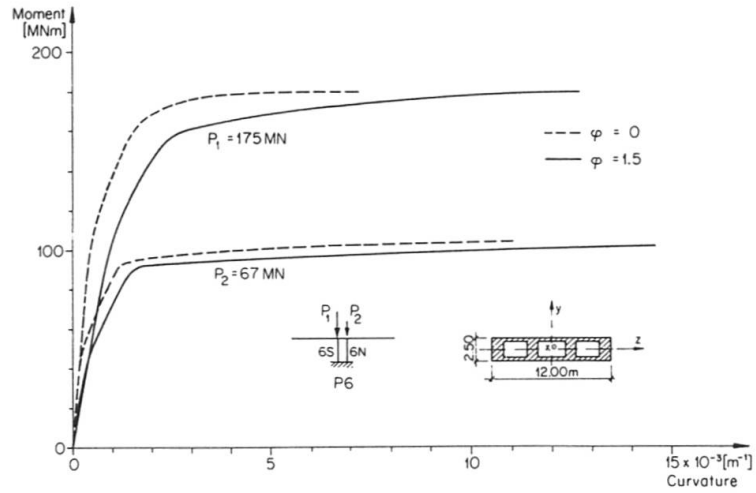
Fig. 11: Bridge Pier,
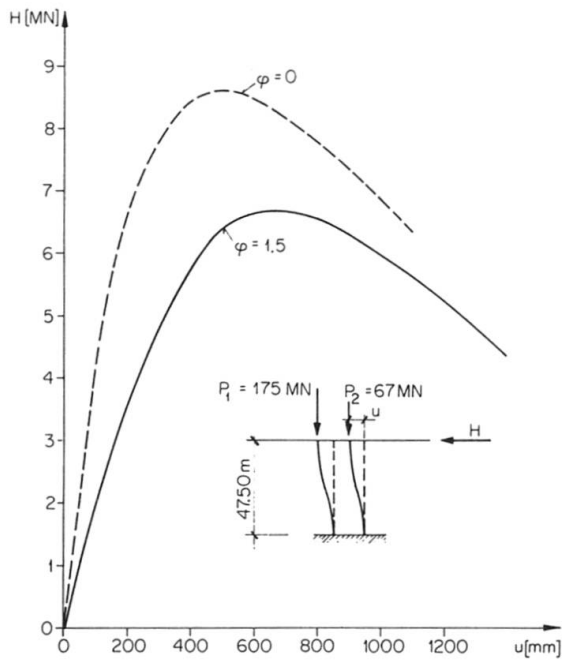Moment-Curvature Diagram
(Gateway Bridge)

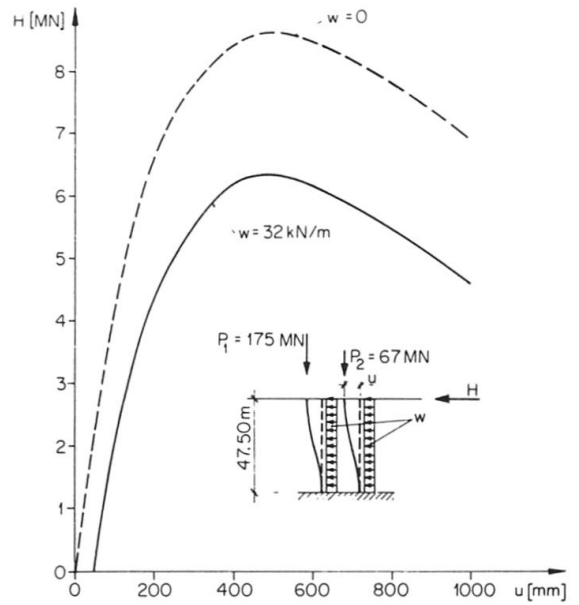Fig. 12: Bridge Pier:
Load-Displacement Diagram
(Gateway Bridge)

Fig. 13: Bridge Pier:
Load-Displacement Diagram
(Gateway Bridge)

is considered. On the basis of such curves the stability and the flexibility
of the support system of the Brisbane bridge have been judged and designed.

As indicated in Fig.10 the placement of hinges at the ends of columns may be-
come necessary. Elaborate steel and neoprene bearings have been developed and
are presently used. However, in many instances "Plastic Hinges" or "Concrete
Hinges" could accomplish the desired effect equally well or even better. Fig.14
compares a column (A) with real hinges with a column (B) with plastic hinges.
Due to a horizontal displacement c the column (A) is loaded at the top with a
driving force H destabilizing the system. On the other hand the plastic hinge
moments M_p of column (B) produce an opposing, stabilizing horizontal shear.

