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Autor: Narayanan, Rangachari / Der Avanessian, Norire G.V.

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Design of Slender Webs Containing Circular Holes

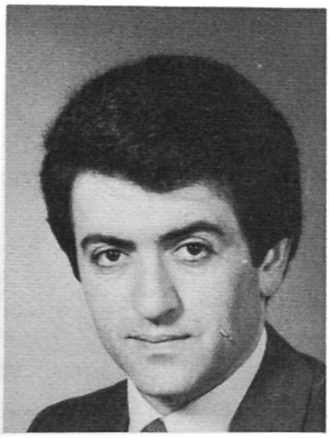
Calcul des âmes élancées présentant des ouvertures circulaires

Berechnung von schlanken Stegen mit kreisförmigen Aussparungen

R. NARAYANAN
University College,
Cardiff, U.K.

Rangachari Narayanan, born 1931, obtained his Civil Engineering Degree from Annamalai University (India) and his Doctorate from the University of Manchester. After spending over 25 years in Universities, Industry and Consulting work, he is now on the academic staff of the University College, Cardiff.

N.G.V. DER AVANESSIAN
University of Lancaster,
Lancaster, U.K.



Norire Der Avanessian, born 1954, got his Civil and Structural engineering degree in 1979 and his Doctorate in 1983 from the University of Wales. After researching into the behaviour of perforated plates for 3 years at University College, Cardiff, he is now on the staff of the University of Lancaster.

SUMMARY
The paper describes an approximate design procedure which can be used to predict the ultimate shear capacity of slender webs containing centrally placed circular holes. The ultimate shear is evaluated as the sum of the elastic critical load, the load taken by the membrane tension in the post buckled stage and the load taken by the flanges. A simplified method of designing ring reinforcement is suggested; the reinforcement so provided is adequate to restore the strength lost by cutting the hole.

RÉSUMÉ
Une méthode de calcul approchée permet de déterminer la résistance ultime au cisaillement d'âmes élancées présentant des ouvertures circulaires. La résistance ultime approchée est égale à la somme de la charge critique élastique, de la charge reprise par la membrane sous tension dans l'état de post voilement et de la charge prise par les semelles. Une méthode de dimensionnement simplifiée des renforts annulaires est proposée afin de récupérer la résistance perdue lors de la coupe des ouvertures circulaires.

ZUSAMMENFASSUNG
Dargestellt wird eine Näherungsmethode zur Erfassung des Schubwiderstandes von schlanken Stegen mit kreisförmigen Aussparungen. Dabei wird der Schubwiderstand gleich der Summe der kritischen, elastischen Schubkraft, der über Zugfeldwirkung im überkritischen Bereich und der über Flanschbiegung übertragbaren Kräfte festgelegt. Eine einfache Methode zur Bemessung ringförmiger Verstärkungen wird vorgeschlagen; durch derartige Verstärkung lässt sich der durch die Aussparungen bedingte Tragfähigkeitsverlust kompensieren.

1. INTRODUCTION

Inspection openings are frequently required in plate girder webs, diaphragms of box girders and in the floors and intercostals of ships. Current methods of estimating the design loads on such webs, based on simplified elastic analysis, are inappropriate when the ultimate limit state has to be considered.

A systematic study of the collapse behaviour of thin webs containing holes has been in progress at Cardiff since 1977, with generous financial support from the U.K. Government. The study has concentrated on thin webs, of the type used in plate girders with webs having a slenderness (h/t) of 200 to 360, and subjected to predominant shear loading. A full account of tests carried out on over 70 perforated web panels, and the suggested theoretical analysis are contained in references |1| to |5|.

It is the purpose of this paper to present an approximate design procedure, which can be used to obtain a quick estimate of the shear ultimate capacity of webs containing centrally located circular holes. If the loss of strength associated with the introduction of the hole is unacceptable, adequate reinforcement has to be provided around the hole and this has to be designed; the latter part of this paper is concerned with this aspect of design.

2. REVIEW OF PREVIOUS WORK

An equilibrium method has been suggested by the authors |2| to predict the ultimate shear capacity of a plate girder containing a centrally placed web hole and comprises of three contributions, viz.

