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Partial Prestressing of Concrete Structures

Précontrainte partielle des structures en béton

Teilweise Vorspannung von Betontragwerken

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SUMMARY

Partial prestressing of concrete structures offers several advantages. The ratio of non-prestressed to prestressed reinforcement in the flexural tensile zone can be varied at will. Fine cracks may occur under service loads and are permissible.

A recent international enquiry showed that partial prestressing has so far been included only in a few national codes and that these contain varying design concepts. A design method which could find general acceptance is proposed in this paper. It is based on the fulfilment of two main design conditions of safety, viz. against the ultimate limit state and satisfactory serviceability. The design method is simple and practical enough for use. The method is discussed by applying it to a design example.

RÉSUMÉ

La précontrainte partielle des structures en béton offre de nombreux avantages. Elle permet dans la zone de traction par flexion des proportions variables d'armature ordinaire et précontrainte. De petites fissures pour des cas de charge de service sont possibles et acceptables.

Une récente enquête internationale a montré que la précontrainte partielle n'est jusqu'ici que très peu introduite dans les codes de construction nationaux et que les critères de dimensionnement sont très différents. L'article propose une méthode de dimensionnement, qui devrait être acceptable d'une façon générale. Elle est basée sur deux conditions essentielles: sécurité à la rupture et serviciabilité. La méthode est très simple et peut être utilisée dans la pratique courante. Elle est illustrée au moyen d'un exemple.

ZUSAMMENFASSUNG

Der teilweise vorgespannte Beton bietet zahlreiche Vorteile. Er weist in der Biegezugzone schlaffe und vorgespannte Bewehrungen in beliebigem Verhältnis auf. Unter Gebrauchslasten sind feine Risse möglich und zugelassen.

Eine internationale Umfrage ergab, dass die teilweise Vorspannung erst in einigen wenigen Normen eingeführt ist, und dass hiefür sehr unterschiedliche Bemessungskonzepte bestehen. Daher wird ein mögliches, allgemein akzeptierbares Bemessungskonzept dargestellt. Es basiert auf zwei Hauptbedingungen: Bruchsicherheitsbedingung und Gebrauchstüchtigkeitsbedingung. Das Bemessungskonzept ist sehr einfach und direkt auf die Bedürfnisse der Praxis ausgerichtet. Es wird anhand eines wegleitenden Beispiels eingehend diskutiert.

1. TYPES OF CONCRETE CONSTRUCTION

Reinforced concrete is suitable for a variety of structures and is reinforced only with non-prestressed reinforcement. Under service loads cracks usually develop in the tensile zone of a cross-section subject to bending. Design of the section is carried out assuming a cracked tensile zone using the modular ratio method (n-method), or more recently, the ultimate method. The occurrence of relatively wide cracks can lead to excessive deflections in the structure.

Prestressed concrete is suitable for applications similar to those of reinforced concrete and permits more slender and larger span construction. It is almost always reinforced entirely with prestressed reinforcement. Under service loads cracking is not usually permitted in the tensile zone of a cross-section subject to bending. Design of the section is carried out assuming an uncracked cross-section and either zero or very small values of permissible extreme fibre tensile stresses. In spite of this, cracking can occur due to self-stresses and stresses from external restraints on the structure. The high degree of prestressing may result in undesirable deformations e.g. upward deflections.

Partially prestressed concrete structures are usually reinforced with a combination of non-prestressed and prestressed reinforcement. Under service loads fine cracks may develop and are permissible in the tensile zone of a cross-section subject to bending. Design of the section is carried out assuming a cracked tensile zone. Partially prestressed concrete may also be considered to be reinforced concrete strengthened by the addition of some prestressed reinforcement. This form of construction possesses certain advantages over both reinforced and fully prestressed concrete construction. Due to the presence of only fine cracks the deflections are smaller than in a reinforced concrete structure. The absence of high prestressing forces avoids the danger of undesirable deformations occurring in the structure. In most cases partial prestressing offers an optimal solution regarding materials employed and cost of structure.

2. INTERNATIONAL INQUIRY ON PRESTRESSING IN VARIOUS CODES OF PRACTICE

In order to assess and compare the design methods for prestressed concrete structures recommended in various national codes of practice, a questionnaire was circulated among the member nation groups of the IABSE. The questionnaire was formulated to include all the classes of prestressing covered in these codes of practice. In spite of this several member groups failed to return a completed questionnaire on the sole ground that partial prestressing has not yet been introduced in their countries. Completed questionnaires were returned by the following countries:

CH	Switzerland
NL	Netherlands
A	Austria
SF	Finland
HG	Hungary
S	Sweden
GB	Great Britain
D	West Germany
Y	Yugoslavia
CS	Czechoslovakia
IN	India

The codes of practice used in these countries together with the dates when they came into force are listed in column b) of Table 1.

The remainder of section 2 contains a summary of the most important findings of the enquiry.

a	b	c		d	e	f							
country	Short title of national standard (Year)	CLASSES OF PRESTRESSING				Values used for calculation [N, mm]							
		as described in relevant standard (reproduced from replies to questionnaire)		Abbre= viation	Classifi= cation according to Tab. 2	f_{ck}	σ_c^-	σ_c^+	f_{sk}	$\Delta\sigma_s$	f_{pk}	σ_{po}	s
CH	SIA 162 (1968)	Full prestressing (Volle Vorspannung)	$\sigma_c^+ = f(f_{ck})$	V	F+L	30	12	1.0	460		1500	1190	1.8
		Partial prestressing (Teilweise Vorspannung)	$\Delta\sigma_s = 150 N/mm^2$	T	P					150			
NL	Regulation for concrete (1974)	Class I	$\sigma_c^+ = 0$	I	F	33	*	0	550		1760	1320	1.7
		Class II	$\sigma_c^+ = f(f_{ck})$	II	L			1.65					
	CUR 87 (1977)	Class III w = 0.1 mm w = 0.2 mm w = 0.3 mm	$\Delta\sigma_s = 75 N/mm^2$ $\Delta\sigma_s = 175 N/mm^2$ $\Delta\sigma_s = 250 N/mm^2$	III	P					75 175 250			
A	ÖNORM B 4250 (1974, generally) ÖNORM B 4252 (1975, bridges)	Full prestressing (Volle Vorspannung)	$\sigma_c^+ = 0$	V	F	30	12	0	500		1600	1080	1.7
		Limited prestressing B 4250 (Beschränkte Vorspannung) B 4252	$\sigma_c^+ = f(f_{ck})$	B	L		12	3 2.4					
		Light prestressing B 4250 (Schwache Vorspannung) B 4252	$\Delta\sigma_s = 150 N/mm^2$	S	P		11.3			150			
SF	BV 5 (1973)	Crack class I (Rissklasse I)	$\sigma_c^+ = f(f_{ck})$	I	F+L	30	12	1.5	400		1000	1170	~167
		Crack class II (Rissklasse II)	$\sigma_c^+ = f(f_{ck})$	II	L			4.0					
		Crack class III (Rissklasse III)	s = 1.67	III	P								
HG	MSZ 15022/2 (1973)	No cracking (Rissefreiheit)	$\sigma_c^+ = f(f_{ck})$	a	F+L	*	*	*	*		*	*	*
		Limited cracking (Rissbeschränkung)	w = 0.0mm w = 0.1mm w = 0.15mm	b	P					*			
S	VV (1965, bridges)	No cracking (Rissefreiheit)	$\sigma_c^+ = f(f_{ck})$	a	F+L	*	9.5	1.0	*		*	*	1.75
	SBN-S 25:21 (1969, buildings)	Limited cracking (Rissbeschränkung)	w = 0.4mm	b	P		10.0			~230			
GB	CP 110 (1972)	Class I	$\sigma_c^+ = 0$	I	F	30	10.2	0	433		1277	*	1.73
		Class II	$\sigma_c^+ = f(f_{ck})$	II	L			2.1					
		Class III	$\sigma_c^+ = f(f_{ck}, \varrho)$	III	L			<7.65					
D	DIN 4227 (1972)	Full prestressing (Volle Vorspannung)	$\sigma_c^+ = 0$	V	F	25	11	0	500		1600	990	1.75
		Limited prestressing generally (Beschränkte Vorspannung) bridges	$\sigma_c^+ = f(f_{ck})$	B	L			3.0					
Y		Full prestressing (Volle Vorspannung)	$\sigma_c^+ = f(f_{ck})$	V	F+L	*	13	1.5	*		*	*	*
CS	ČSN 73 1251 (1969)	Full prestressing (Volle Vorspannung)	$\sigma_c^+ = f(f_{ck})$	V	F+L	*	*	0.8	*		*	*	2.0
		Limited prestressing (Beschränkte Vorspannung)	$\sigma_c^+ = f(f_{ck})$	B	L			2.0					
IN	IRC : 18 (1965)	Full presstressing (Volle Vorspannung)	$\sigma_c^+ = 0$	V	F	*	*	0	*		*	*	*

