Non linear analysis of cable-stayed bridges eccentrically loaded

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Analyse non linéaire des ponts à haubans chargés en flexion et torsion

Eine nicht lineare Untersuchung von Schrägseil-Mittelträger-Brücken unter exzentrischer Belastung

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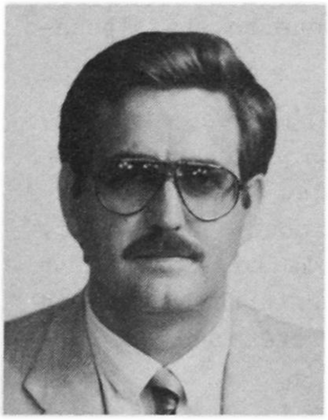

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SUMMARY

In this paper the non linear behaviour of long-span cable-stayed bridges under eccentric live loads is analyzed. Two different structural models are presented. The first is obtained by assuming a continuous distribution of the stays along the deck, the second one is a discrete model corresponding to the actual stays spacing. The numerical results obtained show the significance of nonlinear effects and the accuracy of the structural models used.

RÉSUMÉ

Cet article étudie le comportement non linéaire des ponts à haubans de grande portée chargés en flexion et en torsion. L'analyse est dévelopée, en étudiant par modèles d'une façon continue la distribution longitudinale des haubans, ainsi qu'avec l'emploi d'un modèle discret, qui prend en compte la position réelle des haubans sur la poutre. Les résultats numériques montrent l'importance des effets non linéaires et l'efficacité des modèles employés.

ZUSAMMENFASSUNG

In dieser Arbeit wird das nicht lineare Verhalten von Schrägseil-Mittelträger-Brücken unter exzentrischer Belastung untersucht. Die Untersuchung erfolgt einerseits unter Benützung eines Modells mit kontinuierlicher Verteilung der Seilabspannung längs der Brückenachse und anderseits eines diskreten Modells, das die tatsächliche Verteilung der Seilabspannung längs der Brückenachse berücksichtigt. Die auf diese Weise erhaltenen numerischen Resultate zeigen die Bedeutung der nicht linearen Einflüsse und die Effizienz der angewandten Modelle.

1. INTRODUCTION

The considerable progress in the field of structural engineering, material technology and methods of construction has brought about great interest in the study of long-span cable-stayed bridges. Therefore, it's a common view to consider the cable-stayed bridge scheme an efficient alternative to the suspension bridge. Basic studies about fan-shaped cable-stayed bridges are given in [1-3].

Usually, the investigation methods about the statical behaviour of this scheme are based upon a linear analysis, by using Dischinger's fictitious tangent modulus for the stays. Consequently, torsion and vertical bending are examined separately. This approach is unsuitable for bridges whose central span is longer than 700-800 mt. In this case, in fact, a non-linear analysis, for a more accurate evaluation of some significant effects, is required. Recent contributions about non linear behaviour of cable-stayed bridges are given in [4-7]. In particular, the influence of non-linearities, due to stays'behaviour and geometry changes, is shown in [6-7].

A numerical and analytical investigation is developed in the present work, including the main nonlinearity due to the stays'behaviour. Thus, torsion and vertical bending of the girder are examined in a coupled nonlinear analysis. At the beginning the analysis is made by using a continuous model, already used in [3,6,7]. Solutions, approximate but suitable to account for the stays'main non linear effect are obtained by using a perturbative method of solution. Then, the analysis goes on by using a discrete model of the bridge, corresponding to the actual stays' spacing.

2. STRUCTURAL SCHEME AND STATICAL BEHAVIOUR

The structural scheme we are going to examine is shown in Fig. 1: the stays are characterized by a fan-shaped arrangement, the pylons are composed of two parallel independent towers and the deck is supported by the stays.

The statical behaviour of such scheme is based on two typical characteristics [3,6,7], namely:

- diffused stay arrangement along the deck ($\Delta/L \ll 1$)
- truss-like statical behaviour

Under dead loads g, according to the commonly erection methods used, the girder has a straight configuration and is free from bending moments. So that the stresses are given only by axial forces both in the stays and in the deck. When live loads are applied, deformation and stress increase will be analyzed by taking into account the non-linearity due to the stays' behaviour.

We assume that the whole central span is under a uniform live load (Fig. 1):

$$p^*(\lambda) = \lambda p \qquad (2.1)$$

where λ is the load parameter and p is the design value of the live loads.