(i) the elastic critcal load of the perforated web
(ii) the load carried by the membrane tension in the post-critical stage and
(iii) the load carried by the flange at the instant of collapse. (See Fig. 1).
The ultimate shear load (V_{ult}) can be obtained from the following equation :

$$V_{ult}=(\tau_{cr})_{mod}.h.t + \sigma_t^y.t.h\left[\frac{c}{h}\sin^2\Theta + (\cot\Theta-\cot\Theta_d)\sin^2\Theta - \frac{d}{h}\sin\Theta\right] + \frac{4M_p}{c} \qquad (1)$$

where h depth of the plate girder
t thickness of the web
c distance between the hinges formed in the flanges given by

$$\frac{2}{\sin\Theta}\sqrt{\frac{M_p}{\sigma_t^y.t}}$$

$(\tau_{cr})_{mod}$ modified critical shear stress of the perforated plate
Θ angle of inclination of the tensile membrane stress
Θ_d angle of inclination of the panel diagonal
σ_t^y post buckling membrane tension
M_p fully plastic moment of the flange

The above equation can be written in a non-dimensional form in terms of V_{yw}, the shear which would cause the entire web to yield.

$$\frac{V_{ult}}{V_{yw}} = \frac{(\tau_{cr})_{mod}}{\tau_{yw}} + \left[\sqrt{3}\sin^2\Theta(\cot\Theta-\frac{b}{h})-\sqrt{3}\sin\Theta\frac{d}{h}\right]\frac{\sigma_t^y}{\sigma_{yw}} + \left[4\sqrt{3}\sin\Theta.\sqrt{\frac{\sigma_t^y}{\sigma_{yw}}.M_p^*}\right] \qquad (2)$$

where
b width of the web plate

M_p^* flange stiffness parameter, $M_p^* = \dfrac{M_p}{h^2 t\ \sigma_{yw}}$ (3)

σ_{yw} yield stress of the web in direct tension

τ_{yw} yield stress in shear given by $\dfrac{\sigma_{yw}}{\sqrt{3}}$

V_{yw} $\tau_{yw} \cdot h.t$ (4)