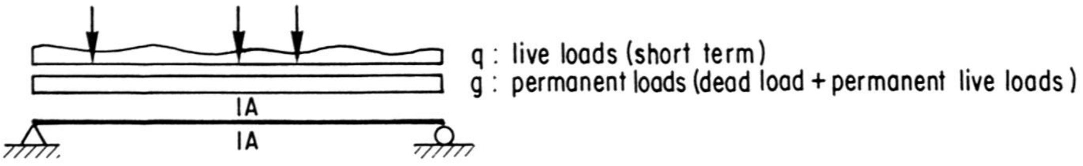
*: details not given in reply

Table 1 National standards, classes of prestressing and data assumed in replies to questionnaire

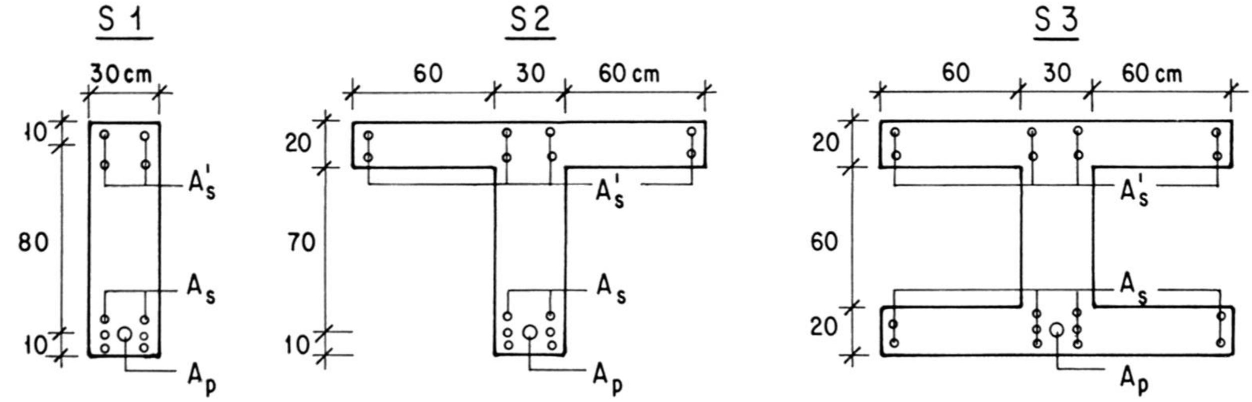
2.1 Standard data supplied with questionnaire

The questionnaire sent out required the member countries to apply the clauses of their national codes in calculations for 3 standard cross-section types S1, S2 and S3. These cross-section types together with the loads, structural system and material properties to be used in replying to the questionnaire are shown in Fig. 1. In this way a proper comparison of the replies to the questionnaire was made possible. A key to notations and symbols used is to be found at the end of this paper.

Loads and Structural System



Cross-section types



Material properties

In-situ concrete: Weight of cement 300 kg/m^3, maximum aggregate size 30 mm, f_{cm} = 33 N/mm^2, f_{cpm} = 27 N/mm^2, δ = 10 %

Prestressed reinforcement: Wires or strands in ducts subsequently grouted, f_{pm} = 1600 N/mm^2, f_{ptm} = 1800 N/mm^2, δ = 5 %, λ = 2 %

Non-prestressed reinforcement: Deformed bars f_{sm} = 500 N/mm^2, f_{stm} = 560 N/mm^2, δ = 5 %, λ = 5 %

Fig. 1 Standard data supplied on questionnaire to IABSE member countries

2.2 Classes of prestressing

The results of the enquiry showed that a variety of descriptions exist to define the classes of prestressing. This is seen in column c) of Table 1 where the descriptions used by the member countries in their replies to the questionnaire are reproduced. Their corresponding abbreviations used in Figures 2a) and 2b) are reproduced in column d). In several countries the division into various classes of prestressing is not based on the degree of prestressing but on the cracking behaviour under service loads as specified by the permissible crack widths. It is also seen that considerable differences exist between the various codes even in the definitions used.

In order to have a common base for comparing the answers provided to the questionnaire, the division into classes of prestressing was subsequently based on the maximum tensile fibre stress occuring under service loads and is shown in Table 2.

Class F	Full prestressing (Volle Vorspannung)	Uncracked section	$\sigma_c^+ = 0$
Class L	Limited prestressing (Beschränkte Vorspannung)	Uncracked section	$\sigma_c^+ > 0$
Class P	Partial prestressing (Teilweise Vorspannung)	Cracked section	$\Delta\sigma_s$, $\Delta\sigma_p$

Table 2 Classes of prestressing as defined by the maximum tensile fibre stress under service loads

The division into classes of prestressing according to these definitions is shown in column e) of Table 1.

It is seen that the prestressing class P is allowed only in the codes of the first six countries of Table 1. The Swiss (CH), Dutch (NL) and Austrian (A) codes specify a limit to the stress increase in the non-prestressed and prestressed steel after cracking occurs. On the other hand the Finnish (SF), Hungarian (HG) and Swedish (S) codes specify only ultimate load factors together with specific detailing requirements.

2.3 Permissible moments on cross-sections with specified amounts of reinforcement

Figure 2 shows the permissible bending moment $M_{g+q\ perm} = M_{perm}$ on the standard cross-section types S1, S2 and S3 for specified amounts of prestressed and non-prestressed reinforcement, A_p and A_s respectively calculated by the member countries in accordance with their national codes. The heights of the bars in this figure correspond to the magnitudes of the permissible moments. The class of prestressing as described by the member countries in their replies to the questionnaire is shown in the left half of each bar (refer also to Table 1, columns c) and d)). The right half of each bar shows the corresponding prestressing class as defined on Table 2 (refer also to Table 1, column e)).

