Deformation of concrete structures: theoretical basis for the calculation

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Objekttyp: **Article**

Zeitschrift: **IABSE surveys = Revue AIPC = IVBH Berichte**

Band (Jahr): **5 (1981)**

Heft S-16: **Deformation of concrete structures: theoretical basis for the calculation**

PDF erstellt am: **22.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-46574

Deformation of Concrete Structures
Theoretical Basis for the Calculation

Déformation des structures en béton
Base théorique pour le calcul

Formänderungen von Stahlbetontragwerken
Theoretische Grundlagen für die Berechnung

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SUMMARY

This document presents the theoretical basis for a calculation of deformation consistent with the 1978 CEB/FIP Model Code. It has been elaborated by the authors within Commission V of CEB in view of a "Cracking and Deformation" manual and was approved by the General Assembly of CEB in Budapest in June 1980. It deals with the calculation of the curvatures and deformations resulting from bending moments with possible axial load. Cracking, creep and shrinkage are taken into account. A further publication will present practical applications and a comparison with laboratory test results.

RÉSUMÉ

Ce document présente la base théorique pour un calcul des déformations conforme au Code-Modèle CEB/FIP 1978. Il a été élaboré par les auteurs au sein de la Commission V du CEB en vue de la rédaction d'un manuel «Fissuration et Déformation» et a été approuvé par l'Assemblée générale du CEB à Budapest en juin 1980. Il traite le calcul des courbures et des déformations par suite de moments de flexion avec un effort normal éventuel, tout en tenant compte de la fissuration, du fluage et du retrait. Une publication ultérieure présentera des applications pratiques et une comparaison avec des résultats d'essais en laboratoire.

ZUSAMMENFASSUNG

Im vorliegenden Bericht werden die theoretischen Grundlagen für die Berechnung von Formänderungen gemäss CEB/FIP-Mustervorschrift dargestellt. Diese Grundlagen wurden von den Verfassern im Rahmen der Kommission V des CEB im Hinblick auf ein Handbuch «Rissbildung und Verformung» erarbeitet und von der Generalversammlung des CEB in Budapest im Juni 1980 bestätigt. Sie behandeln die Berechnung von Krümmungen und Verformungen, welche aufgrund von Biegemomenten eventuell mit einer Normalkraft zusammen entstehen. Der Rissebildung sowie dem Kriechen und Schwinden wird dabei Rechnung getragen. Eine später erscheinende Publikation wird Anwendungsbeispiele und einen Vergleich mit Resultaten aus Laborversuchen beinhalten.

INTRODUCTION

The 1978 CEB-FIP Model Code gives in chapter 16 the principles for the calculation of deformations.

In order to enable the practising engineer to use these principles, it is important to supply him with as clear and simple tools as the subject allows. The object is to introduce a physical model of very general validity by means of which the entire cracking problem can be reduced to the simple state I and state II_o cases. The distribution ratio c between these states is given by formula [15.5] of the Model Code. With the physical model proposed, the calculation of curvatures remains the same whatever the value of c. Any improvement to formula [15.5] in future years will not change in any way the principle of the method.

The considerations apply to linear systems subjected to bending; indications with regard to expanding to plane systems are also given. The influence of shear and torsion will be dealt with separately.

1. BASIC ASSUMPTIONS

1.1 Mean strain of the reinforcement

Let $\Delta\ell$ be the total elongation of a reinforced concrete tie member of length ℓ, subjected to a constant axial tensile force $N = \sigma_{s2} \cdot A_s$.

The mean strain of the reinforcement is equal to (fig. 1.1):

$$\varepsilon_{sm} = \frac{\Delta\ell}{\ell} = \varepsilon_{s2} - \Delta\varepsilon_s \tag{1.1}$$

where $\Delta\varepsilon_s$, representing the contribution of the concrete in tension between the cracks, follows a hyperbolic law above σ_{sr}, which has been confirmed experimentally:

$$\Delta\varepsilon_s = \Delta\varepsilon_{smax} \cdot (\sigma_{sr}/\sigma_{s2}) \tag{1.2}$$