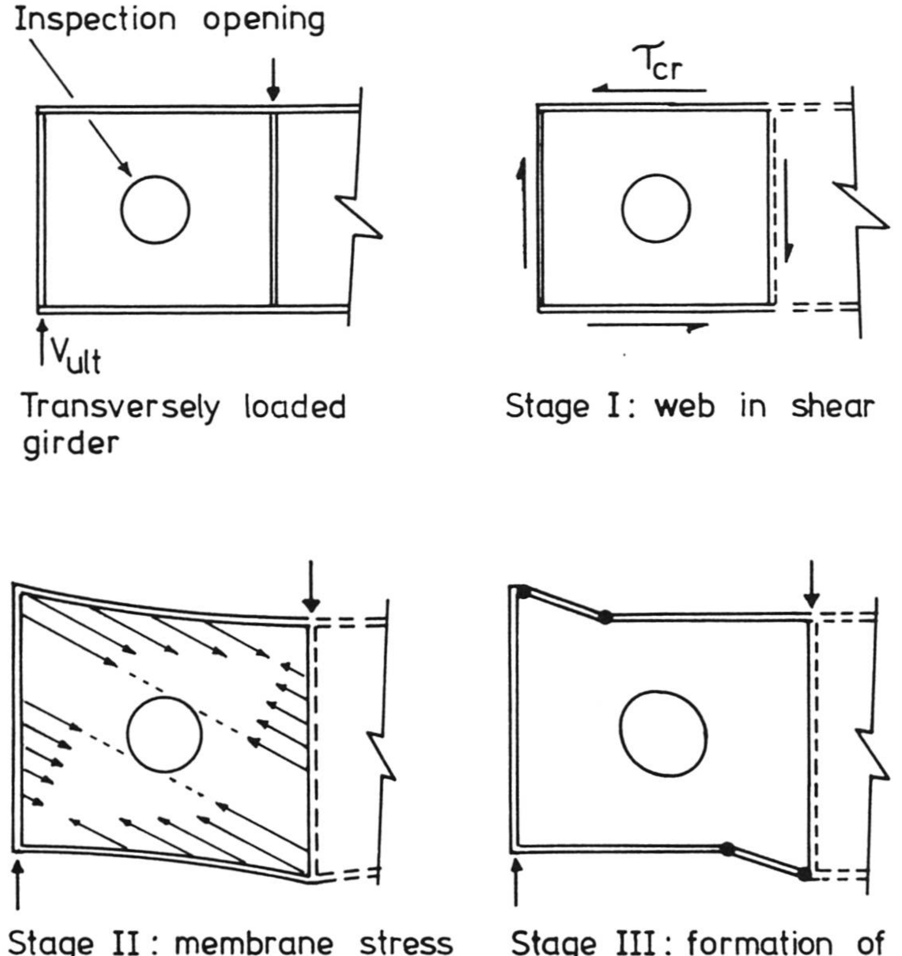


FIG. 1 FAILURE MECHANISM

Equation (2) is valid for all practical sizes of holes defined by $d/h \leq (\cos\Theta - b/h \sin\Theta)$. Holes of larger sizes are unlikely to be met in practice and are outside the scope of this paper. A method suitable for such a web is outlined in reference |2|.

The term in the first bracket in equation (2) is the component due to the buckling stress. The second term is the component due to the post buckling membrane tension supported by the flanges and vertical stiffeners; the reduction due to the presence of the hole is accounted for by the negative quantity containing d/h. The third term in the equation represents the contribution by the flanges. The tensile membrane stress (σ_t^y) is evaluated from

$$\frac{\sigma_t^y}{\sigma_{yw}} = -\frac{\sqrt{3}}{2}\frac{(\tau_{cr})_{mod}}{\tau_{yw}}\sin 2\Theta + \sqrt{\{1 + \frac{(\tau_{cr})_{mod}^2}{(\tau_{yw})^2}(\frac{3}{4}\sin^2 2\Theta - 1)\}} \quad (5)$$

The only unknown quantity in equations (1), (2) and (5) is the value of Θ; the value of V_{ult} obtained from (1) or (2) is dependent on its choice. Since this is an "equilibrium" solution, the maximum value of V_{ult} is obtained by trial and error, by varying Θ; the optimum angle to give a maximum value of V_{ult} is termed Θ_m .

Equations (2) and (5) are, therefore, adequate to obtain the ultimate shear for a web having any centrally placed circular hole.

3. APPROXIMATE EVALUATION OF $(\tau_{cr})_{mod}$

The elastic critical stress $(\tau_{cr})_{mod}$ is very small compared with σ_t^y or σ_{yw} ; any error in evaluating it by approximate methods has little effect in the calculated value of the ultimate shear. To avoid tedious computations based on finite element (or similar) methods, the authors have suggested |4| that $(\tau_{cr})_{mod}$ appropriate to the perforated web can be computed from the following :

$$(\tau_{cr})_{mod} = \kappa_o \left[1 - 1.5\frac{d}{\sqrt{h^2+b^2}}\right] \cdot \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{h}\right)^2 \quad (6)$$

where κ_o = shear buckling coefficient of the unperforated web, encastre at the edges, given by

$$\kappa_o = 8.98 + 5.6\left(\frac{h}{b}\right)^2 \quad (7)$$

Computed values of the elastic critical stress using eq. (6) have been found to agree within ± 2% of the finite element values |4|.

4. EVALUATION OF THE ULTIMATE SHEAR

A parametric study of the influence of the flange stiffness factor M_p^* , on the ultimate capacity of the perforated girder was carried out for a range of aspect ratios and web slenderness values. Figure 2 is a typical relationship obtained showing the variation of the ultimate shear and the angle of inclination of the membrane tension due to a change in the flange stiffness factor M_p^* . Two web slenderness ratios (h/t = 250 and 360) are chosen for illustration in this figure and comparisons have been made between a web with no cut-out and one with a cut-out of diameter 0.3h . The study showed that for all values of M_p^* , the value of (Θ_m) is independent of the web slenderness (h/t). For a given value of M_p^* , the use of a slender web resulted in a reduced strength compared with a stocky web; the value of Θ_m in both cases remained constant. Comparing a web containing a hole of 0.3h with an unperforated web, it can be seen that both the optimum angle and the ultimate shear are significantly reduced.