This load condition is interesting because in such case the maximum torsional

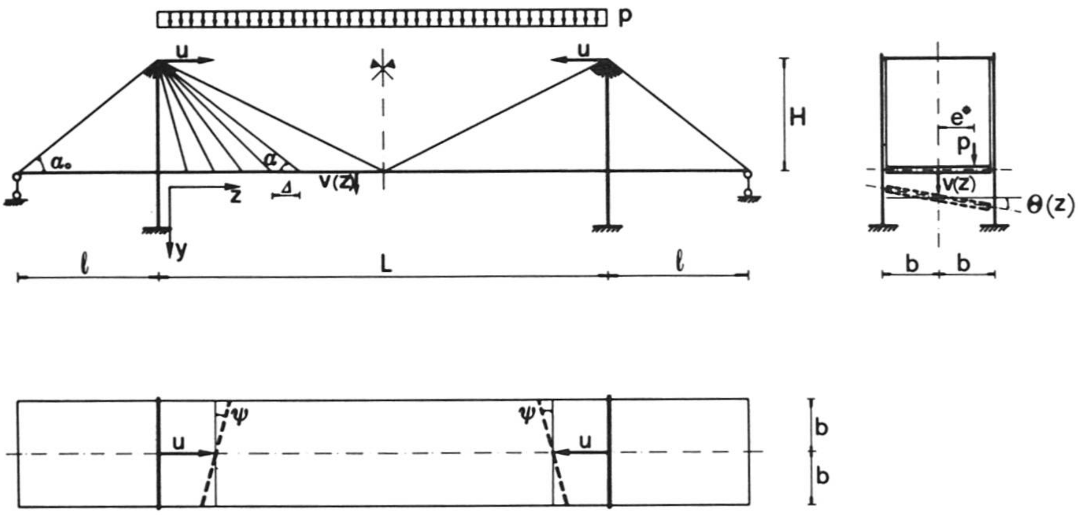

Fig. 1 Structural scheme

rotation and the maximum deflection occur at midspan.

Thus, if the extensional and shear deformabilities of the girder and pylons are neglected, bridge's additional deformation is determined by the following displacement parameters (Fig. 1):

- the girder's vertical displacements $v(z)$
- the pylon tops' horizontal displacement u
- the girder's torsional rotations $\theta(z)$
- the pylon tops' torsional rotation ψ

Before analyzing the non linear elastic response of the bridge under live loads, we shall examine the behaviour of a single stay. Let us consider a generical stay anchored to the left pylon; the deformation increment $\Delta\varepsilon$ produced by v, u, θ, ψ displacements is:

$$\Delta\epsilon = \frac{1}{H}\left[(v \pm \theta b)\sin^2\alpha - (u \pm \psi b)\sin\alpha\cos\alpha\right] \tag{2.2}$$

where + or - signs respectively apply to the left or right stay in respect to the pylons' vertical axis. The $\Delta\sigma$ stay tension increment can be evaluated as:

$$\Delta\sigma = E_s^* \Delta\epsilon \tag{2.3}$$

where E_s^* is the secant modulus of the stay's $\sigma-\varepsilon$ relationship. The tangent and secant moduli are defined in Fig. 2. According to Dischinger's theory these moduli are given by:

$$E_t^* = \frac{E}{1 + \dfrac{\gamma^2 \ell_0^2 E}{12\sigma_0^3}} \quad , \quad E_s^* = \frac{E}{1 + \dfrac{\gamma^2 \ell_0^2 E}{12\,\sigma_0^3}\,\dfrac{1+\beta}{2\,\beta^2}} \tag{2.4}$$

where β represents the ratio between the final value of the stays' tension $\sigma=\sigma_o+\Delta\sigma$ and the initial one σ_o: $\beta=\sigma/\sigma_o$; E is Youngs' modulus, γ the specific weight, ℓ_o the horizontal projection length of the stay.

The cross sectional areas A_s and A_o of the double curtain of stays and of

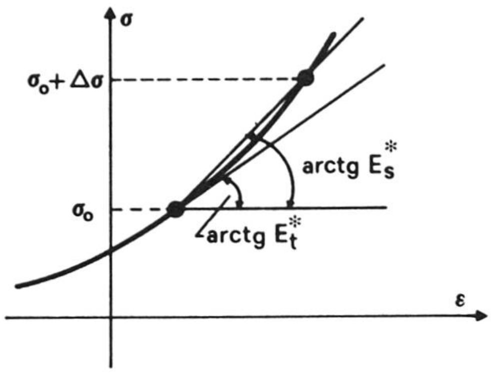

Fig. 2 Elastic response of the single stay

the anchor stays, respectively, are found from the design values σ_g (which we assume constant stay by stay) and σ_{go} of the stays' tension due to dead loads. We have:

$$A_s = \frac{g\Delta}{\sigma_g \sin\alpha} \quad , \quad A_0 = \frac{g\ell}{2\sigma_{g0}}[1+(\frac{\ell}{H})^2]^{1/2}[(\frac{L}{2\ell})^2-1] \tag{2.5}$$

Moreover, operating on the truss-like scheme we get the following values of σ_g and σ_{go}:

$$\sigma_g = \sigma_a \frac{g}{p+g} \quad , \quad \sigma_{g0} = \sigma_a \left[1 + \frac{p}{g}[1-(\frac{2\ell}{L})^2]^{-1} \right]^{-1} \tag{2.6}$$

σ_a being the cable's allowable tension.

According to the assumption of uniform stays' distribution along the deck, the bridge's equilibrium equations in terms of the displacement parameters v, u, θ ψ are:

Girder equilibrium

$$EI\, v^{IV} = q_v + \lambda p \tag{2.7}$$

$$C_t\, \theta'' = -m_t - \lambda p e^* \tag{2.8}$$

Pylon equilibrium

$$-\int_{-\ell}^{L/2} q_0 dz - Ku - S_L^\circ - S_R^\circ = 0 \tag{2.9}$$

$$-\int_{-\ell}^{L/2} m_f\, dz - Kb^2\psi - (S_L^\circ - S_R^\circ)b = 0 \tag{2.10}$$

We observe now that the terms q_v ,q_o ,m_t and m_f, appearing in eqs. (2.7)÷(2.10), are the vertical and the horizontal forces and the torsional and horizontal flexural couples, per unit length, corresponding to the stays-girder interaction produced by the stays' axial deformations [7]. The terms S_L^o and S_R^o are the horizontal components of the anchor cables axial forces due to the displacements u and ψ. Moreover, EI and C_t represent the flexural and torsional stiffnessess of the girder respectively, while K is the pylon tops' flexural stiffness.

