Recent Progress in the Field of Structural Stability of Steel Structures

Récents développements dans la stabilité des structures en acier

Fortschritte auf dem Gebiet der Stabilität von Stahltragwerken

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SUMMARY
The authors give an overall view of the up-to-date state of development in the stability of steel structures, at the light of the extensive recent developments, among which may be especially mentioned the four sessions of the IABSE – ECCS – U.S. SSRC – Structural Stability Research Committee of Japan "Travelling Colloquium" (Tokyo 1976 – Liège 1977 – Washington 1977 – Budapest 1977).

RÉSUMÉ
Les auteurs donnent une vue générale du stade de développement actuel du problème de la stabilité des structures en acier, à la lumière des importants développements de ces dernières années, parmi lesquels il faut compter particulièrement les quatre sessions du « Colloque Itinérant » organisé conjointement par l'AIPC, la CECM, le SSRC des Etats-Unis et le Comité de Recherches Japonais sur la Stabilité des Structures (Tokyo 1976 – Liège 1977 – Washington 1977 – Budapest 1977).

ZUSAMMENFASSUNG
Die Erkenntnisse auf dem Gebiet der Stabilität von Stahltragwerken haben, insbesondere durch die von den Organisationen IVBH, EKS, der SSRC der USA sowie des japanischen Komitees für Tragwerkstabilität organisierten vier gemeinsamen Sitzungen in Tokio 1976, Lüttich 1977, Washington 1977 und Budapest 1977, eine stürmische Entwicklung zu verzeichnen. Die Autoren geben einen Überblick über den heutigen Stand dieser Erkenntnisse.

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Recent Progress in the Field of Structural Stability of Steel Structures

by

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1. HISTORICAL.

This State of Art Report will try to summarize the development of research, in the last twenty years, in the field of Structural Stability of Steel Structures, to give the main progresses accomplished and, in some cases, to present the conflicting ideas of various researchers. The list of references does not pretend to be complete but the authors hope, at least, not to have omitted any book of importance, taking into account that the title subject is considered from the viewpoint of a designer that is responsible for the safety of his structure. The historical review begins with year 1955, when the European Convention for Constructional Steelwork (E.C.C.S.) was founded. Professor H. BEER, from Austria was, at that time, Chairman of the Committee 8 (Instability) and also became later Chairman of Working Commission II (Steel Structures) of the International Association of Bridge and Structural Engineering (I.A.B.S.E.).

In Commission 8, the late J. DUTHEIL was very influential by insisting with tenacity[43,44] that the Commission had to abandon design formulae based on the theory of instability by bifurcation of ideally perfect axially compressed bars and could not continue to correct this approach by "varying sentimentally" the safety factor with the value of the slenderness ratio Kl/r, but had, on the contrary, to take into account the random imperfections of these bars and include them in the design formulae.

Two Subcommittees were formed in Commission 8: The first(buckling experiments) under the leadership of Dr. SFINTESCO[59,99] , launched an extremely large series of experiments on axially loaded columns. Seven countries were involved and about thousand buckling tests have been performed. The second subcommittee, (Theory of buckling) under the leadership of the senior reviewer, discussed several years about the possibility to develop analytical computations taking account of the elasto-plastic behaviour of straight columns having a certain crookedness and various distributions of residual stresses. The answer came when BATTERMAN and JOHNSTON showed [22] how to use the electronic computer to solve above problem[56]. An improved version of BATTERMAN and JOHNSTON's program was used at Liège University to solve several problems [48,49], while

BEER and SCHULZ developed at the Technical University of Graz another type of program that was much more efficient in giving the collapse load. The many (σ_{cr},λ) curves produced at Graz were compared with the experimental results obtained by the first Subcommittee and became the basis of the so-called "european buckling curves" [5,24,25] . These curves were then very slightly modified to take account of some new information provided by Dr. B.W. YOUNG in the United Kingdom.

In 1973, as a result of the links of some of its members with the Column Research Council of the United States, Commission 8 was reorganized and divided in nine task groups; the fields and chairmen of these task groups are given in [7].

Professor BEER had the idea to organize in 1971 an international Colloquium in order to compare the ECCS approach of buckling curves with those prevailing in Eastern Europe, United States of America and Japan.
After the sudden death of Professor BEER, in 1972, it was decided to hold above Colloquium in Paris. The corresponding proceedings were published by Working Commission II of IABSE in 1975 |6|. From 1972 up to now, the chairmanship of Commission 8 was assumed by Dr. SFINTESCO.

With the gradual development of rules for designing against instability emerged, in London, in 1974, the idea to hold an International Colloquium treating every aspect of structural instability of steel structures. Dr. SFINTESCO and Professor L.S. BEEDLE proposed to enlarge the geographical scope of the Colloquium and transform it into a "Travelling Colloquium" to be held successively at Tokyo (9 Sept. 1976), Liège (13-14-15 April 1977), Washington (17-18-19 May 1977) and Budapest (19-20-21 October 1977). An Introductory Report, explaining the research behind the ECCS Stability rules, was published in August 1976 and was conceived as a common scientific basis for above four meetings. This book is usually called Manual on Structural Stability |7|. One book of proceedings |8| was published after the Tokyo Colloquium. At the Liège Colloquium, a Preliminary Report containing 92 contributions |9| was issued end of January 1977 and a Final Report |10| containing the general reports, the prepared and free discussions and the conclusions of the general reporters appeared end of October of the same year. The proceedings of the Washington Colloquium |11| were published at the end of 1977. Finally, a volume of Proceedings containing 56 reports |12| was issued in September 1977 for the Regional Colloquium held in Budapest and Balatonfüred. A Final Report, containing the general reports, the prepared and free discussions, will appear in the first months of 1978 |13|.

To these books devoted to the general theme of structural instability of steel structures, we should add three other books devoted - after the accidents that occurred to four box girder bridges - to the particular problem of design of plate and box girders (or, more generally, plated structures) for ultimate strength. The first book contains the contributions and discussions of a small Colloquium organized in London on invitation by Professors BEEDLE, ROCKEY and MASSONNET (Chairman) |1|. The second book is the proceedings of a conference on Steel Box Girder Bridges held in London in 1972 |2|. The third book contains the proceedings of an International Symposium on Steel Plated Structures organized at Imperial College (London) on July 1976 |3|.

Last, it is worthwhile to mention that, in the United States, a Guide for Stability has been prepared by the Structural Stability Research Council |4|, and edited by B.G. JOHNSTON, whilst, for West European Countries, the European Convention for Constructional Steelwork has edited a first draft of Recommendations for Steel Constructions |5|. A second draft of these latter is in progress.

2. GENERAL PRINCIPLES OF STRUCTURAL SAFETY AND THEIR APPLICATION TO MEMBERS IN DANGER OF INSTABILITY.

2.1. GENERAL PRINCIPLES OF SAFETY.

As is well known, design methods may be distinguished in the following ways :

1) By the way the coefficients related to safety are introduced :
   (a) "allowable stress method", in which the stresses occurring under the maximum service loads are compared with fractions of the strengths of the materials ;
   (b) "limit state methods", in which the stress redistribution due to plastic yielding is taken into account in the cross sections (statically determinate structures) or eventually both in the cross sections and in the structure as a whole (hyperstatic structures). A certain margin of safety is required between design loads and limit state loads.
2) By the type of safety conditions :
   (a) deterministic design methods, in which the basic parameters are treated as non-random ;
   (b) probabilistic design methods, where basic parameters are considered as random.

The design method adopted by E.C.C.S. is a limit state method combining approaches 1b) and 2b) above. More accurately, it is called a semi-probabilistic limit state method of level one. It considers the structures as they are, with their geometrical and structural imperfections, and takes account of the elasto-plastic behaviour of the material. This basic concept seems to be more and more generally accepted (see e.g. the General Report by HALASZ |13|). The bifurcation theory remains the basis of the knowledge, but loses increasingly its central position. This evolution settles obviously the question of validity of traditional concepts like that of effective length. Above refined approach is rendered possible by the use of computers and international cooperation. Nevertheless, everyday's work requires still simple interaction formulae, tables and diagrams.

This design method is gaining acceptance presently within all international organizations. It has been adopted by the I.S.O., for concrete structures by the Comité Européen du Béton (C.E.B.) and the Féderation Internationale de la Précontrainte. All european specifications are progressively transformed to be based on this doctrine and, at the same time, a similar evolution is being observed in the United States and in Canada.

Present state of art Report is entirely based on this safety concept. The authors consider the adoption of the semi-probabilistic limit state method as an important step forward, particularly in the study of instability phenomena, because these are intrinsically non-linear. For this reason, only (non linear) limit state methods put the design rules for instability in their correct perspective.
It is now largely agreed that, in order to avoid unnecessary complications, the limit states may reasonably be reduced to two :

a) the ultimate strength limit state, which is that corresponding to the maximum load-carrying capacity ;
b) the serviceability limit state , which is related to the criteria governing normal use or durability; usually, it will entail a control of the deformations under working load.

2.2. APPLICATION OF THE GENERAL PRINCIPLES OF SAFETY TO MEMBERS IN DANGER OF INSTABILITY.

Usually, the first-order theory is a sufficiently accurate basis for the design of steel structures. In such a theory, not only the effect of the displacements of the structure on the mode of action of the forces may be neglected, but - at least in the case of a quasi-static loading - the residual stresses can be forgotten too, because they are wiped out by yielding. However, this is no more true in second order theory, because not only - by definition - the effect of the displacements on the mode of action of the structure must be considered, but the effect of residual stresses must be considered too, because it may deeply affect the shape of the load - displacement relationships. Otherwise speaking, (cf. U. VOGEL |7|, pp.15-17) the actual stress pattern cannot be calculated because of the presence of residual stresses due to rolling, welding, straightening and erection, and other similar local defects, as well as because of unavoidable imperfections and deviations from the mathematical model used in the structural analysis. Such a structure is only safe because of the ductility of the material.