Rockey et al |6| have shown that in the case of an unperforated web, a mean value for Θ_m can be assumed to be $0.67\Theta_d$ for design purposes, without any significant loss of accuracy; this approximate value was shown by them to be valid for a wide range of values of web slenderness and flange stiffness. The parametric study referred above on the influence on Θ_m due to variations in hole diameter, flange stiffness and panel aspect ratio found that Θ_m varied linearly with the diameter and can be represented by a straight line (see Fig. 3) :

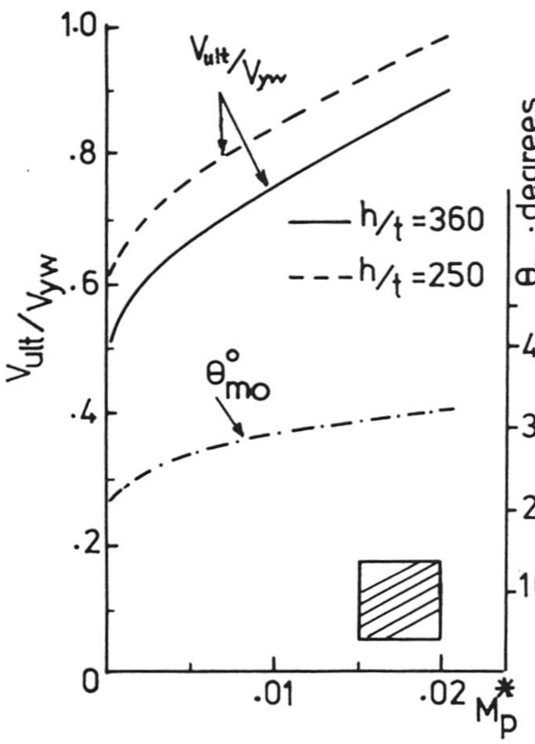


FIG. 2(a) PANEL OF ASPECT RATIO 1 WITH NO CUTOUT.

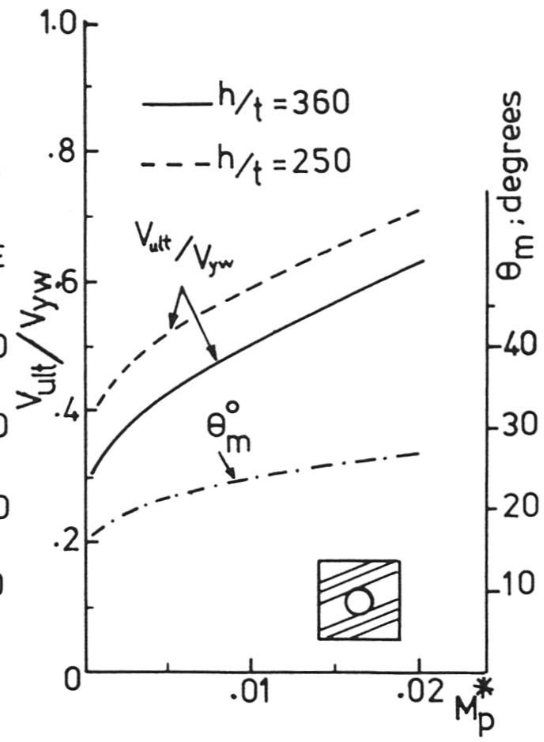


FIG. 2(b) PANEL OF ASPECT RATIO 1 HAVING A CUTOUT d/h = 0.3

$$\frac{\Theta_m}{\Theta_d} = \frac{\Theta_{mo}}{\Theta_d} - \eta_c \left(\frac{d}{h}\right) \qquad (8)$$

where Θ_{mo} = optimum angle of tension field for an unperforated web

η_c = non-dimensional constant

An average value for Θ_{mo} may be taken to be $0.67\Theta_d$ without any significant loss of accuracy.

The parametric studies referred above have shown that η_c is generally independent of web slenderness (h/t) and flange stiffness (M_p^*) for all practical girders and varies only very slightly with the aspect ratio (b/h). Based on these studies, a mean value of 0.435 is suggested for η_c for obtaining a quick estimate of Θ_m and thence V_{ult}.

Using the above concepts, V_{ult} can be evaluated in five simple steps detailed below :

(i) Calculate the elastic critical stress, $(\tau_{cr})_{mod}$ from eq. (6)
(ii) Calculate Θ_m from eq. (8)
(iii) Using the above value of Θ_m for Θ in equation (5), compute the membrane tension σ_t^Y.
(iv) Calculate M_p^* from (3) and V_{yw} from (4).
(v) Compute V_{ult} from (2).

A more exact value of V_{ult} may be obtained by trial and error, i.e. by maximising V_{ult} with respect to Θ , chosen in the region of Θ_m given from Step (ii) above. An example of the use of the above method is given in the Appendix.