By replacing $\Delta\varepsilon_s$ in (1.1) one obtains successively:

$$\begin{aligned}\varepsilon_{sm} &= \varepsilon_{s2} - \Delta\varepsilon_{smax} \cdot (\sigma_{sr}/\sigma_{s2}) \\ &= \varepsilon_{s2} - (\varepsilon_{sr} - \varepsilon_{cr}) \cdot (\sigma_{sr}/\sigma_{s2}) \\ &= \varepsilon_{s2} \cdot (1 - (\sigma_{sr}/(E_s \cdot \varepsilon_{s2})) \cdot (\sigma_{sr}/\sigma_{s2})) + \varepsilon_{s1} \cdot (\sigma_{sr}/\sigma_{s2})^2 \\ &= \varepsilon_{s2} \cdot (1 - (\sigma_{sr}/\sigma_{s2})^2) + \varepsilon_{s1} \cdot (\sigma_{sr}/\sigma_{s2})^2\end{aligned}$$

wherefrom

$$\boxed{\varepsilon_{sm} = (1-c) \cdot \varepsilon_{s1} + c \cdot \varepsilon_{s2}} \tag{1.3}$$

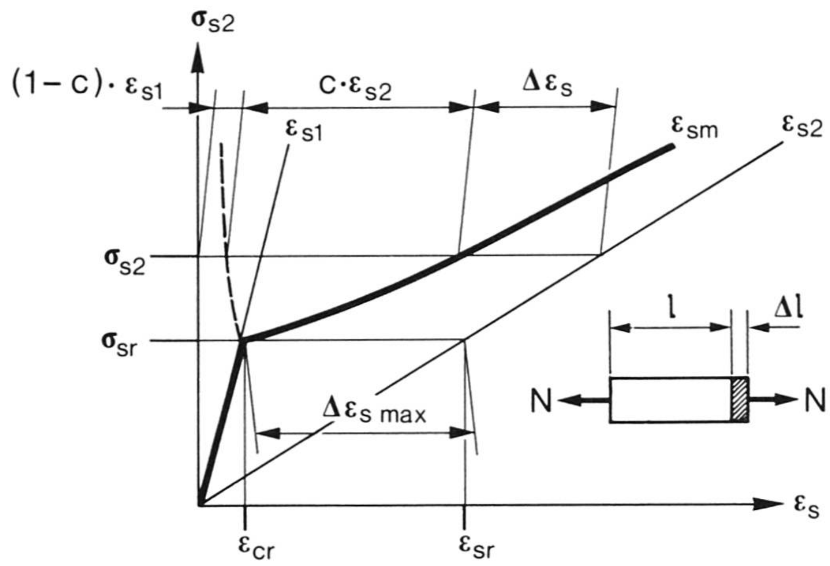


Fig. 1.1 Stress-strain curve for the reinforcing steel

Notation :

$c = 1 - \frac{\sigma_{sr}^2}{\sigma_{s2}^2}$ - distribution ratio; (1.4)

σ_{s2} - stress in the reinforcement in the cracked section, under the combination of actions under consideration;

σ_{sr} - stress in the reinforcement calculated on the assumption of a cracked section, where the maximum tensile stress in the concrete (uncracked section) is taken equal to (clause 16.2.1 of the Model Code):

$f_{ctk\,0,05}$: in order to prevent damage

f_{ctm} : in order to calculate the camber;

ε_{s1} - strain in the reinforcement situated in the embedment zone in state I, i.e. taking into consideration the uncracked section;

ε_{s2} - strain in the reinforcement situated in the embedment zone in nacked state II, i.e. neglecting the contribution of the concrete in tension between the cracks;

ε_{sr} - strain in the reinforcement ε_{s2}, corresponding to σ_{sr};

ε_{cr} - strain in the concrete at the level of the reinforcement, corresponding to σ_{sr};

A_s - area of the reinforcement.

The formulation (1.3) of the mean strain ε_{sm} in the reinforcement enables us to calculate *the deformations* of an element subjected to tension; the same expression is proposed for the calculation of deformations due to bending.

Note: In order to calculate the crack widths, one only needs the increase of the mean strain of the steel with respect to that of the adjacent concrete ($c \cdot \varepsilon_{s2}$), denoted by ε_{sm} in eq. [15.5] of the Model Code.

In order to take into account the bond properties of the bars and the influence of the duration of the application and the repetition of loads, the Model Code proposes in clause 15.2.3 to introduce two coefficients β_1 and β_2 which correct the expression giving c:

$$\boxed{c = 1 - \beta_1 \cdot \beta_2 \cdot \left(\frac{\sigma_{sr}}{\sigma_{s2}}\right)^2} \qquad (1.5)$$

with

$\beta_1 = \frac{1}{2,5 \cdot K_1}$ coefficient which characterizes the bond properties of the bars:

$K_1 = 0,4$ for high bond bars,

$K_1 = 0,8$ for plain bars;

β_2 - coefficient representing the influence of the duration of the application and repetition of the loads:

$\beta_2 = 1,0$ at first loading,

$\beta_2 = 0,5$ for loads applied in a sustained manner or for a large number of load cycles.