For all the reasons stated above, plastic analysis constitutes a much better approach than elastic analysis to study the real behavior of a steel structure. In addition to plasticity effects, however, it is absolutely necessary to take into account the influence of deformations (second-order effects) and unavoidable imperfections in all cases where the danger of instability exists, i.e. for columns, beam-columns, frames, plates and shell-type structures, when compressive forces are involved. In all these cases, instability under ultimate load is accompanied by *divergence* of equilibrium - not by *bifurcation*. It must also be stressed in passing that allowable stresses do not make any sense in stability problems.
This concept is also adopted in soviet codes (cf. POTAPKIN |13|); it must be recalled that U.S.S.R. has been one among the first countries to propose the idea of limit state design. In the U.S.S.R., however, the characteristic strength is obtained by reducing the mean strength by three times - instead of two times in the ECCS rules - the standard deviation. On the other hand, this is compensated by lower load factors.

3. AXIALLY COMPRESSED MEMBERS.

As the subject of axially compressed members has been widely publicized since about 1969, we shall be very brief on this item and only stress some salient points.

3.1.

The realization that geometric imperfections exerted a strong effect in diminishing the actual collapse load as compared to the classical Eulerian bifurcation load $P_{cr} = \pi^2 EI / l^2$ is very old. It goes back to Thomas YOUNG (1807) |121], who was the first to take account of "imperfect columns", to bridge the gap between experimental results and EULER's theory. In 1886, AYRTON and PERRY showed that, for current use, an initial curvature of the bar had a similar effect to that due to an eccentricity of the axial load |16] ; they introduced the concept of collapse criterion. In 1925, ROBERTSON proposed to adopt, in the AYRTON-PERRY formula, an imperfection parameter η proportional to the slenderness ratio λ |87]; later , GODFREY suggested to substitute a value of η proportional to the square of the slenderness |53]. GODFREY's idea is near to that of DUTHEIL, who finds that the parameter depends on λ^2, the yield stress σ_r and a factor C which is calibrated on basis of experimental results.[43,44].

3.2. The demonstration that residual stresses exerted an effect as deleterious as the geometric imperfection was given by the research men of Lehigh [14,23,56,105, 126,127] and the corresponding theory taking account of them was given by OSGOOD [124] and by THURLIMANN [125]. The history of the early development of this modified tangent modulus approach is given in the Guide of B.G. JOHNSTON ([4], pp. 50 to 53). The first published evidence of this thinking in late 1940's is [130]. On the European continent, it seems that the research workers of Liège were the first to demonstrate the paramount effect of these residual stresses [32,69,77, 81]. Some members of Commission 8 of ECCS disagreed during several years to include the residual stresses into the computer simulation. They argued - rather rightly - that residual stresses were rather random, due to the possibility of cold-straightening (by gagging or rotarizing) after the rolled profile was taken from the cooling bed. This argument disappeared when FREY [48,49] demonstrated that cold-straightening was always beneficial and that it was safe, therefore, to introduce the *cooling* residual stresses into the simulations.

3.3.

According to the general safety doctrine of E.C.C.S. and I.A.B.S.E., the numerical simulation was executed by BEER and SCHULZ by adopting characteristic values of the main parameter (σ_r, initial crookedness, residual stresses,etc..) and the theoretical curves $\overline{N} = f(\overline{\lambda})$ where $\overline{N} = \sigma_{collapse}/\sigma_r$ and $\overline{\lambda} = \lambda/\lambda_E$ with λ_E (= EULER slenderness ratio) $= \pi\sqrt{\frac{E}{\sigma_r}}$ were then found to be safe by comparison with the experimental values obtained by the experimental group of Commission 8 [99] . This procedure was, however, criticized afterwards by BJORHOVDE [30] AND by STRATING and VOS [104] , on the ground that they were not in line with the statistical theory. A statistical paper of the same vein was presented at Washington by D.H. HALL |11|. Curiously enough, it appears that above three statistical studies, while similar, present some marked divergences in the conclusions, which shows that conclusions always depend on the basic assumptions. Italy, Norway, Belgium, Yougoslavia, the Federal Republic of Germany have adopted the E.C.C.S. curves, while Tchecoslovakia has adopted a similar PERRY-ROBERTSON type of approach that is described in the paper by CHALUPA, DJUBEK and SKALOUD (|12| pp. 3 ÷ 11).

3.4.

As many countries wanted to take account of the ECCS research effort in writing their own specifications and because of - quite understandable "nationalistic" feelings, several authors have shown successfully that results equivalent to the european buckling curves could be found by adapting the imperfection parameters. DWIGHT [45] shows that the european buckling curves may be represented by a modified PERRY formula, whilst VOGEL [116] evaluates the "representative imperfection", the only parameter in an approximate ultimate strength method, which covers the influence of all possible imperfections.
It seems that soviet code (cf. POTAPKIN [13]) is still in favor of a dimensional presentation of buckling tables which give values of $\overline{N}$ with respect to some ranges of eccentricity $i = e_0/\rho$, expressed as multiple of the core radius of the cross section ρ, with

$$e_0 = 0,125 + 0,0018\,\lambda\,, \qquad (3.1.)$$

3.5.

Above discussion brings about the question of the best analytical representation of the european buckling curves, which is evidently useful for computer programs and especially optimization studies. The original european curves were represented analytically by Dr. S. BAAR [17] . As said in the historical introduction, these curves were subsequently slightly modified to take account of the researches of B.W. YOUNG [120] and have presently the following appearance

$$\bar{N} = 1 \text{ for } \bar{\lambda} \leqslant 0.2 \qquad (3.2.a)$$

$$\bar{N} = f(\lambda) \text{ for } \bar{\lambda} > 0.2 \qquad (3.2.b)$$

A very accurate analytical representation has been very recently proposed by MAQUOI and RONDAL and the corresponding report is presently under press [75] . The formulation gives the non-dimensional ultimate load $\bar{N}$ with respect to the non-dimensional slenderness $\bar{\lambda}$ as follows :

$$\bar{N} = \frac{1 + \alpha\sqrt{\bar{\lambda}^2 - 0{,}04} + \bar{\lambda}^2}{2\,\bar{\lambda}^2} - \frac{1}{2\bar{\lambda}^2}\sqrt{\left[1 + \alpha\sqrt{\bar{\lambda}^2 - 0{,}04} + \bar{\lambda}^2\right]^2 - 4\,\bar{\lambda}^2} \qquad (3.3.)$$

where α is a numerical coefficient,each value of which characterizes one of the five European buckling curves, according to following reference table:

European curve	a_0	a	b	c	d
value of α	0.093	0.158	0.281	0.384	0.587

Another interesting approach endeavoring to simplify the number of tables required for using of the European buckling curves is due to FINZI and URBANO |11|. They refer to a single nondimensional buckling curve completely unaffected by structural imperfections and they define a fictitious slenderness depending on the type of profile; thus the five buckling curves and corresponding tables are replaced by only one table and four formulae of fictitious slenderness. Such an attempt is worthwhile for its search of simplicity but is perhaps not very interesting for an automatic treatment on the computer.

Of course, axially loaded columns hardly exist in practice, and the main value of the study referred to above is to provide a simple way to design beam-columns via adequate interaction formulae or diagrams (see section 8 hereafter) and the bifurcation concept of effective length. The limited validity of this concept and the errors due to its use have been studied by many authors. This validity is discussed here, rather than in the sections 8 and 10 devoted respectively to beam-columns and frames, because it has some feedback on the value of the research about centrally loaded members.

At Washington, the paper by KUHN and LUNDGREN |11| shows that the K approach may be in error, either on the safe or on the unsafe side, depending on the frame and loading. At this Colloquium, the main paper on this topic was that of CLARK |11|, who made a parametric study of eccentrically loaded and end-restrained beam-columns obeying a RAMBERG-OSGOOD type of stress-strain diagram. His main conclusion, based on clever simple lower and upper bound solutions of above problem, is that "For design purposes, it should be satisfactory to assume that a given eccentricity has the same reduction effect on a restrained - end columns as on a hinged - end columns, as long as the effective length factor K lies between 0.7 and 1.0. For lower K values, the strength reduction is less". YURA(see|4|for ref.) has shown how to handle the effective length method for cases where the assumptions underlying the development of the alignment chart were violated.

At the Budapest Colloquium, two studies,respectively by KORONDI(|12|,pp. 87-94)and by KORDA (|12|,pp.111-119)bring new information to the same problem.

A more fundamental question is whether several buckling curves are really necessary. The European (ECCS) researchers believe that accurate and economical design requires the use of five official curves (a_0,a, b, c, d). The Americans are perhaps divided on this question. Whilst B.G. JOHNSTON, in his

Guide (|4|, pp. 38 to 80) discusses the advantages of using several curves, American specification writers prefer presently (1977) to stick to one single curve for obvious reasons of simplicity. One very serious argument in favor of one single curve was presented by G. WINTER:(various writings and a personal letter to the senior reviewer)it is that the effects of accidental crookedness and eccentricity are very much reduced in actual structures like rigid jointed frames. The reviewers believe that this controversy may be settled only by simulation on computers of a subassemblage made of "imperfect" members.

4. COMPOSITE COLUMNS.

4.1. DESIGN METHODS PRESENTED IN JAPAN AND IN EUROPE.

Two methods have been presented for the design of composite columns, the first at the Tokyo Conference |8| and the second in the Introductory Report of the Liège Colloquium (ECCS approach) |7|.
The two methods cover the two basic types of composite columns, i.e the encased column and the concrete-filled tube, and both they assume full interaction between steel and concrete.

4.1.1. Axially loaded composite columns.

The new Japanese formulae use as a basis for design the superposed strength method according to which the strength of the composite column is given by the sum of the buckling strengths of the steel column with imperfection and of the concrete or reinforced column with imperfections. The value of the buckling strengths of the steel and the concrete column may be determined respectively from the column curves of AISC or ECCS and by ACI or CEB methods.
No difference is made between the treatment of encased columns and concrete filled tubes.

The ECCS approach is based on an ultimate load design philosophy and adopts the European buckling curves as the basic design curves for composite columns.