Again, present codes do not treat such cases, whereas experiments as well as
theoretical studies show that considerable lateral displacements c are possible
without a loss of the axial carrying capacity N if adequate detailing, i.e.
lateral confinement of the concrete in the hinge zone, Fig. 15 (A), is provided
[5], [6].

In the early age of reinforced concrete the use of concrete hinges had been
quite common. Today such hinges are generally considered to be inappropriate
for major structures such as bridges. Again, a properly designed and detailed
concrete hinge, Fig. 15(B) will offer in many cases a cheaper and maintenance-
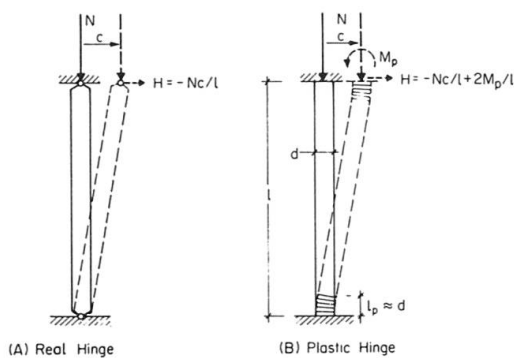free solution.

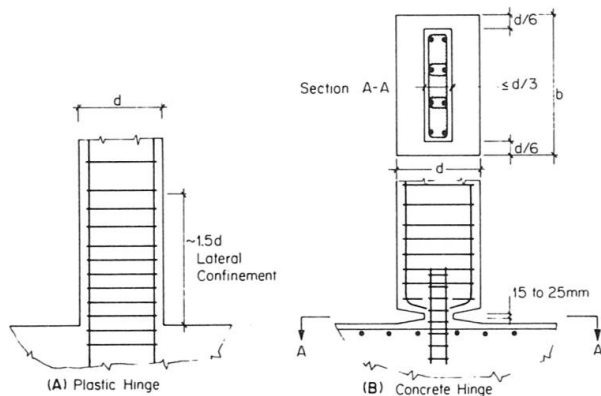

Fig. 14: Hinged vs. Fixed Column

Fig. 15: Hinges in R.C. Columns

7. SECOND EXAMPLE: BIASCHINA BRIDGE

The Biaschina bridge presently being built in Switzerland will have a main span
of 160 m. It is a twin highway bridge with a total jointless length of 645 m
(Fig. 16). The two tapered main piers D and E have a thin walled box section
and are rigidly connected with the superstructure (Fig. 17). The cross sectional
dimensions of the main girder with variable depth are shown in Fig. 18.

For the erection of the piers climbing formwork is used. The spans are con-
structed with the free cantilevering method with in place cast elements. The
prestressing cables are essentially placed and anchored in the deck slab
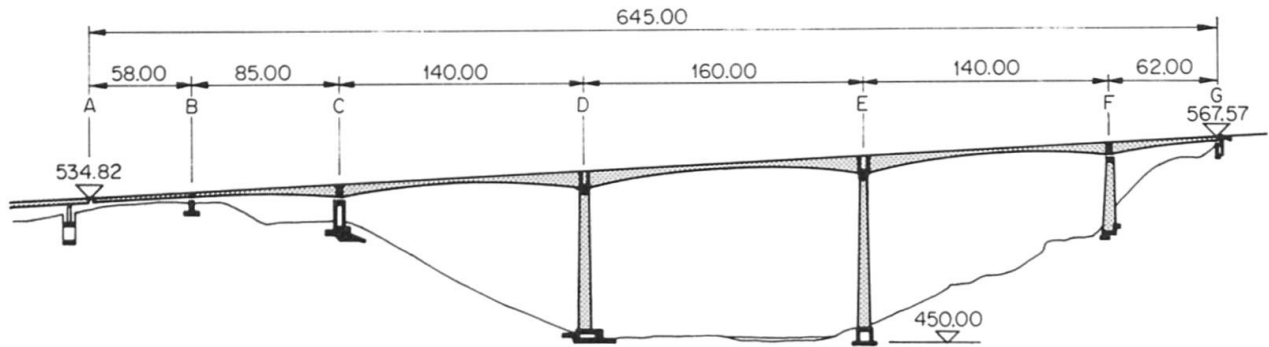(Fig. 19). Continuity at midspan is assured by some additional draped cables.