1.2 Calculation models

From the expression (1.3):

$$\varepsilon_{sm} = \frac{\Delta \ell}{\ell} = (1-c) \cdot \varepsilon_{s1} + c \cdot \varepsilon_{s2}$$

calculation models are established, which are equally valid for elements subjected to pure tension or to bending.

Elements subjected to pure tension (N = const. along the length ℓ)

The real element can be represented by means of a model composed of two parts (fig. 1.2):

- one acting in state I (uncracked sections),
- the other in state II_0 (nacked state II, cracked sections: only the concrete in compression and the reinforcement are considered).

The respective lengths ℓ_1 and ℓ_2 are to be defined:

As

$$\varepsilon_{sm} = \frac{\Delta \ell}{\ell} = \frac{\varepsilon_{s1} \cdot \ell_1}{\ell} + \frac{\varepsilon_{s2} \cdot \ell_2}{\ell}$$

the comparison with expression (1.3) gives:

$$\ell_1 = (1-c) \cdot \ell \qquad \ell_2 = c \cdot \ell$$

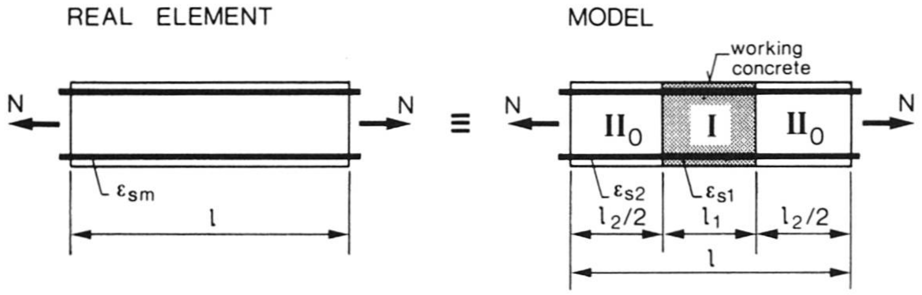


Fig. 1.2 Calculation model - pure tension

Note: It is also possible to consider a model composed of two elements of length ℓ, one of which acting in state I, is subjected to (1-c)-times the load effects and the other, acting in state II_o, subjected to c-times the load effects (fig. 1.2a):

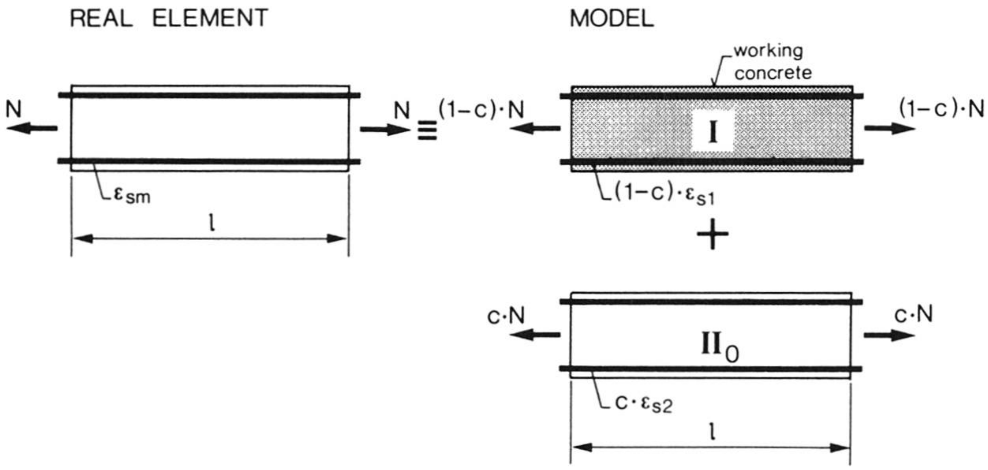


Fig. 1.2a Calculation model - pure tension

<u>Elements subjected to pure bending</u> (M = const. along a length s)

The real element of length s, along which the bending moment can be assumed constant,is once more replaced by a model composed of two parts (fig. 1.3):

- one acting in state I,
- one acting in state II_o.