In the case of encased steel sections and concrete filled rectangular tubes, the ultimate strength of a stocky axially loaded column is given by the sum of the strengths of the steel and concrete section and it is called "squash load" or N_u. The ultimate load of a long composite column depends not only on the cross sectional properties but also on the column slenderness, as failure occurs by the buckling of the column. The column slenderness factor is defined as the ratio of the column length to an unit critical length ($\overline{\lambda} = l/l_c$) and this unit critical length (l_c) is the length for which the EULER load equals the squash load. Having calculated the slenderness factor $\overline{\lambda}$, the designer selects the appropriate basic buckling curve applicable to the corresponding basic metal section. A value of $\overline{N} = \frac{N}{N_u}$ is given directly and N is the ultimate load of the column.

The behaviour of concrete-filled circular hollow sections differs from other types of columns in that, under concentric loading, such columns exhibit an enhanced strength, particularly for short columns. This is explained by the fact that the concrete core in such columns is contained triaxially. The effect of the triaxial containment of the concrete is taken into account by calculating an augmented strength of concrete and a corresponding reduced strength of steel.

4.1.2. Composite beam column.

In the Japanese approach, the strength of a composite beam-column is again considered as the sum of the strengths of the component material columns, but the bending strength is obtained by reducing the value given from the simple superposition. One feature of this approximation is that, in some cases, it may err on the unsafe side.
In the European approach, the uniaxial load-moment interaction expression has been produced by curve fitting the analytical exact interaction curves for composite columns of the two types. The approximated interaction curve is very simple to determine and gives good results which are always on the safe side.

4.1.3. Biaxially loaded composite columns.

A current interaction formula, obtained by generalizing the BRESLER formula, is adopted in the design method for calculating the ultimate strength of biaxially loaded composite column. However, this interaction equation has been shown to give consistently conservative results.
No informations are given in the Japanese design method concerning the treatments of the biaxially loaded composite columns.

4.2. CONTRIBUTIONS TO THE PRELIMINARY REPORT OF THE LIEGE COLLOQUIUM.

The contributions proposed in the Preliminary Report |9| give some interesting information concerning the last developments in the field of composite columns.

The German design method is based on the same general principles as those adopted in the ECCS method. For the axially loaded columns, the same form of non-dimensional expression is used to determine the slenderness of the composite column. There are only some differences in the choice of the values of the modulus of elasticity of the concrete. It must also be pointed out that the german approach does not account for the advantage of the enhanced concrete strength due to the effective triaxial containment of the concrete core.

The interaction curve between load and moment in the case of eccentrically loaded composite columns is given by an interesting formula. This analytical function represents the interaction diagram with accuracy but,unfortunately,it is only valid in the case of concrete filled tubes. No proposals are given for encased steel sections.

There is also a difference in the treatment of the long term loading effects. In the E.C.C.S. method, these effects are mentioned, but they are not retained in the design method for the reasons given. In the german approach, these effects are taken into account by giving different values of the modulus of elasticity of the concrete for short-term and long-term loading.

An accurate and economical approach to the biaxial bending strength of composite columns presented by DOWLING, CHU and VIRDI |9|, pp. 165 - 174 represents a very interesting and useful complement to the E.C.C.S. method, as this last is shown to be particularly conservative.

In the E.C.C.S. design approach, the risk of failure by local buckling of the walls of the hollow section in the case of concrete filled tubes is controlled by imposing limit values for the wall slenderness. This procedure is quite restrictive and a new one is proposed. It improves the E.C.C.S. method by the introduction of a reduction factor of the ultimate load of a concrete filled tube, to be applied whenever the slenderness of walls falls beyond the limit values.

4.3. CONTRIBUTIONS TO THE WASHINGTON COLLOQUIUM |11|.

Concerning the application of the equation of superposition proposed in Japan to determine the ultimate strength of composite beam-columns, consideration of effects of creep and end moment ratio have been presented; also, modified design formulae applicable to the allowable strength design. In the same way, application of this method for a seismic design is discussed by WAKABAYASHI |11|.

Another Japanese paper by TOMII and al. is dealing with test results on concrete filled steel tubes; its most interesting content is related to the effect of containment of the concrete in the case of octagonal steel tubes.

FURLONG's proposal |11| tries to comply with a continuous variation for the allowable load between the recommendations concerning the American AISC structural steel design and the ACI regulations, respectively.

5. LATERAL BUCKLING.

During the last seventy-five years, numerous studies have been devoted to this stability problem. The first of them use the bifurcation theory and HOOKE's law, and reviews dealing with this subject are available in many countries [36, 107, 96, 91, 83, 110]. More recently, inelastic behaviour [51, 84, 85, 76, 68, 63, 40, 115] has been considered and effects of restraint by adjacent members investigated by several authors (see references in |7| pp. 141 and 142).

It is well known that the ultimate strength by lateral torsional buckling depends on a lot of parameters such as material and geometrical imperfections, types of loading and of cross-section, support conditions, web deformation, restraints,.... The ECCS Recommendations |5| give a design curve in a non-dimensional way, which may be written

$$\frac{M}{M_{pl}} = \left(\frac{1}{1+\bar{\lambda}_{LT}^{2n}}\right)^{1/n} \qquad (4.1.)$$

where $\bar{\lambda}_{LT} = \sqrt{M_{pl}/M_{crD}}$ is a modified slenderness which depends on the full plastic moment M_{pl} and on the elastic critical moment M_{crD} for lateral torsional buckling. It would be necessary to choose different values for n in order to cover all the above mentioned parameters, but, because of the lack of informations in the inelastic range and for simplicity, only one value, n = 2,5, has been chosen. This choice ensures that the resulting design curve corresponds to a mean value rather than a lower bound, this attitude being justified by the fact that the proposed formulation neglects some favourable effects |7||10|; the design curve is in good agreement with tests results by MASSEY [76], KLÖPPEL and UNGER [63], DIBLEY [39] and, more recently, by KITIPORNCHAI and TRAHAIR [61] and with theoretical solutions studied by LINDNER [67][68] .

The choice of one curve characterized by n = 2,5 is mainly controverted by the Japanese researchers, who propose to use several design curves with respect to the type of beam: rolled, welded or annealed. They have made tests on a lot of welded beams, which show that the experimental results may be far below the E.C.C.S. design curve. YOSHIDA (|9| pp.191 to 196) develops a theoretical solution and takes account of two different types of residual stresses; he investigates yield stress levels, loading conditions, support conditions, and cross-section dimensions. He proposes to use a modified design curve, different from that of ECCS, for welded beams built-up from flame cut plates, of the type

$$\frac{M}{M_{pl}} = \left[\frac{1}{1 + \bar{\lambda}^{1/2n} + \bar{\lambda}^{2n}} \right]^{1/n} \qquad (4.2.)$$

A lot of tests on rolled and welded beams have been made by FUKUMOTO and his collaborators (|9|, pp.233-240), the results of which lead these researchers to criticize the value n = 2,5 proposed by ECCS and to substitute lower values, peculiarly for welded beams. It must be emphasized that FUKUMOTO uses in his calculations the measured yield stress instead of the nominal yield stress as used for the ECCS design curve. It is obvious that, if parameter n is varied, formula (4.2.) yields a series of lateral buckling curves similar to the ECCS curves for centrally loaded bars. It seems that there is again a controversy about using a single or several curves.

The reviewers are of the opinion that, in present field, more test results and/or computer simulations are needed before complete clarity is obtained. Many researchers are pursuing their effort in studying the effect of some parameters, such as practical loadings, restraints ; in this viewpoint,reports |8| |9| |10| |11| |12| contain valuable informations on this subject.

The Soviet approach is based on the concept of equivalent slenderness ratio from which the carrying capacity is estimated from curves similar to buckling curves, which take account of the elastoplastic effects. (cf. POTAPKIN |13|).

HORNE and MORRIS |11| show how it is possible to take advantage of the fact that, in single storey steel frames, columns and rafters are usually restrained at intervals, respectively by sheeting rails and by the purlins.

SOCHOR (|12| , pp. 184-192) also studies the stabilizing effect on a purlin, due to the torsional rigidity of the cover. The ECCS Recommendations(|5|, Rule R. 7.5.) give similar rules, due to the work of PELIKAN, OXFORT and VOGEL. SOCHOR considers the suction effect of the wind which can subject the free flange of the purlin to compression and his results bring an interesting complement to the ECCS Rules.

Various papers, mainly Japanese, are presented at Liège to show that in-plane and lateral buckling of hinged, fixed or tied arches can be analyzed realistically by finite elements. Some of these papers contain parametric studies very useful in actual arch bridge design.

The classical theory of instability of bars and arches, in particular of their lateral buckling, assumes that the cross section of the bar is absolutely rigid. Some years ago, S. BAAR [18] , SEDLACEK [98] and others developed more refined approaches taking account of the cross section distorsion. This distorsion, which is especially important for plate girders or trusses with tubular flanges or chords, may lower the critical load by very many percent. In |12| , pp.177-183 , TARNAI studies this type of problem, but assumes that the truss - or the equivalent web - remains plane and is articulated on the flanges, which seems perhaps an unrealistic assumption.

Various recent contributions concern the lateral stability of the upper chord of a "pony truss", that is a bridge without upper wind bracing. Whilst the recent American work is summarized in B.G. JOHNSTON's Guide (|4|, pp. 394 to 409), P. DUBAS (|9|, pp. 469 to 474) develops a computer program to analyze realistically this complicated problem and CHLADNY (|12|,pp.129-135)develops an analytical solution taking account, as DUBAS, of the initial deformations of the transverse half-frames in their plane.

6. PLATE AND BOX GIRDERS.

An intensive research program was launched in the United Kingdom, afterwards in the Federal Republic of Germany, and in various other nations, as a result of the four collapses of long span box girders that occurred between 1969 and 1971 (Vienna 6/11, 1969 ; Milford Haven 2/6, 1970 ; Melbourne 15/10, 1970 ; Koblenz 10/11, 1971).