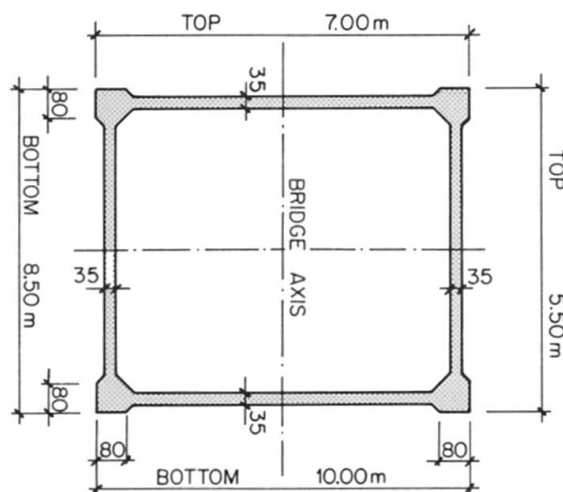Fig. 16: Biaschina Bridge (Ticino, Switzerland)
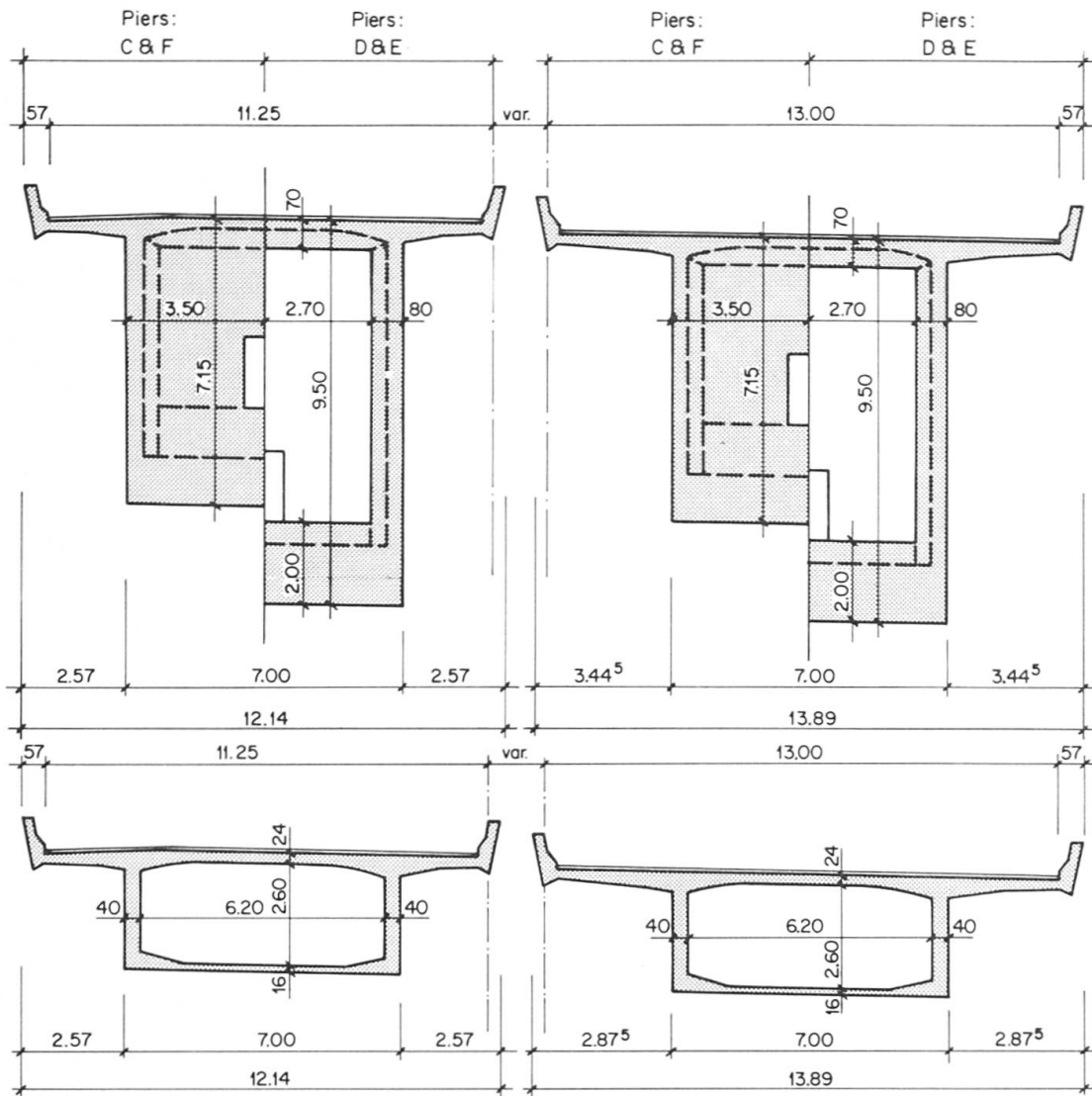