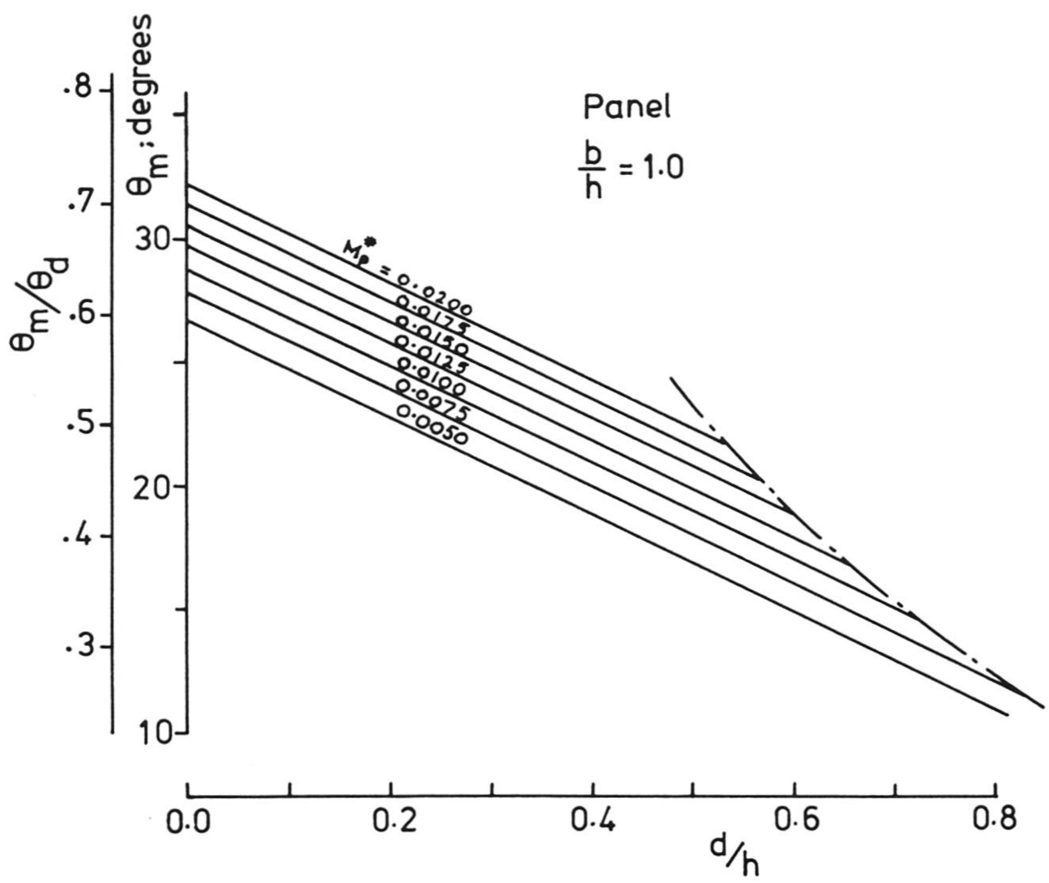


FIG. 3 VARIATION OF OPTIMUM ANGLE θ_m

5. DESIGN OF REINFORCEMENT

A well-designed reinforcement would restore the strength of a perforated web to a value obtainable from the corresponding unperforated web. An equilibrium solution for predicting the strength of webs containing reinforced circular openings is suggested in reference |5| and is summarised below :

The reinforcement to be provided is in the form of a circular ring welded to the edge of the hole. The plastic moment of resistance of a ring having a section of t_r x w_r is given by

$$M_{pr} = 0.25\ t_r \cdot w_r^2\ \sigma_{yr} \tag{9}$$

At the ultimate stage, the ring would collapse by the formation of four plastic hinges as shown in Fig. 4. If the uniform membrane tension appropriate to an unperforated web is assumed to act along the tension band of the reinforced web, the required value of M_{pr} is obtained from the equilibrium of the forces acting across a quarter of the ring (between plastic hinges) :

$$M_{pr} = \frac{\sigma_t^y \cdot d^2 \cdot t}{16} \tag{10}$$

The choice of w_r and t_r should be such that the value of M_{pr} developed should be equal to or in excess of the required value given in equation (10).