The following two cases were considered:

Fig. 2a): Non-prestressed reinforcement predominating
(low to medium degree of prestressing)

Fig. 2b): Prestressed reinforcement predominating
(medium to high degree of prestressing)

It is seen that for any given cross-section there is a large variation in the calculated values of the permissible moments depending on which national code has been used. The largest variation occurs, as is to be expected, for the cases where non-prestressed reinforcement predominates (Fig. 2a). In such cases the codes of practice which permit the prestressing class P have an advantage and lead to higher calculated values of permissible moments. Where the codes do not provide for such a class of prestressing the presence of a large amount of non-

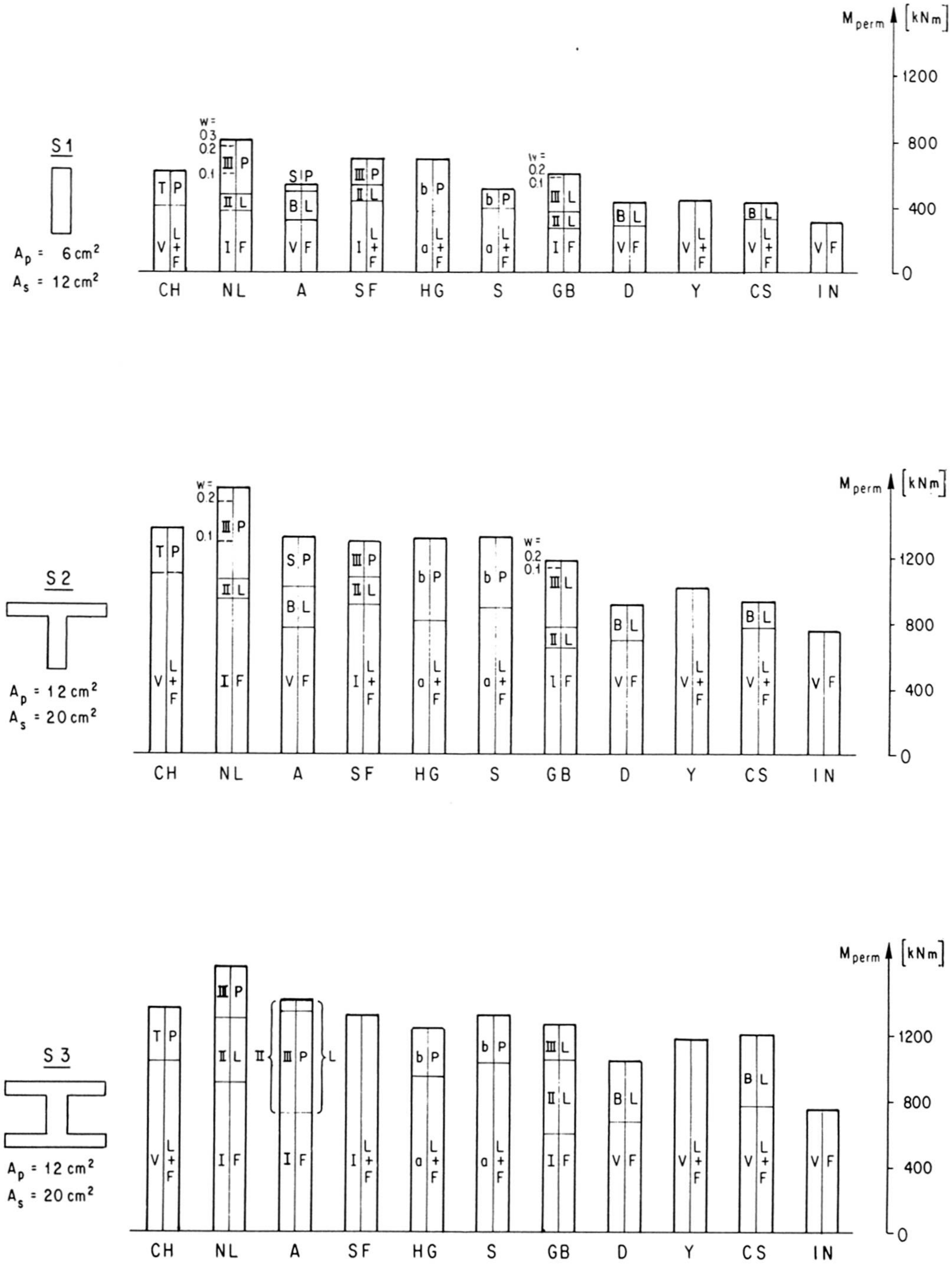


Fig. 2a) Permissible moments on cross-sections S1, S2 and S3 for given values of A_p and A_s - with a high proportion of non-prestressed reinforcement A_s