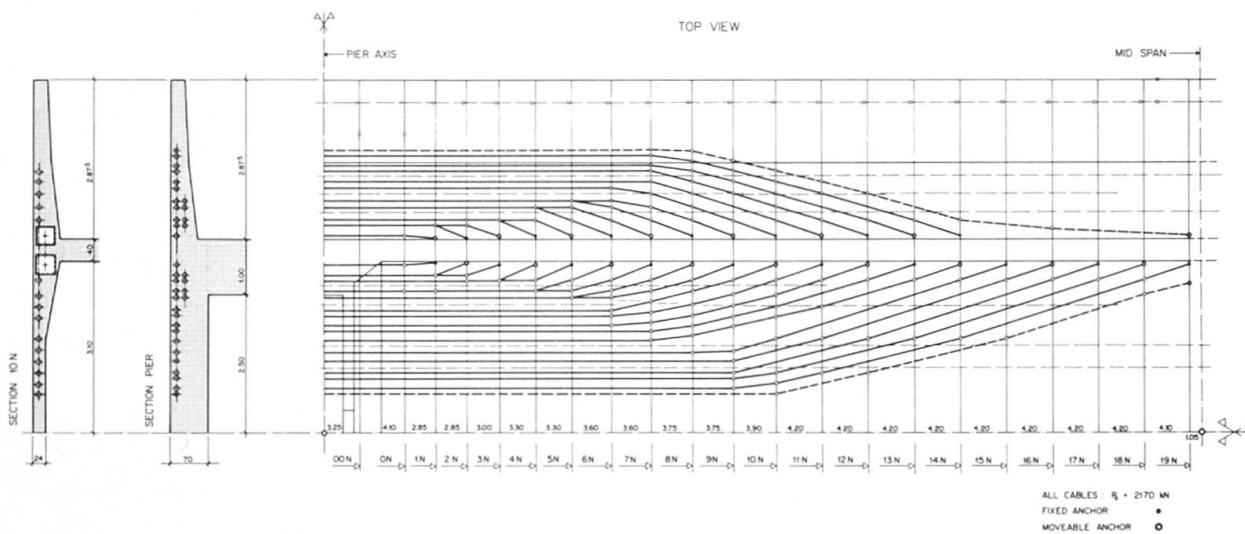Fig. 17: Biaschina, Pier Section

Fig. 18: Biaschina, Cross Section

Fig. 19: Biaschina, Cable Lay-Out

8. CONCLUSIONS

Girder bridges with spans of over 20 m are generally prestressed. Partial pre-
stressing, where part of the required yield force is provided by prestressing
cables and the remaining part by ordinary reinforcement, offers many advantages.
Deflection control is the main criteria for making the appropriate choice of
the degree of prestress λ, i.e. the ratio of the yield force of the prestressing
reinforcement to the yield force of the total (prestressed and ordinary) rein-
forcement. A reasonable disposition of the ordinary reinforcement ensures a pro-
per control of cracks from secondary stresses due to temperature and/or volumet-
ric changes. If the steel stress variation $\Delta\sigma$ due to traffic loads, is kept be-
low about 150 N/mm^2 - a condition which is generally achieved if $\lambda > 0.7$ -
no fatigue limitations exist.

The form of the cross section should be simple and efficient, providing integral
action of all parts to resist longitudinal and transverse forces. Clean and
simple geometrical dimensions are desired for a proper placement of the cables
and the reinforcement. Box sections with a single cell and a deck slab of up to
25 m width have already been used. Cross girders to stiffen the section trans-
versely should only be used when necessary.

More importance should be given to an appropriate choice of the support system.
The number of joints and bearings should be kept at a minimum as they are
the source for the most frequent maintenance and repair problems. With a float-
ing support jointless decks of over 500 m length can be realized. The ability
of R.C. columns to form plastic hinges without losing their axial strength as
well as concrete hinges offer in many cases equal or even better solutions
than elaborate and costly steel or neoprene bearings.

Even in the age of electronic computers a designer should be guided by intuition,
knowledge, experience and last but not least common sense. The critical attitude
"If something is too complicated, it cannot be right" will help him to conceive and
design a bridge which is not only statically sound but also pleasing in appearance.

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