It is convenient to rewrite equations (2.7)÷(2.10) in dimensionless form. Therefore, let us introduce the following quantities:

$$\zeta = \frac{z}{H}, \qquad V = \frac{v}{H}, \qquad U = \frac{u}{H}, \qquad \xi = \frac{e^*}{b}, \qquad t = \frac{H}{b} \tag{2.11}$$

$$a = \frac{\gamma^2 H^2 E}{12\sigma_g^3}, \qquad \overline{a}_{L,R}(\zeta) = a\,\frac{1+\beta_{L,R}(\zeta)}{2\beta^2_{L,R}(\zeta)}, \qquad \varphi_{L,R}(\zeta) = \frac{1}{2(1+\overline{a}_{L,R}\zeta^2)\,(1+\zeta^2)} \tag{2.12}$$

$$\epsilon = \left(\frac{4\,I\,\sigma_g}{H^3 g}\right)^{1/4}, \qquad \tau = \left(\frac{C_t\,\sigma_g}{Eb^2Hg}\right)^{1/2}, \qquad P = \frac{p\sigma_g}{Eg} \tag{2.13}$$

Hence, equations (2.7)÷(2.10) become:

$$\frac{\epsilon^4}{4}V^{IV} + (\varphi_L + \varphi_R)\,V + (\varphi_L - \varphi_R)\frac{\theta}{t} - \zeta\,(\varphi_L + \varphi_R)\,U - \zeta\,(\varphi_L - \varphi_R)\frac{\psi}{t} = \lambda P \tag{2.14}$$

$$\tau^2\,\theta'' - t\,(\varphi_L - \varphi_R)\,V - (\varphi_L + \varphi_R)\,\theta + t\zeta\,(\varphi_L - \varphi_R)\,U + \zeta\,(\varphi_L + \varphi_R)\,\psi = \lambda t P \xi \tag{2.15}$$

$$\int_{-\ell/H}^{L/2H} \zeta(\varphi_L+\varphi_R)V d\zeta + \frac{1}{t}\int_{-\ell/H}^{L/2H} \zeta(\varphi_L - \varphi_R)\theta d\zeta - (\varrho+\chi)\,U - \frac{1}{t}\,\overline{\varrho}\,\psi = 0 \tag{2.16}$$

$$t\int_{-\ell/H}^{L/2H} \zeta(\varphi_L - \varphi_R)V d\zeta + \int_{-\ell/H}^{L/2H} \zeta(\varphi_L+\varphi_R)\,\theta d\zeta - t\overline{\varrho}U - (\varrho+\chi)\psi = 0 \tag{2.17}$$

where

$$\varrho = \int_{-\ell/H}^{L/2H} \zeta^2\,(\varphi_L+\varphi_R)\,d\zeta + \chi_L^o + \chi_R^o\,; \qquad \overline{\varrho} = \int_{-\ell/H}^{L/2H} \zeta^2\,(\varphi_L-\varphi_R)\,d\zeta + \chi_L^o - \chi_R^o\,; \tag{2.18}$$

and

$$\chi_{L,R}^o = \frac{1}{2}\,\frac{E^{*o}_{L,R}\,A_0\,\sigma_g}{EgH}\sin\alpha_0\cos^2\alpha_0\,, \qquad \chi = \frac{K\,\sigma_g}{Eg} \tag{2.19}$$

In the previous equations the indexes "L" and "R" are respectively applied to the left or right stays with respect to the pylons' vertical axis.

An approximate solution of equations (2.14)÷(2.17) which reflects the truss-like behaviour will be given. We will show how this solution, though approximate, can represent the main qualitative and quantitative aspects of the bridge's structural behaviour.

3. CONTINUOUS MODEL

We observe that the flexural and torsional girder stiffness parameters ε and τ which appear in equations (2.14), (2.15) are very small for long-span cable stayed bridges, usually $\varepsilon = 0.1 \div 0.3$, $\tau = 0.05 \div 0.1$. This circumstance allows us to apply a perturbative approach to obtain an approximate solution of equation (2.14)÷(2.17). Actually, the solution is exact when $\varepsilon \to 0$ and $\tau \to 0$. Therefore, the general solution (V, U, θ, ψ) of equations (2.14)÷(2.17) can be expressed as:

$$V(\zeta) = V_0(\zeta) + V_1(\zeta), \qquad U = U_0 + U_1, \qquad \theta(\zeta) = \theta_0(\zeta) + \theta_1(\zeta), \qquad \psi = \psi_0 + \psi_1 \tag{3.1}$$

where $(V_o, U_o, \theta_o, \psi_o)$ is a particular solution obtained by setting $\varepsilon = \tau = 0$ in equations (2.15), (2.16), and characterizing the bridge's dominant truss behaviour, while $(V_1, U_1, \theta_1, \psi_1)$ is an approximate solution of the homogeneous system (2.15)÷(2.18), of local nature.