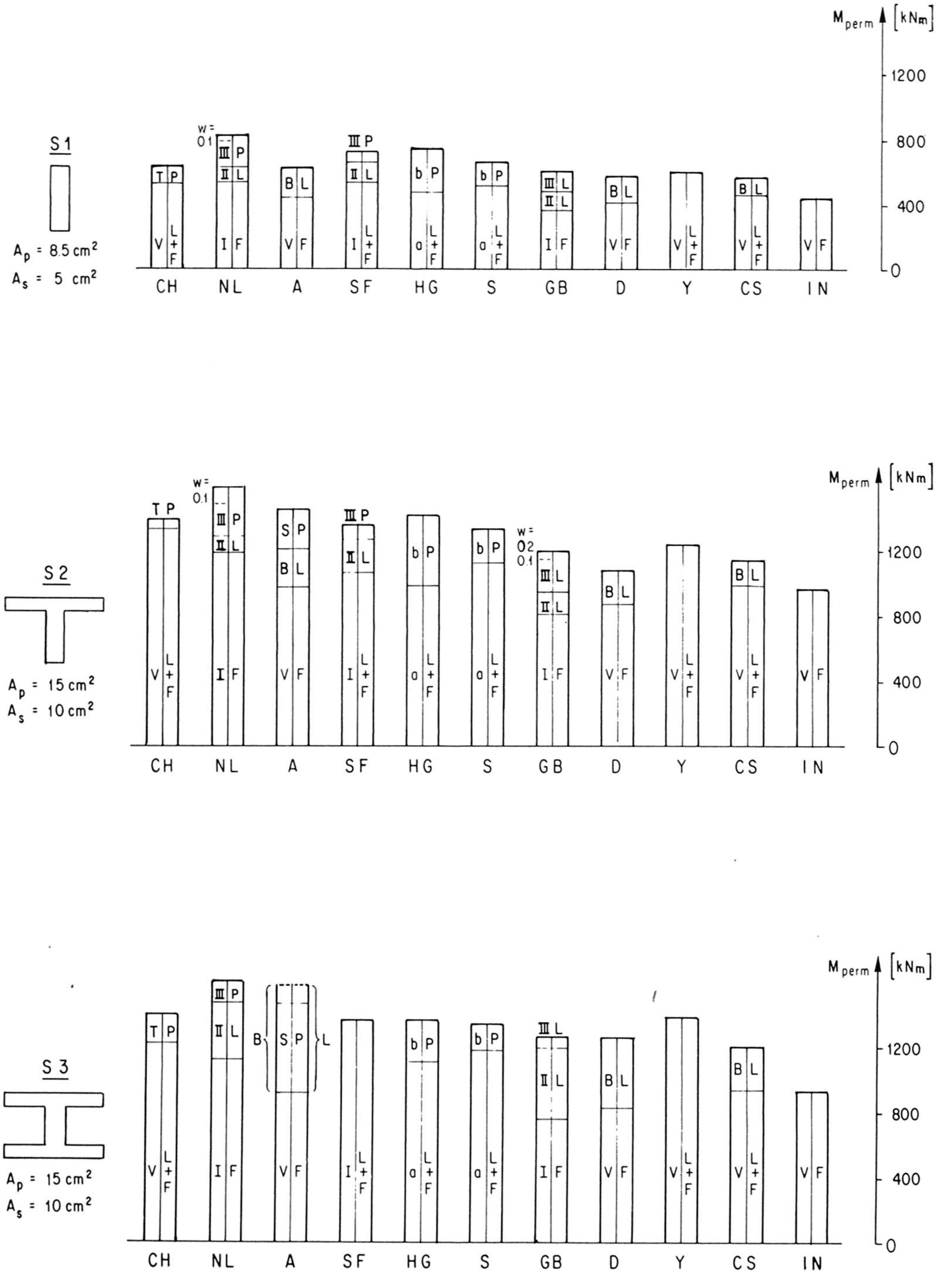


Fig. 2b) Permissible moments on cross-sections S1, S2 and S3 for given values of A_p and A_s - with a high proportion of prestressed reinforcement A_p

prestressed reinforcement has a little or no effect on the calculated permissible moment values. The values of the permissible moments according to the Dutch code (NL) are the largest for all the standard cross-section types; it should be noted, however, that material properties superior to those specified on the questionnaire have been assumed in the calculations (refer to Table 1, column f) and Fig. 1). The German code (D) gives relatively low calculated values of permissible moments, and the Indian code (IN) gives the lowest values in the entire group. In the case of cross-section type S1 the lowest calculated value of permissible moment (code IN) is only 39 per cent of the highest calculated value (code NL).

A more satisfactory picture emerges for those cases where prestressed reinforcement predominates (Fig. 2b). The variation in the calculated permissible moment values is smaller because of the smaller influence of the non-prestressed reinforcement.

2.4 Reinforcement required to resist specified applied bending moments

Figure 3 shows the cross-sectional areas of reinforcements A_p and A_s required for cross-section types S1, S2 and S3 to resist a given value of bending moment calculated by member countries in accordance with their national codes. The amount of prestressed reinforcement A_p to be used was specified and it was only the additional amount of non-prestressed reinforcement A_s which had to be calculated by the member countries.

It is seen that the codes of Germany (D), Yugoslavia (Y), Czechoslovakia (CS) and India (IN) do not permit structures with the relatively low values of A_p specified. The remaining codes (excepting that of GB) which permit the prestressing class P deliver values of A_s which vary considerably. (See Figure 3 next page.)

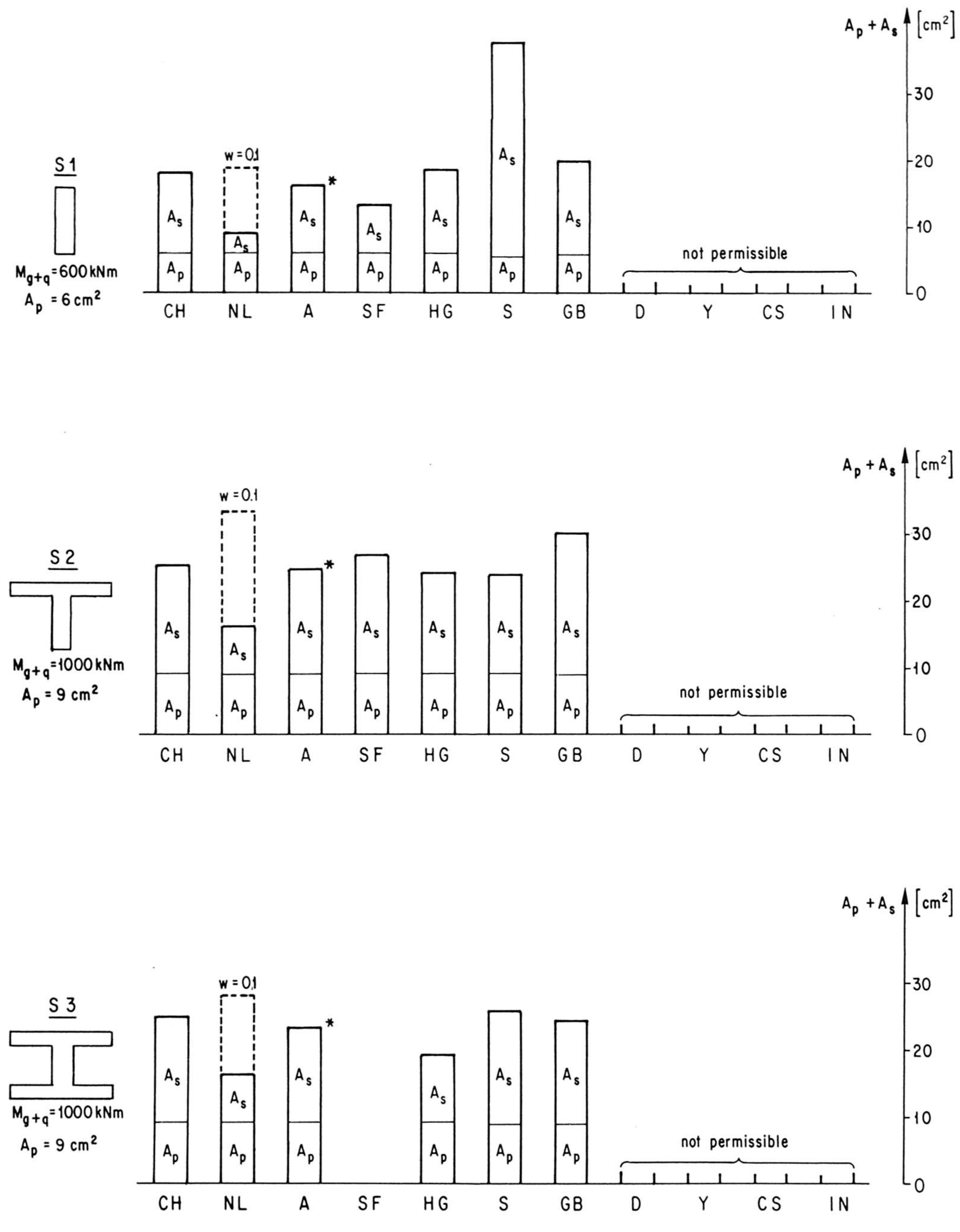
2.5 Conclusions of the enquiry

The results of the enquiry show the large differences existing between the various national codes in their approch to calculations for prestressed concrete construction. This can be attributed to a small extent to the different safety factors and material properties used in the calculations. The main reasons however for the different calculated values are twofold:

1. Partial prestressing (calculations assuming a cracked tensile zone) has only recently been introduced into the codes of a few countries. Most codes have so far only permitted an extension of full prestressing to allow small extreme fibre tensile stresses (limited prestressing).

2. In codes where partial prestressing has been included, the design rules vary considerably.

It would therefore appear necessary to formulate design principles for partially prestressed structures which would find international acceptance.