The common opinion of the specialists is that these accidents are partly due to local errors (see in this respect the references [70][37][92][93] [29] [58] [86]) but also to the fact that these bridges were designed on the basis of linear theory of buckling of plates, by using very low factors of safety against buckling (generally 1.35) [42] [71] [72] [73] [74] . Now, if, in the case of shear, a diagonal tension field can built-up and guarantee a very large effective safety against collapse, this is not true for heavy stiffened longitudinally stiffened box girders, of which the collapse load may very well be less than the critical load given by the linear buckling theory. This was demonstrated first experimentally by P. DUBAS [42] and theoretically by MASSONNET and MAQUOI [42,71] in 1971. This conclusion was confirmed at the Budapest Colloquium by LUTTEROTH and KRETZSCHMAR (|12|, pp. 231-236). After these accidents, the british researchers tried to develop ultimate strength design methods, the first one, named as MERRISON rules [58] , fulfilled its main task, which was to avoid further collapses. Many progresses were obtained by the british researchers, the main ones being in the fields of evaluation of shear lag and design of transverse diaphragms. However, the MERRISON rules made it compulsory to execute a lot of measurements in the fabrication shop and were accused (even inside the United Kingdom) to put a strong brake on the development of large span box girder bridges. Discussions among specialists are underway in the U.K. to write a new british standard about these bridges, that would be much simpler and still safe enough.
Here, we should mention the general report about "inelastic analysis and design of plate and box girders" presented at Washington by DOWLING |11|. This report gives a very good and up-to-date picture of the ultimate strength of stiffened steel plates, as influenced by various types of residual stresses, and underlines the problems needing further research.
Anyway, more than twelve ultimate strength design methods have been developed since 1968 and especially after the accidents; all of them are based on the consideration of a simplified collapse model.

A short description of these methods and the full references up to August 1976 can be found in the Introductory Report of ECCS (|7|, pp. 153 to 177). It is visible that two different schools of design are developing :

a) the British-Australian school, which takes as a postulate that optimal box girder bridges must possess thick flanges and stocky stiffeners; in that case, the postbuckling strength is small and is neglected in the methods. These consider therefore the buckling deflection of the flange to be cylindrical and the stiffeners as working independently as isolated columns formed of the stiffener itself and the effective width of the adjacent part of the sheet, influenced by its initial crookedness and its welding residual stresses

b) the continental approach, which favors lighter steel boxes where the metal is more expanded. The first method of this kind was published by MASSONNET and MAQUOI in 1971 [71] and was found later in good agreement with large scale model tests. [73][74] . Several more refined theories have appeared since that time. The main one was developed at Karlsruhe University by Prof. STEINHARDT, VALTINAT and Dr. RUBIN [94][102] [103] . It was found in good agreement with large scale tests.

Various other improvements have been obtained by BILSTEIN [28] . Dr. CHANG SUNG PIL extended the Karlsruhe method to closed section stiffeners [33] . Prof. DJUBEK showed how the MAQUOI-MASSONNET method could be improved by analysing more carefully the non linear coupled equations of the membrane plate (|12| pp. 263 to 272).

Up to now, the Task Group 8/3 (Plate Buckling) has been unable to select, from these twelve models, the more appropriate for practical design. As many plate and box girders are now under construction, this Task Group, working in connection with the West german subcommittee on plate buckling, developed a set of simple provisional rules, still based on the linear buckling theory of plates, but where the varying magnitude of the postbuckling strength reserve is taken account of by some suitable correction factors applied to the critical buckling stress (see Appendix 4 of |5|). These provisional rules impose also technological requirements to avoid the reccurrence of the accidents referred to above. These rules have been applied, in Belgium, to the design of large span bridges totalling several billion of belgian francs. In his general report about the tendencies in the eastern European countries (CMEA), DALBAN |13|, believes also that it is possible to adapt the classical buckling theory to avoid too complex postcritical solutions. Some of these countries continue to use the linear theory in their specifications and offer in option a postcritical approach. In the computation of ultimate shear web panels, they take into account the postcritical effects but are more cautious than BASLER's basic approach. For plate girders, the various ultimate strength models presented are around ten. All of them are improvements of the semi-diagonal tension field theory used by BASLER and THURLIMANN in 1960 [20] , which itself may be tracked back to H. WAGNER [117] and even RODE [90] . The most general- and possibly also most correct-seems to be the last version of the Cardiff model [89][47] Precisely, this type of model raises another fundamental question: the Cardiff ultimate strength design method , at least in its part devoted to the shear strength of plate girders, considers an ultimate strength situation in which, it is taken account of the strength of a "panel mechanism" involving four plastic hinges.

Present reviewers are in favor of plastic design for building girders subjected to quasi-static load, but not for continuous bridges where a certain danger of fatigue is always present. They agree therefore with the viewpoint presented in 1976 [41] by the Tchecoslovakian researchers, professor SKALOUD and DJUBEK, which has been incorporated in the latest edition of the Tchecoslovakian Specifications: while the ultimate design rules for buildings (developed by Prof. SKALOUD) are plastic, those applicable to bridges (developed by Prof. DJUBEK) are elastic, but take account of the beneficial effect of the membrane stresses and are therefore non-linear.

Another problem of interest is related to tolerances of fabrication, because any imperfection may affect the serviceability and the behaviour till the ultimate strength. At present time, it has not received a complete answer, but a task group of Commission II of IABSE, under the leadership of the senior reviewer, has in charge the preparation of a state-of-art report on the subject; the viewpoints of many countries are summarized and measurements of any kind are gathered. In the same vein, SKALOUD (|12|, pp. 219 to 227) announces extensive measurements of the geometrical imperfections of several bridges in Tchecoslovakia.

An important research work was coordinated since 1973 by the Task Group 8/3 (Plate Buckling) of ECCS, of which the senior and junior reviewers are respectively chairman and secretary. This work consisted in the review of the proposed ultimate strength methods for plate and box girders, their description, and their statistical comparison with the full set of available valid tests made throughout the world. For the full details of this study, the reader is referred to |7|, pp. 153 to 177. Here, we shall only give, in two tables 4.1. and 4.2., the main characteristics of the various methods and explain the statistical evaluation was made. The ratio

$$r = \frac{\text{ultimate strength computed according to method X}}{\text{experimental observed ultimate strength}}$$

was computed for all tests and all methods ; the mean $m = \bar{r}$ of all values r and their standard deviation was computed. The results obtained are summarised in tables 4.3. and 4.4.

Among the problems that were unsolved until recently, let us mention the choice of the relative rigidity $\gamma = \frac{EI}{bD}$ of the stiffeners, in order to keep these straight under the critical stress and, eventually, in the postbuckling range up to failure. More than twenty years ago, the senior reviewer proposed to use a rigidity $\gamma^{**} = m\,\gamma^{*}$, (where γ^{*} is the "strict rigidity" given by the linear buckling theory) and to choose m equal to 4 - 5. This value has recently be justified by theoretical computations using large displacement finite element programs (|9| pp. 273 to 278). A parameter study can now be imagined, that would furnish the value of m to use as a function of the type of stiffened panel, type of stressing, initial imperfection and postbuckling performance requested. Unfortunately, under present conditions, this study would be extremely costly. In the mean time, we must mention the appearance of remarkable papers, such as that of CRISFIELD (|9| pp.427 to 432) on the ultimate behaviour of an imperfect box girder containing an imperfect diaphragm as well as the study of FUJITA and YOSHIDA |11| on the ultimate strength of the stiffened haunched angle of a portal frame. Therefore, there is place for simplified methods, like HOYER's study (|12|, pp. 243 to 250) which is based on the concept of drift force. The 12 new large size tests executed in Prague by SKALOUD, LHOTAKOVA and KARNIKOVA (|12|, pp. 219 to 229) are especially interesting, because they show that, in order to remain straight up to collapse, the stiffeners must have their theoretical strict rigidity multiplied by 4 to 5.

In this period of high salaries, the workmanship expenses produced by the adjustment and welding of transverse stiffeners should be avoided if possible. For this reason, a new type of plate girder without stiffener was developed in Sweden ([54],[26]) . Due to the efforts of Commission 11 of ECCS, the Swedish design method has become an European method [60] . The corresponding rules have recently been tested experimentally by FREY [50] and found safe. This type of girder was found to be very slightly sensitive to rather large initial geometrical imperfections. SZATMARY, by 16 tests on this type of girder - but with rectangular tubular, very stiff flanges - has shown (|12|, pp. 251 to 261) that, in that case, the postbuckling range was especially extended. SKALOUD remarks, in his general report of the Budapest Colloquium |13|, that the beneficial effect of this flange rigidity is automatically accounted for in the ROCKEY - SKALOUD design method for plate girders.

DJUBEK and BALÁZ (|12| pp. 268 - 272) show that the MAQUOI - MASSONNET method [71] may be improved by using several terms (instead of one) in the FOURIER expansion of the transverse deflection w and adopting, as collapse criterion, the criterion $\sigma_{max}^{membrane} = \sigma_r$ instead of $\sigma_{mean}^{membrane} = \sigma_r$.

According to POTAPKIN |13|, the soviet viewpoint on the practical design of compressed flanges of steel box girders is based on the fact that the critical stress can only be fully developed if the stiffeners remain rigid up to collapse. Formulae are presented in order to comply with that requirement for the inertia of the longitudinal - as well as for the transverse stiffeners. It seems that two ways are open for a design: a first one based on the concept of strut and a second one which refers to the behaviour of an orthotropic plate. Recommendations are also given for the torsional buckling of the stiffeners themselves. The problem of simultaneous loadings is solved by means of interaction curves.

The problem of the ultimate strength of corrugated webs was analyzed by GACHON in Liège (|10|, pp. 339 to 342) and by LIBOVE in Washington |11|. DWIGHT discussed |11| the effect of welding on the ultimate strength of plates.

The ultimate strength of plate girders to localized transverse loads applied to one flange (case of the girders supporting overhead cranes and of bridge plate girders launched on rollers) was studied theoretically and experimentally by BERGFELT [26], P. DUBAS [42], K.C. ROCKEY [88] and M. SKALOUD [100]. New tests and studies are under way in Liège to study the case where the web thickness is increased in the vicinity of the flange, as is customary in Belgium.