For the particular solution, putting $L/2H=r$ in eqs. (2.14)÷(2.17), we get

$$U_0 = \frac{1}{2}\lambda P r^2 \frac{(\chi_L^\circ + \chi_R^\circ + \chi) + \xi(\chi_R^\circ - \chi_L^\circ)}{(2\chi_L^\circ + \chi)(2\chi_R^\circ + \chi)} \tag{3.2a}$$

$$\psi_0 = \frac{1}{2}\lambda P r^2 t \frac{(\chi_R^\circ - \chi_L^\circ) + \xi(\chi_L^\circ + \chi_R^\circ + \chi)}{(2\chi_L^\circ + \chi)(2\chi_R^\circ + \chi)} \tag{3.2b}$$

$$V_0(\zeta) = \frac{1}{4}\lambda P \frac{(\varphi_L + \varphi_R) + \xi(\varphi_R - \varphi_L)}{\varphi_L \varphi_R} + \zeta U_0 \tag{3.2c}$$

$$\theta_0(\zeta) = \frac{1}{4}\lambda P t \frac{(\varphi_R - \varphi_L) + \xi(\varphi_L + \varphi_R)}{\varphi_L \varphi_R} + \zeta \psi_0 \tag{3.2d}$$

An approximate solution of the homogeneous system can be expressed as [3,6,7]:

$$V_1(\zeta) = c_1 e^{-f(\zeta)} \sin f(\zeta) + c_2 e^{-f(\zeta)} \cos f(\zeta), \quad \theta_1(\zeta) = k_1 e^{-f_1(\zeta)} + k_2 e^{-f_2(\zeta)}, \quad U_1 = \psi_1 = 0 \tag{3.3}$$

where

$$f(\zeta) = \frac{1}{\epsilon}\int_\zeta^r \varphi^{1/4}\, d\zeta, \qquad f_1(\zeta) = \frac{1}{\tau}\int_\zeta^r \varphi^{1/2}\, d\zeta, \qquad f_2(\zeta) = \frac{1}{\tau}\int_0^\zeta \varphi^{1/2}\, d\zeta, \tag{3.4}$$

and

$$\varphi(\zeta) = \varphi_L(\zeta) + \varphi_R(\zeta) \tag{3.5}$$

Hence for the given load condition we get:

$$V_1(\zeta) = \frac{1}{2}\frac{\epsilon}{\varphi^{1/4}(r)} \Big\{U_0 + \lambda P\,[r\{2 + r^2(\overline{a}_L + \overline{a}_R)\} + r(1 + r^2)(\overline{a}_L + \overline{a}_R)] +$$
$$+ \lambda P \xi\,[r^3(\overline{a}_L - \overline{a}_R) + r(1 + r^2)(\overline{a}_L - \overline{a}_R)]\Big\} \cdot [e^{-f(\zeta)} \sin f(\zeta) - e^{-f(\zeta)} \cos f(\zeta)] \tag{3.6}$$

$$\theta_1(\zeta) = -\frac{\tau}{\varphi^{1/2}(r)}\Big\{\lambda t P[r(\overline{a}_L - \overline{a}_R) + 2r^3(\overline{a}_L - \overline{a}_R) + \\ + \xi\{r(2 + \overline{a}_L + \overline{a}_R) + 2r^3(\overline{a}_L + \overline{a}_R)\}] + \psi_0\Big\} e^{-f_1(\zeta)} - \lambda P t \xi e^{-f_2(\zeta)} \quad (3.7)$$

The maximum deflection δ and maximum torsional rotation $\theta(r)$ at midspan are expressed by:

$$\frac{\delta}{H} = \frac{1}{2}\lambda P\Big\{(1 + r^2)[(1 + \overline{a}_L r^2) + (1 + \overline{a}_R r^2)] + \xi r^2(1 + r^2)(\overline{a}_L - \overline{a}_R)\Big\} + \\ + rU_0 - \frac{\epsilon}{2\varphi^{1/4}(r)}\Big\{U_0 + \lambda P[(2 + \overline{a}_L + \overline{a}_R)r + 2r^3(\overline{a}_L + \overline{a}_R)] + \lambda P\xi[r(\overline{a}_L - \overline{a}_R) + 2r^3(\overline{a}_L - \overline{a}_R)]\Big\} \quad (3.8)$$

$$\theta(r)/t = \frac{1}{2}\lambda P\Big\{r^2(\overline{a}_L - \overline{a}_R)(1 + r^2) + \xi[2(1 + r^2) + r^2(1 + r^2)(\overline{a}_L + \overline{a}_R)]\Big\} + r\psi_0/t + \\ - \frac{\tau}{\varphi^{1/2}(r)}\Big\{\lambda P[r(\overline{a}_L - \overline{a}_R) + 2r^3(\overline{a}_L - \overline{a}_R) + \xi\{r(2 + \overline{a}_L + \overline{a}_R) + 2r^3(\overline{a}_L + \overline{a}_R)\}] + \psi_0/t\Big\} \quad (3.9)$$

Furthermore, the bending moment at midspan is expressed by:

$$\frac{M(r)}{pH^2} = \frac{\epsilon^3}{4}\varphi^{1/4}(r)\Big\{U_0/P + \lambda[(2 + \overline{a}_L + \overline{a}_R)r + 2r^3(\overline{a}_L + \overline{a}_R) + \\ + \xi\{r(\overline{a}_L - \overline{a}_R) + 2r^3(\overline{a}_L - \overline{a}_R)\}]\Big\} \quad (3.10)$$

and the twisting moment at the section $\zeta = 0$:

$$\frac{M_t(0)}{pbH} = \lambda\xi\tau + \tau^2\frac{\psi_0}{tP} \quad (3.11)$$

The equations (3.8)÷(3.11) are non linear because of terms depending on the value of the secant modulus of the stays. Therefore, these equations have been solved by using a direct iterative procedure.