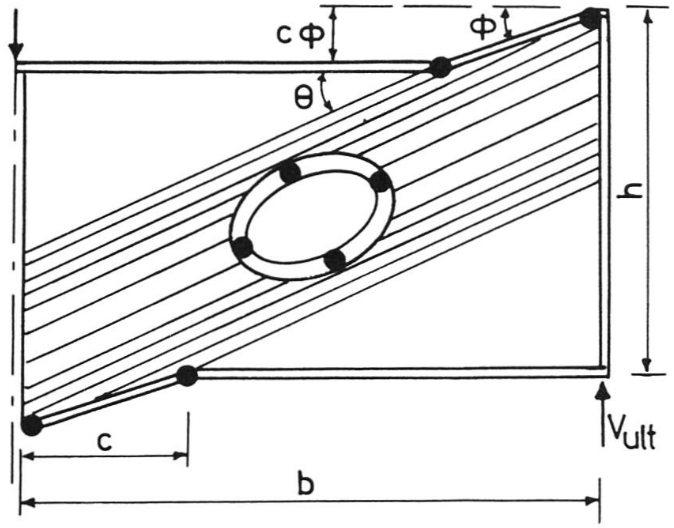


FIG.4 WEB WITH REINFORCED HOLE UNDER SHEAR LOADING

It is essential to verify that the reinforcement is capable of providing the perforated web with a value of shear buckling stress which is at least equal to that of an unperforated web. Based on studies on elastic buckling behaviour of perforated plates |4|, the following minimum requirement of reinforcement is suggested :

$$\left(1 - \frac{1.5d}{\sqrt{h^2+b^2}}\right)\left[1 + 6\left(\frac{t_r}{t}\right)^2\left(\frac{w_r}{h}\right) \cdot \frac{d}{\sqrt{h^2+b^2}}\right] \geq 1 \qquad (11)$$

The steps involved in the design of reinforcement is summarised below :

(i) Assuming that the opening did not exist, (i.e. d = 0), compute the elastic critical stress from eq. (6) and the membrane tension, σ_t^y , from eq. (5) for a full web.

(ii) Compute the <u>required</u> value of M_{pr} from eq. (10).

(iii) Choose the dimensions of the ring (w_r and t_r) such that eq. (9) is satisfied.

(iv) The adequacy of the chosen dimensions is checked against eq. (11).

An illustrative example using the above method is worked out in the Appendix.

6. CONCLUSIONS

A rapid method of assesing the ultimate shear strength of a perforated web is suggested and is based on the equilibrium solutions developed earlier. A method of designing a suitable circular ring reinforcement which would restore the strength lost due to the cut-out is proposed.

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APPENDIX

The suggested design method is illustrated using a girder with details listed below : $(b \times h \times t)$ = 1500mm x 1500mm x 6mm; $(b_f \times t_f)$ = 300mm x 30mm; d = 500mm; $\sigma_{yw} = \sigma_{yf} = \sigma_{yr}$ = 250 N/mm^2

(1) Calculate the elastic critical stress: From eq. (6), $(\tau_{cr})_{mod}$ = 27.26 N/mm^2

(2) Calculate the optimum angle, Θ_m using the approximate value of $0.67\Theta_d$; $\Theta_m = 23^o$

(3) Calculate the membrane tension σ_t^y from eq. (5): (σ_t^y/σ_{yw}) = 0.823

(4) Calculate M_p^* from eq. (3): M_p^* = 0.005

(5) Compute V_{ult} using $\Theta = 23^o$, in eq. (2) : V_{ult} = 613 kN

The cross-sectional dimensions of the reinforcement should be adequate to restore the strength of the above girder to that of an unperforated web.

(1) Assuming that the opening did not exist, (i.e. d=0), compute the elastic critical stress from (6); $(\tau_{cr})_{mod}$ = 42.2 N/mm^2

Substituting $\Theta = 0.67\Theta_d$ in (5), (σ_t^y/σ_{yw}) = 0.762

(2) M_{pr} is obtained from eq. (10); $M_{pr} = 1786 \times 10^4$ N-mm

(3) The dimensions of the ring are selected to satisfy eq. (9): $1786 \times 10^4 = 0{\cdot}25 \times 250 . t_r . w_r^2$, giving $t_r \times w_r^2$ = 285,760. Suitable values are t_r = 20mm and w_r = 120mm

(4) Finally check that inequality (11) is satisfied; this gives 1.46 > 1 . Hence the chosen t_r and w_r are adequate.