The problem of shear lag was extensively studied in the United Kingdom by DOWLING and MOFFATT [82], in the frame of the MERRISON researches. They showed that the reduction coefficients established for isotropic plates and which are presented in practically usable form e.g. in the German Specification DIN 1073, are still valid for orthotropic stiffened flanges. A study by SCHINDLER (|12|, pp. 287 to 297) shows good agreement with the above british research. HORNE showed in London in 1976 that shear lag effects are reduced by plastic action (|3|, pp. 1 to 23), provided the plates are sufficiently thick, but the reviewers question the fact that such a plastic action can be invoked in the case of bridges. ROIK demonstrates the same fact (|3|, pp. 169 to 195). The reviewers have shown at the same London conference (|3|, pp. 89 to 107) that there were two shear lags : a first-order shear lag due to shearing in-plane deformations and a second-order shear lag due to postbuckling effect. In imperfect stiffened plates, both types of shear lags occur together, but they have shown that it was safe to compute their combined effect by multiplying the two reduction coefficients corresponding to the separate phenomena.

7. INTERACTION BETWEEN VARIOUS MODES OF BUCKLING.

To be brief, we shall here concentrate on the most important problem, that of the interaction between general buckling and local plate buckling in thin-walled members with closed section, subjected to axial compression. According to the classical theory of F. BLEICH [80], the member has optimal dimensions when the two critical stresses are equal

$$(\sigma_{cr})_{\text{buckling}}^{\text{general}} = (\sigma_{cr})_{\text{buckling}}^{\text{local}}$$

from which it is easily seen that, the largest slenderness ratio $\lambda \equiv \frac{Kl}{r}$, the largest the thinness b/t of the walls of the column. (fig. 1 and 2)

This "naive approach" has been strongly criticized in a series of theoretical papers due to VAN DER NEUT [111][112], KOITER and KUITEN [64], THOMPSON [106] and several others. [66]
The general theory of buckling developed in 1945 by Professor KOITER [65], which dis-

cusses in detail the case where two buckling modes occur simultaneously, should in principle be able to predict the collapse load. However, this theory does not take account neither the eventual residual stresses present in the tube nor the plastic action, so that engineers are not convinced of the validity of its conclusions [74] . However, a lot of progress has recently been made to overcome these difficulties, in particular by TVERGAARD [108] [109]. The unstable (explosive) character of the buckling of such members, predicted by above theories, does not seem to have been confirmed experimentally on industrial members.

Practically, the classical approach, which is bifurcation theory based on the concept of effective width, was developed long ago for cold formed light gage steel by G. WINTER, using his well known expression for the effective width and its use was extended in 1969 to heavy construction (see Appendix of the AISC 1969 Specif.) At the Washington Colloquium, WINTER and his collaborators present a bifurcation theory |11| enabling to investigate the overall column stability after local buckling of its plate elements. This remarkable study, based on WINTER's effective width formula, assumes that A_{eff} is not influenced by the general crookedness of the column and is based on the concept of modified tangent modulus discussed in Section 3. It is in good agreement with the test results.
A bifurcation type solution scheme for the analysis of torsional- flexural buckling of locally buckled beams and columns, based on the finite element method, is given by WANG and WRIGHT |11|, who conclude that a considerable amount of post-local buckling strength is available. This conclusion is puzzling to the reviewers, because it seems that all papers of the KOITER - VAN DER NEUT - THOMPSON school tend to demonstrate the contrary.
It must now be recalled that the bifurcation concept of ideally straight members has been abandoned by ECCS (see section 2). Researches by KLÖPPEL [62] and by SKALOUD [100][101] have shown that, due to initial crookedness of the member, the effective widths of the various walls, not only vary continuously under increasing load, but are different for the concave and convex sides of the column. Electronic computer programs have been developed by SKALOUD and NAPRSTEK [101] and by DOWLING, BASU and DJAHANI [21] and CRISFIELD [38] . However, they are too costly for everyday use. There is a need, therefore, for simplified formulae. Such a formula was proposed years ago by BAAR and HICK [19] . Extensive buckling tests are currently under way at University of Liège to help clarify this nasty problem.

8. BEAM-COLUMNS.

The subject of beam columns has been extensively studied since the early days of BIJLAARD's studies [27] . From 1936 to 1977, the senior reviewer has referred in his synthesis paper [79], to more than 250 papers discussing this subject.
In most specifications, the formula in use is that developed in parallel in Belgium and in the United States, namely

$$\frac{N}{N_K} + \frac{\mu}{\mu-1}\,\frac{M_e}{M_{pl}} \leqslant 1, \tag{8.1.}$$

with $M_e = C\,M$ and $\mu = N_E/N$.

In the case of a trapezoïdal primary moment diagram, the expression

$$C = \sqrt{1 + 0,3\, r^2 + 0,4\, r} \tag{8.2.}$$

with $r = M_2/M_1$ was proposed by the senior reviewer in 1954. [78]. Slightly later, similar expressions were proposed by HORNE [55], SALVADORI [95][96] and AUSTIN [15] Fig. 3 shows that all these expressions are practically equivalent.

Recently, the method of finite elements has been applied to this problem, notably by VINNAKOTA[113][114][115], CHEN and SANTATHADAPORN [34] and the senior reviewer's paper [79] gives a good account of the analytical methods proposed, while CHEN's papers and recent book are more centered on numerical methods for computers, based on finite elements.

In 1975, ECCS has adopted another approach, which is slightly more economical. The new formula is based on the concept of a representative parameter of imperfection. This involves replacing the buckling load N_k by a fictitious eccentricity e^* chosen such that the extreme case of the centrally loaded column is satisfied :

$$\frac{N}{N_{pl}} + \frac{\mu}{\mu-1}\,\frac{Ne^*}{M_{pl}} \leqslant 1.0 \qquad (8.3.)$$

The value of e^* can easily be derived from above formula. Failure of the column is supposed to occur when the yield stress is reached in the outer fibers. The modified interaction formula for a *beam-column* is now :

$$\frac{N}{N_{pl}} + \frac{\mu}{\mu-1}\,\frac{(C\nu M + Ne^*)}{M_{pl}} \leqslant 1.0. \qquad (8.4.)$$

There is an obvious gain over the use of N_k in this formula; indeed, because of the presence of a bending moment M, the axial force N will be reduced and consequently also $N.\ e^*$.

Particularly for larger slenderness ratios, the advantage of using e^* is considerable. The modified interaction formula is completed by introducing a coefficient for lateral torsional buckling ν and a coefficient C which compensates for unequal end moments.

The ECCS interaction formula has been checked with test results and is shown in good agreement as well for elastic design as for plastic design.

For biaxial bending, a simple extension to the uniaxial interaction formula should be given by straightforward adding of the bending stresses :

$$\frac{N}{N_{pl}} + \frac{\mu_x}{\mu_x-1}\,\frac{(M_x + Ne_x^*)}{M_{plx}} + \frac{\mu_y}{\mu_y-1}\,\frac{M_y}{M_{ply}} \leqslant 1.0 \qquad (8.5.)$$

However, the American viewpoint is that the linear combination, while safe, is generally too conservative. The Dutch set of formulae has two versions: an elastic one, based on the collapse criterion $\sigma_{max} = \sigma_r$, and a plastic one, predicting the real elastoplastic collapse of the member (|7|,rules R4 and R5).
On the other hand, by analyzing numerically a large number of cases, CHEN has been able to develop an empirically set of design formulae, which are non linear and seem therefore slightly more economical than the Dutch approach.[35]

A large series of buckling tests (about 100) on compressed and biaxially bent I beam-columns is underway now at University of Liège. The first series of tests indicates a good agreement with the refined analysis of VINNAKOTA, as well as with the interaction formulae proposed by ECCS (|10| pp. 203 to 206). Added to the available biaxial tests of KLÖPPEL - WINKELMANN, CHUBKIN and others, they will decide on a statistical basis about the safety and accuracy of the various interaction formulae presented these last years.

As already mentioned in Section 2, above design formulae apply in all rigor only to "isolated" beam-columns. The rotational restraint at the ends of the column is taken into account by the introduction of an effective length factor.

Of course, this approach is an oversimplification: the beam-column under consideration is usually a member of a frame, and there is an interaction between that column and the remainder of the frame, especially when plasticity is taken into account. The problem of this "restrained" beam.-column has been analyzed by a number of authors, notably at Lehigh University (U.S.A.), and by GENT [52] and R.H. WOOD[119] in Great Britain.
There is obviously here a strong interaction between the instability of the beam-column and the general elasto-plastic behaviour of the remainder of the frame. For more details, the reader should study the ECCS Recommendations about plastic design, which are scientifically explained in the book by professor SAVE and the senior reviewer [97] .
An excellent state of the art review of the recent research - mostly finite element and other numerical simulation methods is given at the Washington Colloquium by Professor CHEN, whose two recent volumes on the same theme should also be mentioned [35] . This review does not cover older researches, either American or European, and should perhaps be complemented by a similar SoA report written by the senior reviewer 2 years ago [79] .
Anyway, CHEN proposes interaction formulae (based on refined parametric numerical computations), which are probably the best available presently and, therefore, their essential formulation will be given hereafter.
At an end or braced location of the beam-column, the following non linear "biaxial plastic hinge formula" must be satisfied :

$$\left(\frac{M_x}{M_{pcx}}\right)^{\alpha} + \left(\frac{M_y}{M_{pcy}}\right)^{\alpha} \leqslant 1 \qquad (8.6.)$$

where $\alpha = 1.6 - (P/P_y)/2\ln\,(P/P_y)$ for wide flange shapes having a width to depth ratio from 0.5 to 1 and $\alpha = 1.9 - (P/P_y)\,/\ln\,(P/P_y)$ for a square box column.