* condition $\Delta\sigma_s$ = 150 N/mm² not satisfied

Fig. 3 Reinforcement required in cross-sections S1, S2 and S3 for a given value of M_{g+q} (values of A_p specified)

3. AIMS OF DESIGN METHOD FOR PARTIALLY PRESTRESSED SECTIONS

Any design method or code requirements for partially prestressed sections under bending should satisfy the following conditions:

1. The design method must be simple and easily applicable. Only the essential design check calculations should need to be carried out.
2. The design method should be an extension of the design steps already well-known and established for reinforced and fully prestressed concrete structures.
3. The design method should enable a smooth transition between the design of reinforced and fully prestressed concrete structures.
4. The design method should lay emphasis on sound detailing, especially of the non-prestressed reinforcement.
5. The code clauses incorporating the design method should not be unnecessarily restrictive and should leave a wide range of possibilities and responsibility to the engineer.

4. A PROPOSED DESIGN METHOD FOR PARTIALLY PRESTRESSED SECTIONS

In the following sections a design method will be proposed which fulfils the requirements set out in section 3. The lessons drawn from nearly 10 years of experience with partial prestressing in Switzerland have been incorporated in the proposed method.

4.1 Main design conditions

Based on the results of the enquiry and the present state of the art ([3] to [11]) it would appear possible to formulate a method of design for the following two conditions:

1. Limit state of collapse: the structure should have an adequate safety against the limit state of collapse.
2. Limit state of serviceability: the structure should behave satisfactorily under service loads.

The first condition is usually fulfilled by ensuring that a minimum global safety factor or partial safety factors are observed. The second condition is fulfilled by limiting extreme tensile fibre strains or crack widths and deflections of the structure.

4.2 Design procedure

The section is first designed to provide the required ultimate safety factor and checked for serviceability in a second design stage. The design steps are listed below:

Step 1: Choice of degree of prestressing
Step 2: Calculation of prestressed reinforcement
Step 3: Calculation of non-prestressed reinforcement
Step 4: Detailing of non-prestressed reinforcement
Step 5: Calculation of extreme fibre stresses based on a cracked tensile zone (only for members subject to fatigue loading)

These steps will be dealt with subsequently in detail. At this stage it is important to stress the following:

Compared with full or limited prestressing, partial prestressing offers considerable freedom in the choice of prestressing tendons. All that is required is to provide sufficient non-prestressed steel to complement the actual area of prestressed steel chosen for the cross-section. For example prestressing tendons in continuous beams may be carried through from end to end without any curtailment.

Step 1: Choice of degree of prestressing

It will be assumed that the dimensions of the cross-section to be designed are known. In almost all cases the maximum possible eccentricity of the tendons in the cross-section will be chosen.

In the first design step a value of bending moment M_d is chosen which - together with a prestressing force P_∞ to be calculated in Step 2 - produces zero extreme tensile fibre stress (Fig. 4). M_d is sometimes referred to as the decompression moment. The degree of prestressing κ is defined [5] to be the ratio of M_d to the total applied bending moment M_{g+q}

$$\kappa = \frac{M_d}{M_{g+q}}$$

The degree of prestressing gives a measure of that proportion of the total bending moment under which the maximum tensile fibre stress is zero. $\kappa = 0$ implies no prestressing, while $\kappa = 1$ signifies full prestressing.

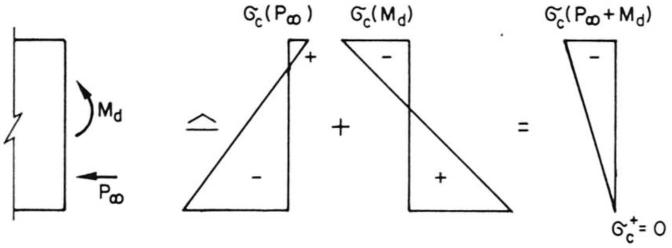


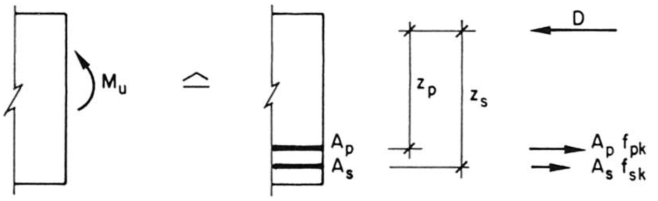
Fig. 4 Stress distribution under the action of prestressing force P_∞ and external bending moment M_d (decompression moment)

Step 2: Calculation of prestressed reinforcement

In the second design step the prestressing force necessary for the bending moment M_d is calculated using the well-known rules for prestressed concrete. The corresponding area of prestressed reinforcement and the type of prestressed tendons can be selected.

Step 3: Calculation of non-prestressed reinforcement

In the third design step the area of non-prestressed reinforcement can be chosen to provide the required ultimate load safety factor (Fig. 5):



$$M_u = s\,M_{g+q} = A_p\,f_{pk}\,z_p + A_s\,f_{sk}\,z_s$$

$$A_s = \frac{1}{f_{sk}\,z_s}\,(M_u - A_p\,f_{pk}\,z_p)$$

Fig. 5 Internal forces at ultimate moment M_u

With some practice it will be easily possible to estimate the values of z_p and z_s with reasonable accuracy.

Step 4: Detailing of non-prestressed reinforcement

The next and in most cases the last step in design will be to pay particular attention to the detailing of non-prestressed reinforcement. In addition to its contribution to the ultimate load safety factor in the case of members with medium and low degrees of prestressing the non-prestressed reinforcement has to serve the following purposes [5], [11]:

- Before prestressing (during construction): to limit crack widths and deformations resulting from settlement of falsework, shrinkage and temperature stresses
- After prestressing (during service): to ensure satisfactory behaviour regarding crack formation, crack widths and corrosion.

A meticulous detailing of non-prestressed reinforcement - longitudinal and stirrups - is essential for all prestressed concrete structures.

The following recommendations relate the longitudinal reinforcement to the degree of prestressing used:

a) For high degrees of prestressing ($\kappa > \underset{=}{\sim} 0.7$), where calculations indicate that the required ultimate load safety factor is achieved without any non-prestressed reinforcement, a minimum area of non-prestressed steel should still be provided. This shall be in the form of several relatively small diameter rods contained by closely spaced stirrups, so as to enclose the prestressing tendons with a narrow grid of reinforcement. The minimum area of reinforcement shall be as follows:

- tensile zone (flexure)	0.2-0.3% of area of cracked tensile zone
- tensile zone (direct tension)	0.5-0.7% of area of cracked tensile zone

The smaller of the above values is to be used for lower grades of concrete.

In the author's opinion this minimum area of non-prestressed reinforcement should also be used in structures with full or limited prestressing.

b) For medium degrees of prestressing ($\kappa \sim 0.4$-0.7), the longitudinal reinforcement especially in the outermost layer should consist of rods of a diameter larger than that required for condition (a) above.

c) For low degrees of prestressing ($\kappa < \sim 0.4$) the non-prestressed reinforcement will dominate the total reinforcement to be provided. Detailing principles used for reinforced concrete structures are to be adopted in this case.

The above recommendations apply to post-tensioned structures prestressed with tendons in ducts which are subsequently grouted with cement grout. For unbonded tendons further research is necessary before similar rules can be formulated.

In the case of pre-tensioned structures it may be possible to do without any non-prestressed steel provided that the prestressing steel is in the form of well-distributed deformed wires and that no additional steel is required to provide the required ultimate load safety factor. Due to the excellent bond that is developed between such wires and concrete, only closely spaced hairline cracks can be expected under service loads.