4. DISCRETE MODEL

In the previous section, the non linear behaviour of the fan-shaped cable-stayed bridge scheme has been analyzed by assuming a uniform stay distribution along the deck. An approximate analytical solution has been obtained which reflects the main truss-like behaviour of such structures. Obviously a more accurate analysis of stresses and deformations may be obtained only by means of a discrete model which accounts for the actual stays' spacing and geometry change. However, the results given in [6-7] shows that with a suitable choice of the girder stiffness parameter ε, the non-linear effects due to geometry changes are insignificant. Therefore, in the present work a discrete model which includes only the nonlinearities due to the stays' behaviour will be analyzed.

The discrete model is defined by the following displacement parameters:

- the pylon tops' horizontal displacement u

- the pylon tops' torsional rotation ψ
- the girder's nodal vertical displacement v_i
- the girder's nodal bending slopes ϕ_i
- the girder's torsional rotations θ_i

The finite element discretization is based on a cubic interpolation of the vertical displacement $v_e(z)$ and on a linear interpolation of the torsional rotation $\theta_e(z)$ for the element of length Δ, that is:

$$v_e(z) = \mathbf{F}_1^T(z)\,\mathbf{d}_e^{(1)}, \qquad \theta_e(z) = \mathbf{F}_2^T(z)\,\mathbf{d}_e^{(2)} \tag{4.1}$$

where $\mathbf{d}_e^{(1)}$ and $\mathbf{d}_e^{(2)}$ are the vectors of the nodal displacement parameters (deflections, bending slopes and torsional rotations of the end sections), and $\mathbf{F}_1$, $\mathbf{F}_2$ are the shape function vectors. The elastic response $\mathbf{s}_e(\mathbf{d}_e)$ of the girder element is defined by means of the virtual work equation:

$$\delta\mathbf{d}_e^T\mathbf{s}_e = \int_\Delta (EIv''\delta v'' + C_t\,\theta'\delta\theta')\,dz \tag{4.2}$$

where $\mathbf{d}_e \equiv \begin{bmatrix}\mathbf{d}_e^{(1)}\\ \mathbf{d}_e^{(2)}\end{bmatrix}$ is the nodal displacement vector of e-th element.

The pylon equilibrium equations, analogous to (2.16) and (2.17), are:

$$-(\varrho + \chi)u - b\,\overline{\varrho}\,\psi + \frac{\Delta}{H}\sum_{i=1}^{N}\zeta_i(\varphi_L^i + \varphi_R^i)v_i + \frac{\Delta b}{H}\sum_{i=1}^{N}\zeta_i(\varphi_L^i - \varphi_R^i)\theta_i = 0 \tag{4.3}$$

$$-(\varrho + \chi)\psi - \frac{\overline{\varrho}}{b}u + \frac{\Delta}{bH}\sum_{i=1}^{N}\zeta_i(\varphi_L^i - \varphi_R^i)v_i + \frac{\Delta}{H}\sum_{i=1}^{N}\zeta_i(\varphi_L^i + \varphi_R^i)\theta_i = 0 \tag{4.4}$$

where $\varphi_L^i = \varphi_L(\zeta_i)$, $\varphi_R^i = \varphi_R(\zeta_i)$ and $N = (L/2+\ell)/\Delta+1$.

The girder equilibrium condition is expressed by:

$$\mathbf{s} - \mathbf{q} - \lambda\mathbf{p} = 0 \tag{4.5}$$

where $\mathbf{p}$ is the global nodal load vector, $\mathbf{s}$ is the assembled elastic response of the girder and $\mathbf{q} = [q_v,\ q_t]$ is the stays' action on the deck:

$$q_v^i = \frac{Eg\Delta}{2H\sigma_g}[(\varphi_L^i + \varphi_R^i)(u\zeta_i - v_i) + b(\varphi_L^i - \varphi_R^i)(\psi\zeta_i - \theta_i)] \tag{4.6}$$

$$q_t^i = \frac{Eg\Delta}{2H\sigma_g}[b(\varphi_L^i - \varphi_R^i)(u\zeta_i - v_i) + b^2(\varphi_L^i + \varphi_R^i)(\psi\zeta_i - \theta_i)] \tag{4.7}$$

For numerical applications, the set of non linear equations (4.3)÷(4.5), has been solved by using the standard Modified Newton-Raphson method.

5. NUMERICAL RESULTS

Numerical results, obtained by using both the continuous model and the discrete model are given. The main parameters which characterize the bridge scheme are:

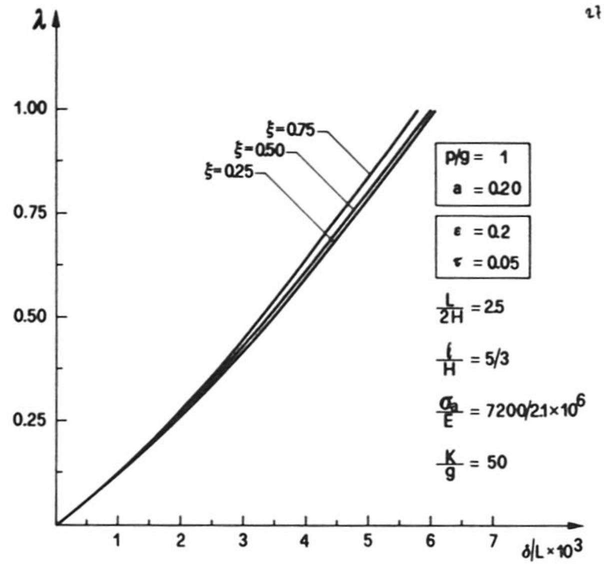

Fig. 3 Continuous model: midspan deflection (ε=0.2, τ=0.05).