To check stability between braced points, the following equation should be satisfied :

$$\left(\frac{C_{mx}\,M_x}{M_{ucx}}\right)^{\beta} + \left(\frac{C_{my}\,M_y}{M_{ucy}}\right)^{\beta} \leqslant 1. \qquad (8.7.)$$

where C_{mx}, C_{my} are the equivalent moment factors discussed above in this section ; M_x, M_y are the greatest of the moments applied at either end of the beam-column and β is given conservatively by formulae

$$\begin{aligned} \beta &= 0.4 + P/P_y + B/D \geqslant 1 && \text{when } B/D \geqslant 0.3 \\ \beta &= 1.0 && \text{when } B/D < 0.3 \end{aligned} \qquad (8.8.)$$

$$\beta = 1.3 + \frac{1000\,(P/P_y)}{(L/r)^2} \geqslant 1.4 \qquad (8.9.)$$

for H columns and a square box column, respectively.
Three automatic column selection programs were developed, in the course of last year by the Canadian Institute of Steel Construction (see paper by P.G. SANDFORD in |11|. Obviously, there are excellent tools for relieving the designer of tedious tasks, but they must always be adjusted to the currently best approach.
An extensive numerical comparison between the ECCS, the CMEA and the Russian rules is given in the report by DALBAN et al (|10| pp. 217-218) for $r = M_2/M_1 = +1,0,-1$ and relative excentricity $m \equiv e/r = 1$ and 5. Unfortunately, the recent American approach by CHEN has not been included in this comparison.

9. BUILT-UP MEMBERS AND TRIANGULATED STRUCTURES.

Rather few new material was presented at the colloquia about built-up members. For the theory adapted to the new principles of safety and the experiments supporting it, the reader should refer to the Manual on Stability (|7| pp. 119 to 126) and to the Preliminary Report of the Liège Colloquium (|9|, pp. 119 to 142).

Triangulated structures are probably among the first in the world which have ever been erected. Nevertheless, in the viewpoint of stability, more sophisticated theoretical and experimental analysis are now adopted in order to improve the design and to have a better knowledge of their behaviour. Transmission towers are probably the most typical example of triangulated structures; it is the reason why stability problems related to angles draw the attention of the researchers in this field.

The reviewers feel that a real engineering effort has been accomplished in this way and that further progress could only be obtained by very costly parametric investigations based on computer programs describing realistically the interaction between all the - imperfect - elastoplastic bars composing the whole structure.
Let us mention briefly the papers presented by DJUBEK |13| and MELCHER |12| at the Budapest Colloquium, that contributes to our knowledge of the buckling of angles used as chords of these towers.

The basic buckling curves adopted for angles - and by extension to tubular members of transmission towers, and proposed in the ECCS Recommendations |5| are valid for equal and unequal legs, rolled or cold formed angles. The influence of local and torsional buckling is taken into account by introducing a conventional yield stress $\overline{\sigma}$ which depends on the thinness of the legs and on the manufacturing procedure - rolled or cold formed. Design rules are proposed to evaluate the effective buckling length and thus the slenderness ratio, as well as for legs and chords as for compound members as legs. In addition, it is proposed to design the unstressed stabilizing members - which reduce the effective length of main legs and bracing - by assuming that they are subject to an hypothetical force equal to a percentage of the leg (or bracing) load and various values of this percentage are given with respect to the slenderness ratio of the leg.

Papers presented and discussed at the several colloquia are devoted either to a comparison between experimental results and design rules, or to a comparison between buckling stresses obtained by a sophisticated theoretical analysis (computer simulation) with those given by more classical solutions. They tend to conclude that classical analysis of stability problems can lead to too conservative solutions.

For transmission towers,it is a common practice to apply an effective length factor for the design of web-members also for buckling out of the plane of the truss; in fact,the restraining effects are more pronounced for slender members such as the diagonals of a tower. English tests could rise some doubts on the buckling stresses given in the Manual |7| for web-members with stays, bending about an axis parallel to the angle leg; such conclusions are not in agreement with current testing practice on actual towers and, even of the difference may occur from the testing conditions, a further examination in this field should be advisable.

In his contribution to the Liège and Budapest Colloquia (|12|, pp.75-83) (|10|, pp. 281-283) SCHULZ has shown that the end restraint accounted for by the

K factors at working load level is not present anymore at ultimate load level and that, therefore, no use of effective length reduction should be accepted. Tests are presently under way in various places to clarify this much up to date problem. An elastoplastic computer program taking account of the welding residual stresses has been developed at Liège and is presently calibrated with the results. A publication is pending.

The limited validity of the concept of effective length as used in the design of rigid jointed frames was already emphasized by the reviewers in section 3. CHEONG SIAT MOY |11| criticizes this concept in the design of frames. He illustrates pitfalls resulting from this concept, presents the basis behind a rational design method not considering the K factor and, last, proposes a fundamental formulation for a possible modification of the design specifications, in order to eliminate this factor. It must, however, be emphasized that each user must keep in mind that the concept of effective length is only a simple and convenient short-cut, which is only used because there is nothing better, which must be used with some intelligence and critical spirit, and that nobody is obliged to use it if he knows a better, yet sufficiently simple, solution .

In connection with the theme "triangulated structures", some other interesting problems have received partly a solution. French researchers have presented, on basis of tests undertaken by CIDECT, useful charts and formulae for the determination of the effective lengths of web members in welded lattice girders in hollow sections; it is clearly shown that torsional stiffness can reduce the effective buckling length in an appreciable manner.

A special field of interest is the scaffold assemblies, which are structures very sensitive to imperfections and for which repeated loads can increase their imperfections and reduce their ultimate load. It must be emphasized that such structures may not be equally successful for identical jobs and that it would be interesting to know more about the discrepancy loads, this discrepancy being probably greater than for other structures.

An unsolved question concerns the effect of the so-called "secondary moments" due to the rigidity of the connections in trusses. This question becomes more and more up-to-date because square and rectangular tubes are increasingly used to fabricate welded trusses without gussets. Whilst the design of the joints has advanced quite well recently through the work of MOUTY (|9|, pp. 491 - 496)and others, full size experiments have recently shown that collapse could be brought about, either by brittle fracture in the connection itself, or by local collapse of one bar of the truss near to a connection, due to the secondary moments.
The various Specifications are very vague in this respect, and either ignore the secondary bending stresses or impose to take account of them through elastic calculations (made easy through classical computer programs like STRESS) or finally impose their consideration, but allow an increased allowable stress, exceeding the normal one by 10 or 15 per cent.

10. FRAMES.

The question of the stability of elasto-plastic frames, especially those of multistorey buildings, involves as much plasticity as stability. It is impossible to give here an exhaustive treatment of the theme.

It seems that the classical *elastic* method, depending on the concept of allowable stress, and using first-order analysis and the concepts of effective length and beam-column interaction formulae, is increasingly abandoned.

WANG |11|, by analysing on a computer a large number of design examples and, particularly, those given in Chapter 15 of JOHNSTON's Guide |4|, concludes that the elastic solution of "a structure with partially plastic column will not only be overly conservative,but can also be unconservative".
Bifurcation theory is now easy to extend to three-dimensional frames, as shown by CHENG |11|, but the assumptions of bifurcation and elastic behaviour restrict very much the practical interest of such computations.

An interesting paper by MAZZOLANI, DI CARLO and PIGNATARO |11| presents the results of the computer analysis of 600 plane elastic multi-storey frames subject to vertical and horizontal loads. The paper is based on KOITER's general theory of stability and uses a finite element model due to BUDIANSKY. The paper concludes that "the design procedures based on an indirect control of the lateral displacements, as considered in codes for checking the overall instability of multi-storey frames, are valid provided that the fictitious horizontal forces equivalent to the PΔ effects are linked to appropriate values of safety factors, which must take into account the additional aspects included in the elastic analysis". Incidentally, let us recall the P-Δ method of DAVISON and ADAMS |128|, who were the first to attack the problem in this way.

Extensive studies of the ultimate strength of frames designed by the allowable stress method, with and without using the effective length factors, have been conducted by LU, and al at Lehigh University 122 , 123 . Design procedures are currently under development.
Commission 5 of ECCS, headed by the senior reviewer, has developed a set of design rules |5|, which seem usable and safe(|9|,pp. 529 to 534), but which are rightly criticable. The very simple way of taking account of second order effects in sway frames by using the MERCHANT-RANKINE formula as modified by WOOD was criticized by CLARK |11| and VOGEL |11|. VOGEL |11| says that this formula, coupled with the "effective length concept", can lead to results which are either too far on the safe or too far on the unsafe side and maintains that complex systems like unbraced frames cannot be covered by a simple formula, but by refined second order methods. At the same time, he warns against using sophisticated computer programs which are a "black box" for the majority of designers.
HALASZ(|12|, pp. 147 - 156) presents a modified version of the MERCHANT-RANKINE where initial deformations can be included. On the other hand, a vast majority of designers agree that treating all multi-storey frames by refined computer methods is going too far.

The most comprehensive account of the elastic approach is that given in B.G. JOHNSTON's Guide (|4|, pp. 410 - 436). The commentary of the ECCS Rules, given in the first draft (1976) of the ECCS Recommendations |5|, seems to be, in certain parts, insufficient or not clear enough. These defects were remedied, to a large extent, by the book of plastic design written by the senior reviewer and prof. M. SAVE [97].

The historical (also called biographical) second order elastoplastic method analysing the frame under increasing loads controlled by increasing multiplier λ is obviously the best of all, but is very time consuming. Therefore, both HALASZ |12| and the senior reviewer find it unnecessary. This point must be cleared up by further researches. Anyway, UHLMANN |13| has shown that, in customary conditions, the classical concept of (concentrated) plastic hinge is sufficient. In other terms, the effect of the spatial extension of these hinges is noticeable only in exceptional cases.

Among the many present controversial points, let us mention :

a) the value of the out-of-plumb to adopt. BEAULIEU and ADAMS, basing on a statistical analysis, recommend 1/500 ([9], pp. 23 to 30 ; [11]) whilst ECCS has adopted 1/200 in [5].

b) the three-dimensional effects, to which VINNAKOTA [7] and KOLLAR ([12], pp. 411 - 422) have brought contribution.