From the mean strain in the reinforcement in tension

$$\varepsilon_{sm} = \frac{\Delta s_s}{s} = \frac{\varepsilon_{s1} \cdot s_1}{s} + \frac{\varepsilon_{s2} \cdot s_2}{s} = (1-c) \cdot \varepsilon_{s1} + c \cdot \varepsilon_{s2}$$

one obtains lengths s_1 (state I) and s_2 (state II_o):

$$s_1 = (1-c) \cdot s \qquad s_2 = c \cdot s$$

Therefore, the mean strain in the concrete in the upper fibre is:

$$\varepsilon_{cm} = \frac{\Delta s_c}{s} = \frac{\varepsilon_{c1} \cdot s_1}{s} + \frac{\varepsilon_{c2} \cdot s_2}{s} = (1-c) \cdot \varepsilon_{c1} + c \cdot \varepsilon_{c2}$$

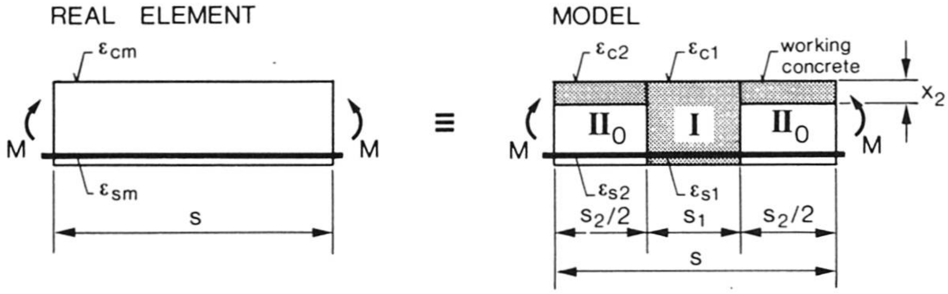


Fig. 1.3 Calculation model - pure bending

Note: One can once again consider a model composed of two elements of length ℓ, one of which, acting in state I, is subjected to (1-c)-times the load effects and the other, acting in state II_o, is subjected to c-times the load effects (fig. 1.3a).

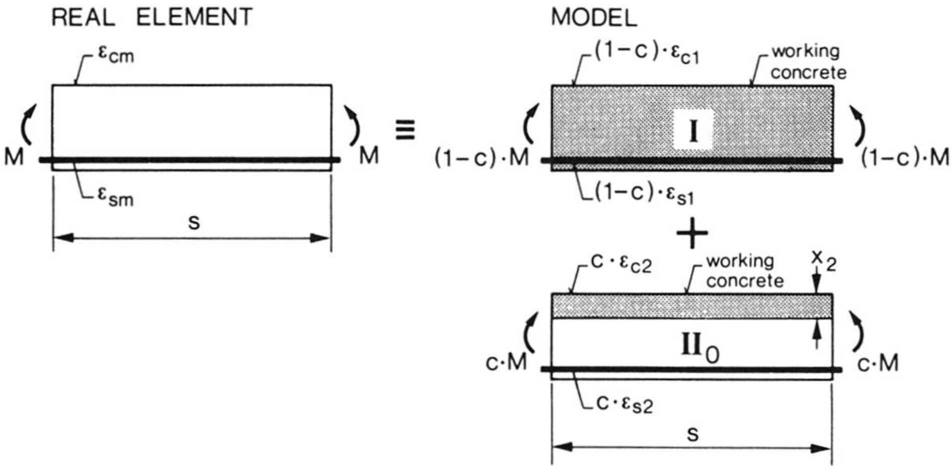


Fig. 1.3a Calculation model - pure bending

Elements subjected to combined bending (M and N = const. along a length s)

The same calculation models can be retained (fig. 1.4). One should however remark that neutral axis depends on the load effects ($x = f(M/N)$); the calculation, especially in state II_o, is therefore more complicated.

In a further article, we will deal in more detail with the problem of combined bending as well as the possible simplifications for practical applications.