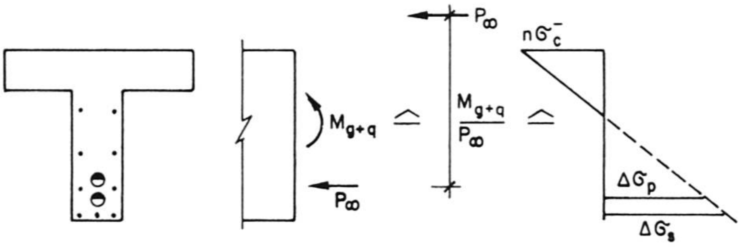


Fig. 6 Stress distribution in cracked cross-section

Step 5: Calculation of extreme fibre stresses (only for members subject to fatigue loading)

In certain special cases it will be necessary to calculate the following extreme fibre stresses based on a cracked tensile zone under the total load:

- stress increase $\Delta\sigma_s$ in the outermost layer of non-prestressed reinforcement
- stress increase $\Delta\sigma_p$ in the outermost layer of prestressed reinforcement
- extreme compressive fibre stress σ_c^- in the concrete

The values of $\Delta\sigma_s$ and $\Delta\sigma_p$ in particular give an indication of the behaviour of the structure under service loads. When the structure is subject to fatigue loads these values are to be limited to certain specified values.

The values of $\Delta\sigma_s$, $\Delta\sigma_p$ and σ_c^- can be calculated as for sections under combined bending and axial loads using the modular ratio method (n-method) (Fig. 6). The value of prestressing force P_∞ after deducting all losses due to shrinkage, creep and relaxation of steel is to be used for the axial load. Strictly speaking, the value of the decompression force P_d in the tendons should be used. The error involved in using the slightly lower value P_∞ is very small and lies on the conservative side. The value of n to be used in the above calculation is not very critical; usually a value of $n = 10$ is used.

4.3 Discussion of the proposed method

A few of the important parameters and aspects of the proposed method will be briefly discussed in the following sections:

a) Influence of degree of prestressing

The influence of the degree of prestressing is of particular interest. Fig. 7 shows for the case of a rectangular cross-section subject to a total bending moment M_{g+q} the influence of the degree of prestressing κ on the following values 1):

- ultimate load safety factor s
- areas of prestressed reinforcement A_p, non-prestressed reinforcement A_s and the total reinforcement A_p+A_s
- stress increase $\Delta\sigma_s$ in the non-prestressed reinforcement
- stress increase $\Delta\sigma_p$ in the prestressed reinforcement
- extreme compressive fibre concrete stress σ_c^- in cracked section, and extreme tensile fibre concrete stress σ_c^+ in uncracked section

The cross-section was at first designed to be fully prestressed. The area of prestressed steel was then successively reduced, and the area of non-prestressed steel increased, if necessary, to maintain the ultimate load safety factor at 1.8.

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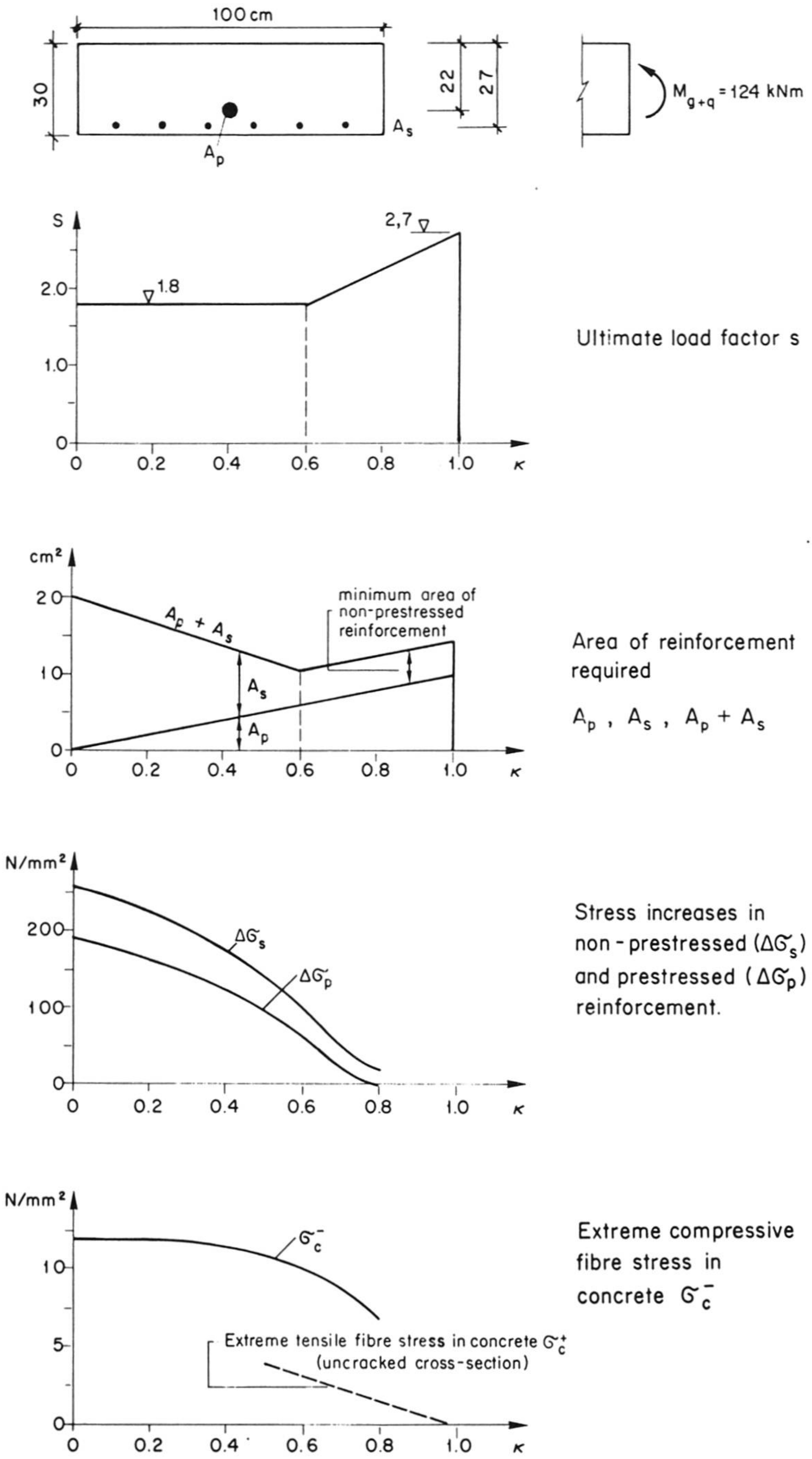
1) The shape of the curves in Figs. 7 to 9 is influenced to a certain extent by the shape of member cross-section. However, the conclusions drawn for the rectangular cross-section are generally valid for all cross-sections commonly used.



Ultimate load factor s

Area of reinforcement required

A_p , A_s , $A_p + A_s$

Stress increases in non - prestressed ($\Delta\sigma_s$) and prestressed ($\Delta\sigma_p$) reinforcement.

Extreme compressive fibre stress in concrete σ_c^-

Data assumed in above calculations :

f_{pk} = 1500 N/mm², σ_{po} ($t = \infty$) = 103 N/mm²
f_{sk} = 460 N/mm², $\sigma^+_{c\ perm}$(tens.) = 0,n = 10

Fig. 7 Influence of degree of prestressing on important parameters

The area of non-prestressed steel was not reduced below a certain minimum. Fatigue loading of the section was considered unlikely.