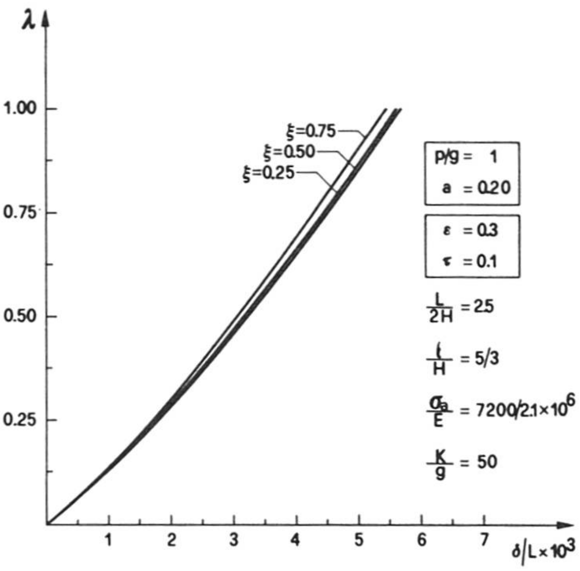

Fig. 4 Continuous model: midspan deflection (ε=0.3, τ=0.1).

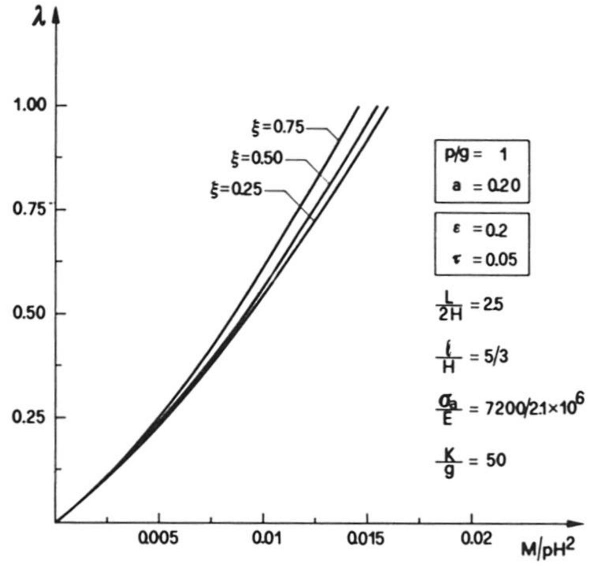

Fig. 5 Continuous model: midspan bending moment (ε=0.2, τ=0.05).

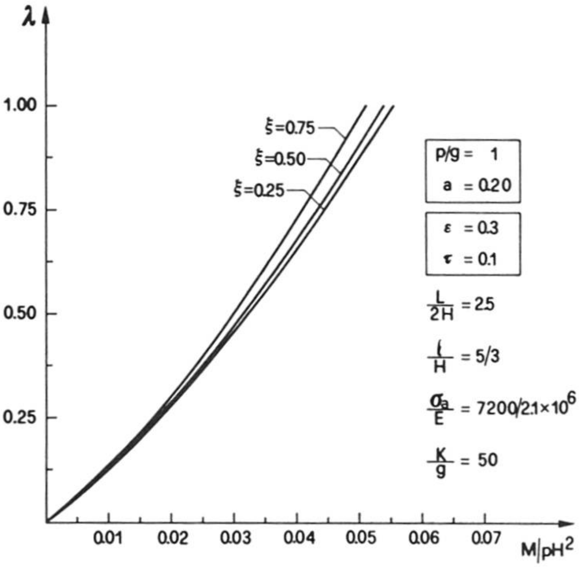

Fig. 6 Continuous model: midspan bending moment (ε=0.3, τ=0.1)

$$\frac{L}{2H}, \frac{\ell}{H}, \frac{\sigma_a}{E}, \frac{k}{g}, \epsilon, \tau, a, \frac{p}{g}.$$

For the first two parameters, according to the stability condition of the anchor stays and to the minimum weight condition of the stays, we assumed the following values [3]: $L/2H=2.5$, $\ell/H=5/3$. The value of σ_a/E is, for steel stays $\sigma_a/E=7200/2.1\text{x}10^6$. For the nondimensional tower bending stiffness K/g parameter, according to its weak influence on the overall behaviour of the bridge,

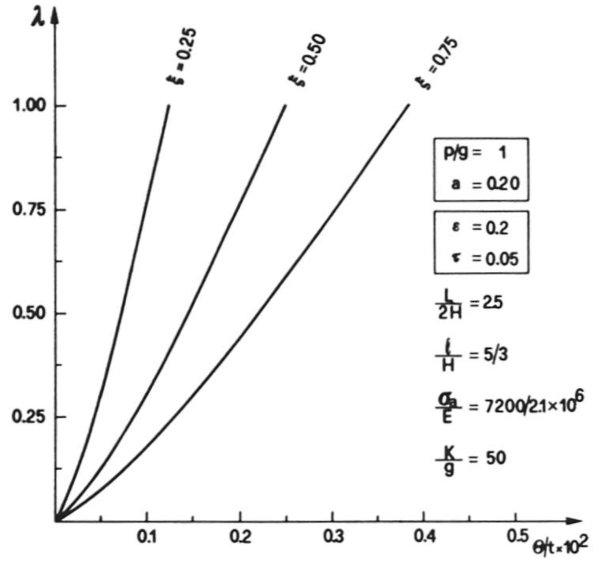

Fig. 7 Continuous model: midspan torsional rotation (ε=0.2, τ=0.05).