11. SHELLS.

Shell stability is one of the fields of structural stability which is the most difficult to be reduced to practical design formulae. The first reason is that the theoretical solution of a range of shell problems, related to complex shapes and loading cases, is nearly impossible to be found; the second one is that, even if this theoretical solution is known, there is a very high discrepancy between experimental and theoretical results. It is usual to find that the experimental buckling load is only 1/3 to 1/5 of the theoretical critical load.

It is now well known that such a discrepancy is due to the fact that the theory is applied on idealized structures, whilst actual shells always present initial randomly distributed imperfections, whose effect - unlike the plates -is to lead to a limit load lower than the critical load, with afterwards a further drop of the load accompanied by large displacements. Thus, in many cases, shells are very sensitive to deviations from the ideal shape and the ideal membrane loading.

A lot of papers have been published on the subject during the past 30 years and some of the most interesting ones may be found through the review papers [57][46] and [31].

In spite of the complexity of shell problems, and of the recent constitution of Task Group 9 of the ECCS, the chairman of this latter, Prof. VANDEPITTE, had the merit to present a first draft of design specifications in Chapter 10 of the Introductory Report [7] This chapter is related to a very limited number of types of shells and of loading cases. The application of these specifications constitutes the pragmatic approach for the designer. In the case of complex shell structures, above design option is no more possible and a successful design requires the availability of an appropriate design program.
At present time, the available programs have different levels of capability: bifurcation analysis, bifurcation buckling from a non-linear prebuckling state, general collapse analysis, imperfection sensitivity, postbuckling analysis, dynamic analysis.

Let us return to ECCS design specifications. They cover (|7|, pp.275-295) the following problems :
unstiffened cylinders subject to axial compression, or to axial compression combined with internal pressure, or to pure bending in the meridional direction, or to an axial compressive load combined with bending, also ring-stiffened cylinders under uniform external pressure, and unstiffened spherical shells under uniform external pressure. The idea is that these tentative specifications will be amended and enhanced in the years to come and will be included eventually in future editions of the "Recommendations for the Design and Construction of Steel Structures" edited by the European Convention for Constructional Steelwork.

The present draft of ECCS specifications mirrors the practically usable attainments of the state of the art, and it provides the practicing designer with a workable tool which, it is hoped, reconciles safety, the very first requirement, with economy.

An important feature of the tentative specifications is that, for each type of shell, a limiting imperfection is given, beyond which the rules are not applicable. This boils down to subdividing shells into two classes, bad ones whose imperfections exceed the limit, and good ones whose imperfections do not exceed the limit. Actually, Task Group 9 would have preferred to classify shells, as constructed, in more than two quality classes, defined by several specific ranges of imperfection, and to allow higher loads and stresses for shells with lesser defects, or, better still, to determine the allowable loads as a function of the actual imperfections. But the task group did not feel able to achieve this, considering that it did not have enough information regarding the relationship between actual buckling loads and imperfections of various magnitudes.

The Preliminary Report of the Liège Colloquium |9| contains eight papers pertaining to shells : in two of them,approximate formulas for the collapse loads of axially loaded cylindrical shells are developed; one studies the stability of buried cylindrical and spherical shells, one tells about buckling experiments on shells with positive Gaussian curvature under *internal* pressure, two contain the results of calculations and of a model investigation regarding two actual failures, and two papers are related directly to the design code proposed in the Introductory Report. |7|

The reviewers believe that the stability problem is frequently so complicated - especially in shell branched structures - that resort should be taken to realistic computer programs,whenever this can be done economically. BALTUS and the senior reviewer have shown (|9| , pp. 609 to 618) how, in the case of axisymmetric shell structures, the most refined BOSOR 5 program developed by Lockheed could contribute to explain two recent collapses that occurred in Belgium and to design safely and economically this type of shell structure. The paper presented by CERNY at Budapest (|12|, pp. 309 to 314) while quite valuable, tries also to solve above type of problem,but the use of BOSOR programs is more convenient from the viewpoint of the designer. Several papers WHITE (|12|, pp. 355 to 361) - OSTAPENKO |11| are devoted to the effect of welding stresses on the stability of stiffened cylinders used in offshore structures. Various papers (TENNYSON |11|,DULACSKA (|12|, pp. 323 - 328) endeavour to find formulae and interaction formulae for spherical and cylindrical shells which are safe, yet sufficiently accurate for practical purposes.

A practically important problem is the stability of open cylindrical shells under the action of self weight and wind; this situation occurs, not only during erection of cylindrical gasoline tanks, but also after the completion of such tanks with a floating roof. A useful analysis of this problem, based on model tests, was given by ZIOLCO (|12|, pp.431 - 438), who proposes simple interaction formulae.

At the Washington Symposium, BEN KATO |11| presents interesting experimental results on local buckling of steel circular tubes, which occur for relatively small D/t ratio and takes place after some plastic deformation. Simple empirical formulae are proposed, which predict the maximum stress and strain in terms of a single nondimensional parameter; a method to evaluate the rotation capacity is suggested.

GOLDBERG |11| describes a computer procedure for calculating critical combinations of longitudinal load and external pressure for ring-stiffened cylindrical shells. This method could be competitive with the BOSOR programs; its advantage is that it seems to be applicable to non-symmetrical buckling under any non symmetrical load but,it treats only one shell at a time and the system of equations directly depends of the geometrical topology of the shell. A comparison of results with other methods should have been appreciated.

BUCHERT |11| emphasizes too the differences between Chapter 10 of the ECCS Manual and Chapter 18 of the SRRC Guide, with respect to spherical shells under external pressure. ECCS chapter covers unstiffened spherical shells only, whereas the scope of SSRC Guide is larger; the first one is based on a lower bound of experimental results. On the contrary, American rules refer to theoretical results and take account of any value of imperfection whilst the validity of European recommendations is limited to a certain amount of imperfection. If the comments brought by BUCHERT are generally right, the reviewers believe, however, that it is necessary to recall that the European Task-Group has been constituted much later than the American group and that its work is still in progress.

Incidentally, what BUCHERT calls in |11| a factor of safety in the ECCS rules seems to the reviewers to be a coefficient taking account of the unfavorable effect of imperfections.

12. DYNAMICS.

Only at the Washington Colloquium a theme was devoted to dynamics. Most of the papers presented are dealing with seismic design. BEN KATO |11| emphasizes the fact that the earthquake loading is frequently the controlling one because of the greater stresses and deflections it induces in the structure. He comments on the inelastic response analysis currently used and on the limit design and the deformability of steel structures. This paper gives in condensed form a comprehensive summary of seismic design in Japan.

KARADOGAN |11| criticizes one of the main assumptions made in studies related to aseismic design, according to which the effect of axial forces is neglected; in order to bridge this gap, he studies the effects on the behaviour of building structures of P-Δ moments and of rotatory inertias. He introduces a coefficient C as a criterion for a decision about the order of importance and the necessity of using a second order analysis. The simplified approach proposed by the author is applicable to any type of structural system which has a similarity between buckling and free vibration mode shapes and excited by lateral loads; it is also, according to the author's opinion, suitable for practical applications as well as for codes because it does not require the use of a computer and any kind of iteration.
The static and dynamic stability of elastic imperfection - sensitive reticulated shells has drawn the attention of HOLZER [11], who describes a numerical solution technique for the determination of nonlinear equilibrium paths and critical loads.

13. SPECIAL PROBLEMS - EXPERIMENTAL METHODS.

Present section is devoted to all kinds of stability problems which are not directly in connection with above mentioned "classical" themes. For this reason, it is difficult to find a common denominator to the papers presented at the different meetings of the "travelling colloquium". Therefore, the reviewers will only try to give here the main trends of research.

The interaction between plate stability and dynamical behaviour becomes important because the trend is to design structural elements taking account of postcritical strength. In this field, the phenomenon of "breathing" of the web under repeated loads may induce fatigue cracks in the HA zone of connecting welds; up to now, it has only been investigated through tests and more research in this field is needed. Earthquake design seems to attract the favor of some researchers, mainly in the countries which are concerned with seismic vibrations.

Fire resistance is another problem which must retain our attention, and namely the stability of braced and unbraced frames at elevated temperatures. It must be emphasized that the study of such behaviour depends on the actual distribution of temperatures inside the building and of the evolution of the temperature - dependent material properties.

Because of the increasing costs of the structures, a new trend of research is to make them more competitive by increasing their efficiency (use of high strength steel, prestressing,...) or by optimizing the dimensions and the properties of the carrying structural elements, by taking account of all the stability constraints. Another way is to study in more detail the stability behaviour of a structure as a whole, instead of that of all their components, considered separately; the general feeling is that such a concept might lead to a more economical construction.

The use of the computer in any kind of stability problem has opened the way to a possible extensive field of research. It must, however, be kept in mind that the use of computer programs, using large displacements and elasto-plastic approaches, is lengthy and costly; therefore, such a way must remain a tool for research conducting to the elaboration of design formulae and design practice rules.

As, according to H. POINCARE, "L'expérience est source de toute vérité", experiments will remain the main valuable test for checking theoretical studies, but the set of measurements desired is often lacking. For the deformation measurements, data-loggers with automatical sweeping of several hundreds of channels are available, and for deflection measurements, a photogrammetric method has been developed mainly in the east-European countries, which may give a complete figure of the deflection field of a plate.

Among the very many other special problems that were studied recently, let us mention briefly :

1) the problem of tapered beam-columns (cf. HORNE and MORRIS |11|).
2) the problem of creep buckling of viscoelastic structures (cf. KRAJNOVIC |11| and BYCHAWSKI |12|) or of sandwich panels having viscoelastic polyurethan or similar core (NAGY |12|, MARSH |11|).
3) the problem of instability of prestressed steel structures, which seems to interest particularly in Eastern Europe (TOCHACEK and FERJENCIK |12|, pp. 401 - 410).