The following features become apparent:

- For the higher values of degree of prestressing the value of ultimate load safety factor is in excess of what is required.
- The total quantity of steel used is minimum for a value of $\kappa = 0.6$.
- Of particular importance is the observation that the stress increases in the non-prestressed and prestressed reinforcement are extremely small for higher values of degree of prestressing. For medium values of degree of prestressing these stress increases are still considerably lower than the steel stresses in conventional reinforced concrete sections.

It is usual in practice to choose the degree of prestressing so that the total amount of steel used is a minimum. This is normally the case for values of κ between 0.6 and 0.7 depending on the shape of the member cross-section.

b) Influence of errors in P_∞ and M_{g+q} on the stress increases in steel

It is extremely important that in prestressed concrete structures the prestressing forces are introduced at their intended values accurately and that the applied bending moments are calculated exactly. However, it is possible that the actual prestressing force introduced into the structure is less than that intended, e.g. owing to an incorrect estimation of the friction losses, or that the design applied bending moments are exceeded.

For the particular example chosen, Fig. 8 shows the variation of steel stress increases calculated for a cracked tensile zone resulting from errors in estimating the prestressing force P_∞ and the applied bending moment M_{g+q}. The values of $\Delta P_\infty = -20\%$ and $\Delta M_{g+q} = +20\%$ have been arbitrarily chosen to illustrate their effect on the value of the variation of stress increases in the prestressed and non-prestressed reinforcement. It can be seen from this diagram that the ratio

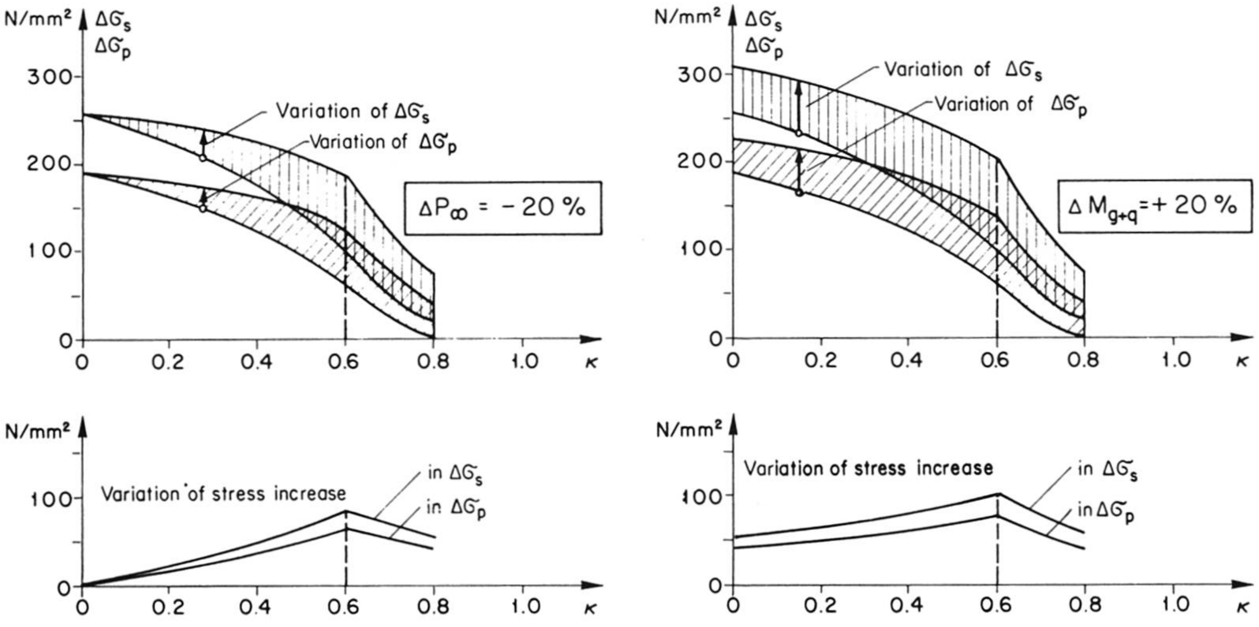


Fig. 8 Variation of stress increase in reinforcement caused by errors in values of P_∞ and M_{g+q}

of the variation of stress increase to its original value is relatively high. The variation of stress increase is maximum for $\kappa = 0.6$ which is also the value of κ for which the total steel requirement is a minimum. The absolute values of steel stress increases in members with a medium to high degree of prestressing are, however, considerably lower than the steel stresses in reinforced concrete members.

The following conclusions may be drawn:

1. An overestimation of the prestressing force or an underestimation of the applied bending moment by the designer (up to 10 or 15 per cent) do not lead to unreasonably large stress increases in structures with a medium to high degree of prestressing.
2. In structures subject to fatigue loading, however, it is of particular importance to ensure that the correct values of prestressing force P_∞ are introduced into the structure and that the applied bending moment M_{g+q} is calculated correctly.

c) Stress increases in steel caused by live loads only

It is of particular interest, and especially so in the case of fatigue loading, to observe the stress increases in non-prestressed and prestressed steel, $\Delta\sigma_s(q)$ and $\Delta\sigma_p(q)$ respectively, caused by live loads only.

$\Delta\sigma_s(q)$ may be calculated as follows:

$$\text{for} \quad \kappa < \frac{M_g}{M_{g+q}} : \qquad \Delta\sigma_s(q) = \Delta\sigma_s(M_{g+q}, P_\infty) - \Delta\sigma_s(M_g, P_\infty)$$

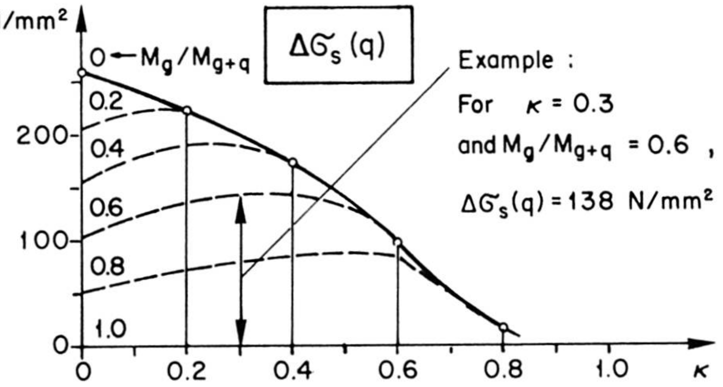
$$\text{for} \quad \kappa > \frac{M_g}{M_{g+q}} : \qquad \Delta\sigma_s(q) \cong \Delta\sigma_s(M_{g+q}, P_\infty)$$

When $\kappa < M_g/M_{g+q}$, it is to be expected that the flexural tensile zone is cracked under the permanent loads and that the non-prestressed reinforcement is under a tensile stress. $\Delta\sigma_s(q)$ is therefore to be obtained by subtracting the stress increase under permanent load g from that caused by the total load (g+q).

When $\kappa > M_g/M_{g+q}$, the stress in the non-prestressed steel is nearly zero under permanent loads, assuming that P_∞ is nearly the same as P_d. $\Delta\sigma_s(q)$ is thus almost equal to the stress increase under the total load (g+q).

$\Delta\sigma_p(q)$ may be calculated using a similar reasoning.

Fig. 9 shows, for the same design example, the stress increase $\Delta\sigma_s(q)$ in the non-prestressed steel caused by live loads only for various values of M_g/M_{g+q}.