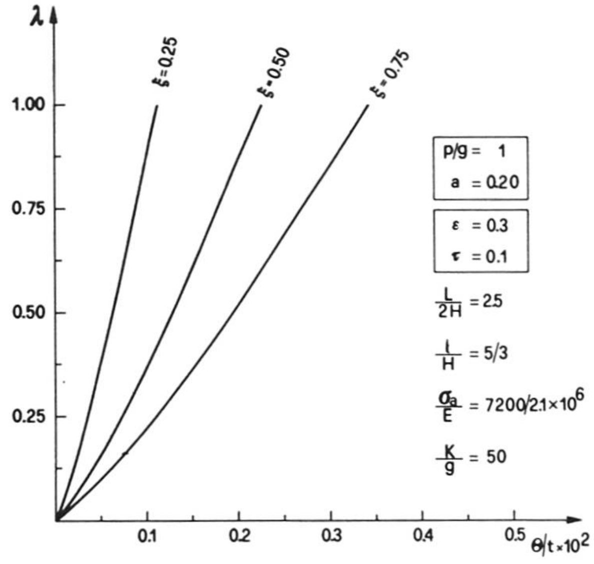

Fig. 8 Continuous model: midspan torsional rotation (ε=0.3, τ=0.1)

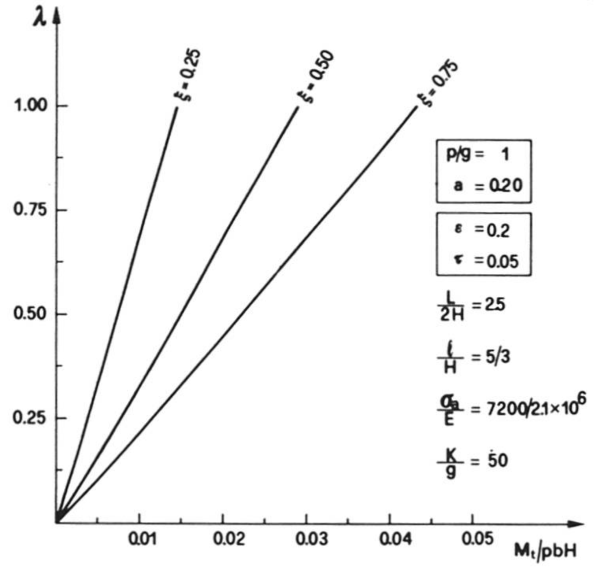

Fig. 9 Continuous model: twisting moment at the section ζ=0 (ε=0.2, τ=0.05).

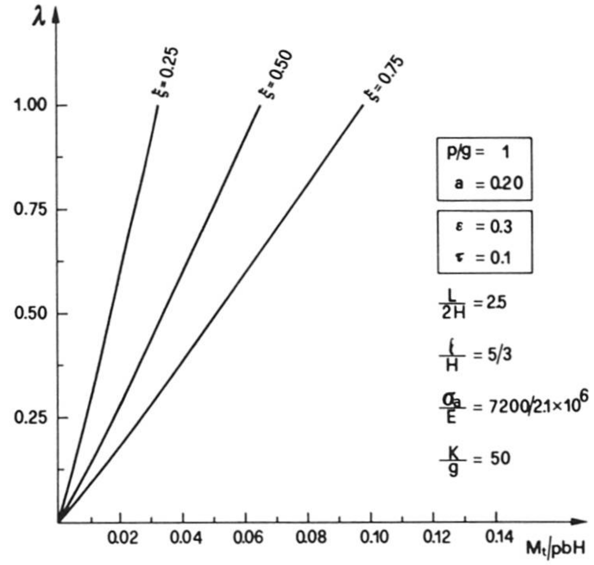

Fig. 10 Continuous model: twisting moment at the section ζ=0 (ε=0.3, τ=0.1).

we assumed the mean value K/g=50. While, for the ε, τ, a, p/g parameters, which have a considerable influence on the overall behaviour of the bridge, some values, corresponding to long-span cable-stayed bridges, have been considered.

In Figs. 3-10 the values of some dimensionless quantities which are more significant to describe the bridge behaviour are given. That is, the maximum midspan deflection δ/L, the midspan bending moment M/pH^2, the maximum midspan torsional rotation $\theta(r)/t$ and the twisting moment M_t/pbH at the section $\zeta=0$. Furthermore, Figs. 11-14 show the comparison between analytical and numerical solutions. Obviously, the approximation of the results obtained by using

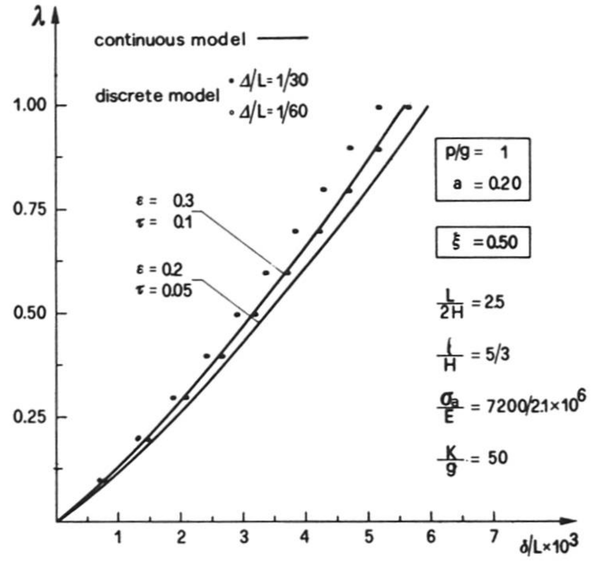

Fig. 11 Midspan deflection: comparison between analytical and numerical results.