14. GENERAL CONCLUSIONS.

Very considerable progresses have been made these last five years in the field of structural stability. The general tendency is towards a more realistic appraisal of the problems with all their complexity (geometric and structural imperfections, elastoplastic or viscoelastic behaviour, large displacements), which may be successful because of the possibilities of the electronic computers and an increased international cooperation.

However, there remain several fields in which the specialists of various continents or nations disagree. As, in addition, technology presents us continuously with new problems (e. q. structural problems involving ocean oil pipelines and off-shore structures), structural stability research does not risk to become inactive in the years to come.

In closing this report, the reviewers wish to mention that it is planned to write a more extensive Summary Report of all four sessions of the Travelling Colloquium, that should abstract, summarize and compare the various contributions.

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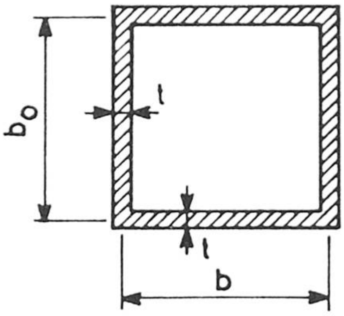

Fig. 1

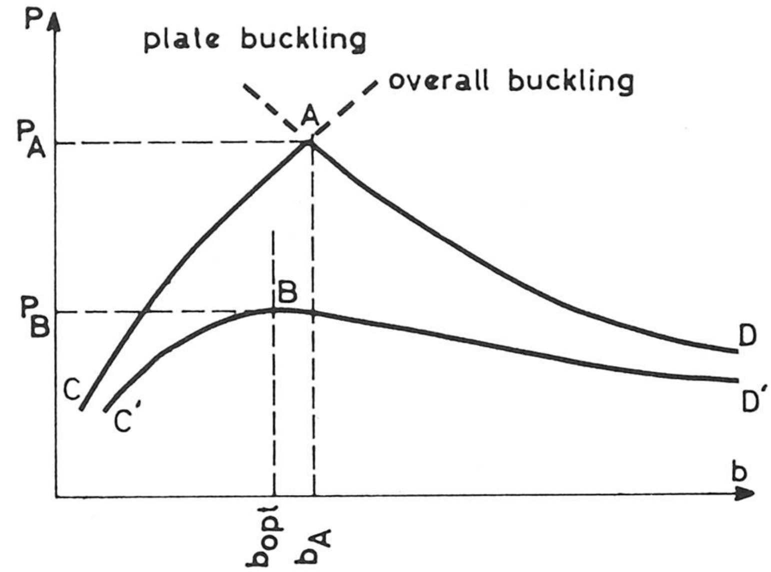

Fig. 2

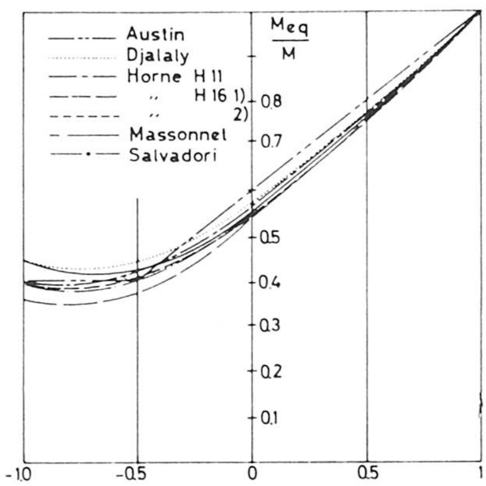

Fig. 3

Table 4.1.

	Aspects	Aarau	Cardiff	Göteborg	Karlsruhe	Lehigh	Osaka	Prague-Cardiff	Stockholm	Tokyo	Zürich
1	Clarity of the method	fair	very good	very good	very good	good	fair	very good	very good	good	very good
2	Simplified method, easy for users	no	yes	yes	yes	no	no	no	yes	no	yes
3	Computer needed	no	charts	no	no	yes/no	yes	no/yes(1)	no	no	no
4	Account taken of longitudinal stiffeners	yes	yes	no	yes(2)	yes/no(3)	no	yes	no	no	no
5	Applicable to hybrid girders	yes	yes	yes	yes	yes	no	yes	yes	yes	yes
6	Applicable to unsymmetrical girders	yes	yes	yes	yes	yes	yes	no	no	yes	yes
7	Applicable to composite girders	no	nothing said	nothing said	yes	nothing said	no	no	no	nothing said	nothing said
8	Details for calculation of bending capacity	yes	yes	yes	yes	yes	no	yes	yes	yes	yes
9	Consideration of normal forces	no	no	yes	yes	no	no	yes	yes	no	yes
10	Details about σ_{cr} of compression flange	yes	mentioned	no	mentioned	yes	no	mentioned	mentioned	no	no
11	Pure shear	yes	yes	yes	yes	yes	yes	yes	yes	yes	yes
12	Normal stresses due to bending	yes	yes	yes	yes	yes	no	yes	yes	yes	yes
13	Combined shear and bending	yes	yes	yes	yes	yes	no	yes	yes	yes	yes

(1) for solving a cubic equation ;

(2) in the serviceability state ;

(3) computer method takes account of longitudinal stiffeners.

Table 4.2.

Aspects of stiffened compression flange, Covered in the Different Methods	Method							
	Merrison Rules	Liège	Cambridge	Karlsruhe	Monash	Manchester	Imperial College	Zürich
1. Asymmetry of cross-section of flange about horizontal axis	V	-	V	V	V	V	V	-
2. Buckling of plate between stiffeners	V	V	V	V	V	V	V	V
3. Torsional buckling of stiffener outstand	V	-	-	-	-	-	V	-
4. In-plane transverse stresses in flange plate	V	-	-	-	-	-	-	-
5. In-plane shear stresses in flange plate	V	-	V	-	-	-	V	-
6. Locally applied lateral loading	V	-	-	-	-	-	V	-
7. Variation in axial load along length	V	-	V	-	-	-	V	-
8. Overall curvature of box girder	V	-	-	-	-	-	V	-
9. Overall postbuckling behaviour of flange	V	V	-	V	-	-	V	-
10. Welding residual stresses	V	-	V	-	-	V	V	-

Table 4.3.

Method	Transverse stifeners only			Transverse and longitudinal stiffeners		
	n	m	sd	n	m	sd
Aarau	42	0,99	0,20	48	1,01	0,21
Cardiff	44	1,02	0,06	66	1,oo	0,07
Göteborg	33	0,99	0,12	-	-	-
Karlsruhe	78	0,97	0,14	102	0,96	0,14
Lehigh	48	1,02	0,12	60	1,02	0,11
Osaka	36	1,07	0,17	-	-	-
Prague-Cardiff	32	0,98	0,08	46	0,99	0,07
Stockholm	33	0,96	0,11	-	-	-
Tokyo	40	0,95	0,15	-	-	-
Zürich	30/26	0,79/1,06	0,25/0,18	-	-	-

n = N° of tests compared with design method

m = Mean value of ratio of predicted to observed load

sd = Standard deviation

N = N° of cases where ratio of predicted to observed load exceeds one.

Table 4.4.

DESIGN METHOD		TEST RESULTS		TOTAL 12	N
		LIEGE 6 tests	IMPERIAL COLLEGE 6 tests		
ZURICH	n	6	6	12	5
	m	0.869	1.004	0.937	
	sd	0.089	0.052	0.099	
MERRISON	n	6	6	12	1
	m	0.702	0.844	0.773	
	sd	0.058	0.138	0.126	
LIEGE	n	6	6	12	0
	m	0.751	0.878	0.815	
	sd	0.108	0.076	0.113	
CAMBRIDGE	n	6	6	12	0
	m	0.704	0.804	0.754	
	sd	0.063	0.048	0.075	
KARLSRUHE	n	6	6	12	1
	m	0.817	0.915	0.866	
	sd	0.107	0.178	0.170	
MONASH	n	6	6	12	0
	m	0.693	0.762	0.729	
	sd	0.168	0.110	0.100	
MANCHESTER	n	6	6	12	0
	m	0.756	0.871	0.814	
	sd	0.096	0.088	0.108	
IMPERIAL COLLEGE	n	6	6	12	0
	m	0.693	0.758	0.726	
	sd	0.086	0.123	0.111	

n = N° of tests compared with design method

m = Mean value of ratio of predicted to observed load

sd = Standard deviation

N = N° of cases where ratio of predicted to observed load exceeds one

4.4. (following)

DESIGN METHOD		TEST RESULTS				TOTAL 48	N
		CAMBRIDGE 12 tests	MONASH 3 tests	NAGOYA 27 tests	DUNFERMLINE 6 tests		
ZURICH	n	12	3	27	6	48	
	m	1.051	0.880	0.983	1.004	0.996	26
	sd	0.420	0.056	0.131	0.093	0.116	
MERRISON	n	12	3	27	6	48	
	m	0.805	0.627	0.863	0.784	0.824	4
	sd	0.043	0.032	0.109	0.163	0.119	
LIEGE	n	12	3	27	6	48	
	m	0.912	0.677	0.965	0.712	0.875	0
	sd	0.047	0.009	0.022	0.143	0.119	
CAMBRIDGE	n	12	3	27	6	48	
	m	0.973	0.764	0.898	0.895	0.908	8
	sd	0.039	0.055	0.100	0.089	0.098	
KARLSRUHE	n	12	3	-	6	21	
	m	1.003	0.790	-	0.917	0.948	8
	sd	0.069	0.050	-	0.120	0.121	
MONASH	n	12	3	27	6	48	
	m	0.943	0.729	0.779	0.726	0.810	3
	sd	0.052	0.053	0.100	0.087	0.117	
MANCHESTER	n	12	3	27	6	48	
	m	1.093	0.938	0.956	0.916	0.984	21
	sd	0.056	0.053	0.186	0.113	0.124	
IMPERIAL COLLEGE	n	12	3	27	6	48	
	m	0.946	0.810	0.902	0.814	0.901	10
	sd	0.056	0.043	0.074	0.151	0.110	

n = N° of tests compared with design method
m = Mean value of ratio of predicted to observed load
sd = Standard deviation
N = N° of cases where ratio of predicted to observed load exceeds one