Example :
For $\kappa = 0.3$
and $M_g/M_{g+q} = 0.6$,
$\Delta\sigma_s(q) = 138$ N/mm²

Fig. 9 Stress increase in non-prestressed reinforcement caused by live loads

The continuous line on the diagram applies for the conditions $\kappa > M_g/M_{g+q}$. The dotted lines are applicable when $\kappa < M_g/M_{g+q}$. A similar diagram will result when the stress increase $\Delta\sigma_p(q)$ in the prestressed steel is plotted as a function of κ.

The following conclusions may be drawn:

- When the live load component is small, say $M_g/M_{g+q} = 0.8$, the steel stress increases caused by live loads are not substantially influenced by the value of the degree of prestressing in the structure.
- When the live load component is large, say $M_g/M_{g+q} = 0.4$, it is preferable to choose a value of κ greater than M_g/M_{g+q} in order to minimize the steel stress increases caused by live loads.

d) Fatigue loading

Fatigue tests on concrete beams containing a mixture of non-prestressed and prestressed steel [12] have shown that the prestressed steel tends to have a better behaviour than the non-prestressed steel. The majority of fatigue failures occurred in the non-prestressed steel. This remark does not apply to tendon couplings, which should in any case not be located in zones subject to large stress fluctuations. Should this be unavoidable, additional non-prestressed steel should be provided in such zones. It can therefore be generally said that fatigue loading is not a problem particular to partial prestressing. On the contrary, in most cases the conditions in a partially prestressed structure are more favourable than in reinforced concrete structures. In the case of the former class of structures, it is only the very rarely occurring live load peaks which leads to a sizeable variation of stress increase in the steel. In any case, owing to the greater probability of fatigue damage to the non-prestressed steel, the problem is not different from that of ordinary reinforced concrete subject to fatigue loading.

4.4 Practical advantages of the proposed design method

a) Calculation of the absolute stress in the prestressed steel is not necessary

The proposed method does not require the calculation of the absolute stress in the prestressed steel. The reasons for this are twofold:

1) The behaviour of the structure under service loads is dependent on the additional strains and as a result on the change in stress in steel, and not on the absolute value of the stress. It must be noted, however, that this is valid only in the region of linear elastic behaviour of the steel.

2) Safety against the limit state of collapse is ensured by step 3 of the design method, where certain minimum global or partial safety factors have to be observed.

The prestressed steel in partially prestressed structures can be prestressed to the same initial value σ_{po} as for fully prestressed structures. This has the advantage that prestressing tendon types can be designated with the same nominal force values regardless of whether they are used in a partially or fully prestressed structure.

b) Calculation of crack widths not necessary

The proposed design method does not require the calculation of crack widths. A comparison of the several crack width formulas currently in use shows wide divergence in the results obtained from them. The effects of the stirrup reinforcement and the shape of cross-section (particularly for shallow beams) are more-over not included in these formulas. These effects can often be more critical than that of the longitudinal reinforcement and need to be studied further. Until further detailed information on these effects is available, the designer would be well-advised to concentrate more on sound detailing of the non-prestressed steel than on calculating crack widths.

c) Partial prestressing for permanent loads is also feasible

Some codes of practice require that limit prestressing should be present for the permanent load condition i.e. $\kappa > M_g/M_{g+q}$. Experience however now shows this to be a narrow restriction. Several applications - such as raft foundations and warehouse floors - to cases where almost the entire load is of a permanent nature have shown that values of $\kappa < M_g/M_{g+q}$ are feasible and serve their purpose well. The author would therefore suggest that the restrictive clause mentioned above should be omitted in future. The designer should, after considering each case on its merits, be free to choose a value for the degree of prestressing which suits the case best.

d) Smooth transition from reinforced to fully prestressed concrete design

The Swiss (CH), Dutch (NL) and Austrian (A) codes of practice require that the stress increases $\Delta\sigma_s$ and $\Delta\sigma_p$ in the steel should be calculated for a cracked tensile zone and that these should not exceed certain permissible values. These are less than the permissible steel stresses in reinforced concrete structures. In partially prestressed beams, however, the fine cracks which develop under the total load almost always close up completely on removal of the live load component [1]. At the time of its inclusion in the Swiss code, partial prestressing was breaking new grounds and the calculation and limitation of the steel stress increases appeared justified. Since then, however, there has been considerable practical experience of this form of construction together with results of additional research work [3], [10] and it is the author's opinion that for normal building and bridge structures not subject to fatigue loading this calculation may be omitted.

It was shown earlier in this paper that provided a partially prestressed structure is designed to have the minimum ultimate load safety factor, the stress increases $\Delta\sigma_s$ and $\Delta\sigma_p$ are always smaller than the steel stresses in a reinforced concrete structure. For structures with a medium to high degree of prestressing the values of $\Delta\sigma_s$ and $\Delta\sigma_p$ are considerably less than the steel stresses reached in reinforced concrete. This effect of the prestressing leads directly to smaller tensile fibre strains and, as a consequence, to smaller crack widths. By omitting the calculations for $\Delta\sigma_s$ and $\Delta\sigma_p$, partially prestressed structures with any value of degree of prestressing may be directly designed by the ultimate strength method (step 3). The method outlined in this paper thus allows a smooth transition between reinforced and fully prestressed concrete design through the intermediate stage of partial prestressing.

5. CONCLUSIONS

Experience so far with partially prestressed structures has been very encouraging. No cases of damage which are directly relateable to the principle of partial prestressing have so far been reported. On the contrary, it can be presumed that many of the signs of distress resulting from excessive prestress in earlier structures may well have been avoided had they been only partially prestressed.

In order to enable partially prestressed structures to be more widely accepted, an internationally acceptable and uniform design method should be employed. This method should be as simple as possible and practical enough for office use. One such method has been put forward in this paper. It would be an error to believe that better structures result only from complicated design methods and code requirements.

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NOTATIONS (following CEB-rules [13])

Concrete

A_c	Area of cross-section
f_{cm}	Cube crushing strength (average)
f_{cpm}	Prism crushing strength (average)
f_{ck}	Characteristic strength
σ_c^+	(permissible) Extreme tensile fibre stress
σ_c^-	(permissible) Extreme compressive fibre stress

Non-prestressed reinforcement

A_s	Area of cross-section in tension zone
A_s'	Area of cross-section in compression zone
f_{sm}	Yield limit (average)
f_{stm}	Tensile strength (average)
f_{sk}	Characteristic strength
$\Delta\sigma_s$	(permissible) Stress increase based on cracked cross-section

Prestressed reinforcement

A_p	Area of cross-section
f_{pm}	Yield limit (average)
f_{ptm}	Tensile strength (average)
f_{pk}	Characteristic strength
σ_{po}	Initial stress
$\Delta\sigma_p$	(permissible) Stress increase based on cracked cross-section

Moments and forces

M_g	Permanent load bending moment
M_q	Live load bending moment
M_{g+q}	Total load bending moment
$M_{g+q\ perm}$	$= M_{perm}$ Permissible moment
M_d	Decompression moment ($\sigma_c^+ = 0$)
M_u	Ultimate moment
P_∞	Prestressing force after deducting losses due to shrinkage, creep and steel relaxation
P_d	Tendon force at decompression

Miscellaneous

s	Global safety factor	δ	Coefficient of variation
ω	Average crack width	λ	Elongation at rupture outside zone of contraction
f(...)	Function	κ	Degree of prestressing
ρ	Percentage of reinforcement		
n	Modular ratio		

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