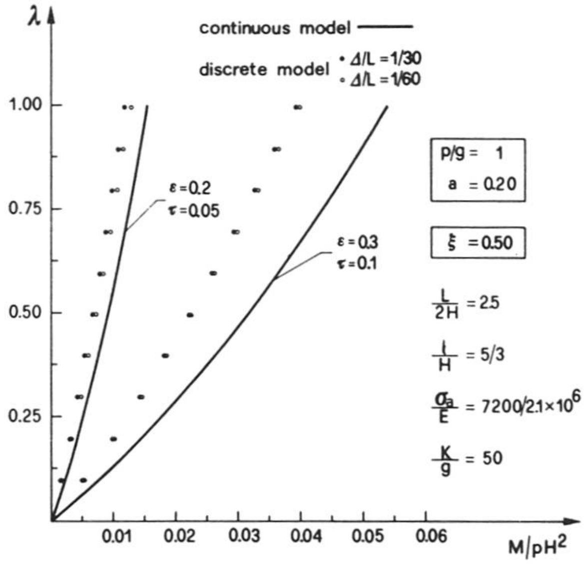

Fig. 12 Midspan bending moment: comparison between analytical and numerical results.

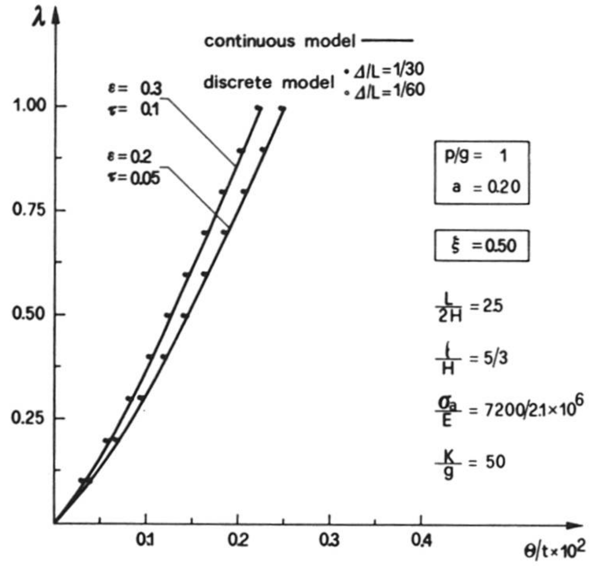

Fig. 13 Midspan torsional rotation comparison between analytical and numerical results.

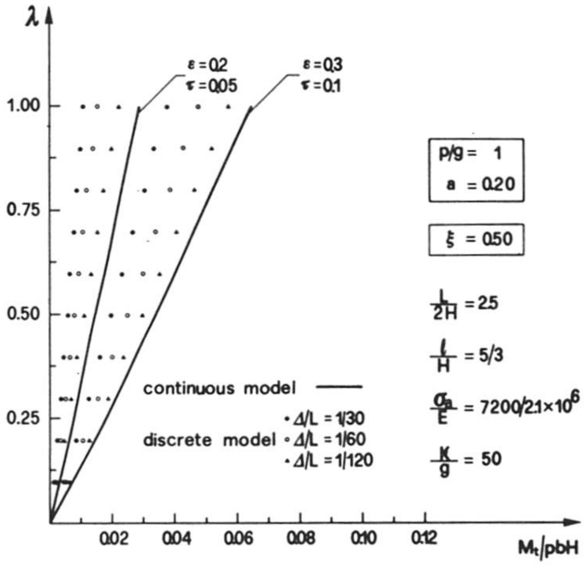

Fig. 14 Twisting moment at the section ζ=0: comparison between analytical and numerical results.

the continuous model in respect to the discrete one is connected with the values of the ε and τ parameters and with the ratio Δ/L. In particular, the approximation of the results relative to the midspan deflection and the midspan torsional rotation obtained by using the continuous model is related to the values of parameters ε, and τ, with errors between 5-8% (Fig. 11) and 0.5-1% (Fig. 12), respectively, for ε=0.2-0.3 and τ=0.05-0.1.

In figures 12, 14 similar results are given for the bending moment at the midspan section and the twisting moment at section ζ=0; the error of the continuous model compared to the discrete one in this case is higher and is roughly between 8-20% (Fig. 12) for ε=0.2 - 0.3 and τ=0.05-0.1 in respect to the bending moment; while as far as concerns the twisting moment, this error is strongly related to the ratio Δ/L and is about between 8-40% for Δ/L=1/30 - 1/120.

In any case, we can observe, from the figures previously shown, that the differences between the results obtained by using a linear analysis, and those derived by a more refined nonlinear analysis, is considerable in particular in regards to the deformations, that is, midspan deflection (≅ 30%) and torsional rotation (≅ 45%).

6. CONCLUSIONS

In this paper the static non linear behaviour of long-span cable-stayed bridges was analyzed. A suitable continuous model of the bridge was developed and an analytical solution of the Statics' basic equations was derived. This solution focuses the prevailing truss behaviour of the bridge and allows us to achieve a synthetic understanding of the statical behaviour of the bridge and to express, by simple formulas, the more significant stress and deformation characteristics, useful for design in practice.

Moreover, the influence of the intrinsec nonlinear behaviour of stays on the overall behaviour of the bridge was examined. In particular the obtained results show the importance of nonlinear effects on the deformability of the bridge, which, especially for long-span bridges, is undergoes severe restrictions.

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