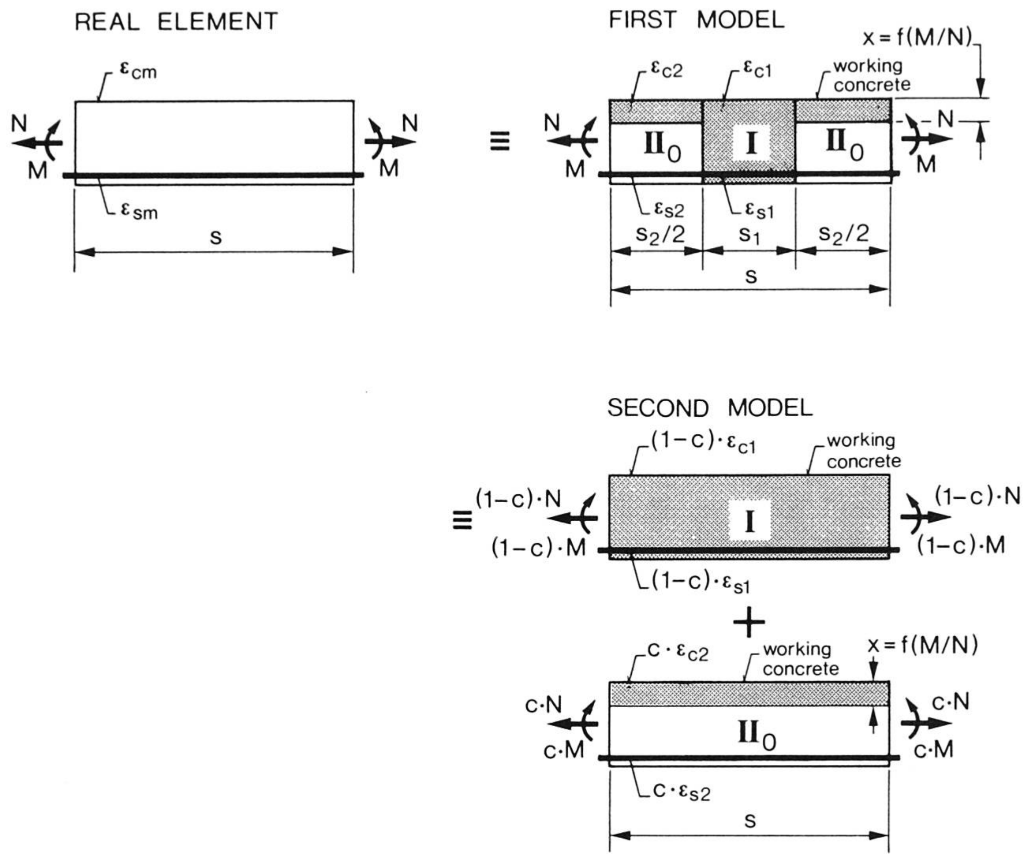


Fig. 1.4 General calculation model

2. CURVATURES

At every point of the structure, the curvature (see fig. 2.1) is given by:

$$\kappa = -\frac{1}{r} = \frac{M}{EI} = \frac{\varepsilon_s - \varepsilon_c}{d}$$

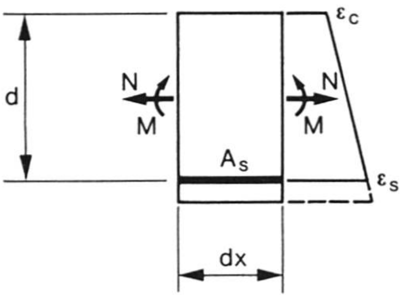


Fig. 2.1

The calculation of the curvatures in states I and II_o is made from the basic curvature

$$\kappa_c = \frac{M}{EI_c} \tag{2.1}$$

where EI_c is the bending stiffness corresponding to the section of concrete alone. The basic curvature κ_c is modified by the correction coefficients k (see § 2.2) in order to take into account the reinforcement and creep; the effect of shrinkage is considered separately.

The calculation of the mean curvatures is carried out by means of the appropriate models (fig. 1.3, 1.4).

2.1 Curvatures in the different states

Curvature in state I: $\kappa_1 = \kappa_{10} + \kappa_{1\varphi} + \kappa_{1s}$

The curvature κ_1 in state I represents the lower limit of the mean curvature κ_m.

Calculated from the basic curvature κ_c by introducing the effect of the reinforcement (correction coefficient k_A), it can be expressed at the time t = 0 by:

$$\kappa_{10} = k_{A1} \cdot \kappa_c \tag{2.2a}$$

At the time t, taking into account only the permanent loads (g), the increase of curvature due to creep (creep coefficient φ, correction coefficient k_φ) can be expressed by:

$$\kappa_{1\varphi} = (k_{A1} \cdot k_{\varphi 1} \cdot \varphi) \cdot \kappa_{cg} \tag{2.2b}$$

The curvature due to shrinkage (strain in the concrete due to shrinkage ε_{cs}, correction coefficient k_s) is given by:

$$\kappa_{1s} = k_{s1} \cdot \frac{|\varepsilon_{cs}|}{d} \tag{2.2c}$$

Curvature in state II_o: $\kappa_2 = \kappa_{20} + \kappa_{2\varphi} + \kappa_{2s}$

The curvature κ_2 in state II_o represents the upper limit of the mean curvature κ_m.

Similarly to what was done in state I, one can write:

$$\kappa_{20} = k_{A2} \cdot \kappa_c \qquad - \text{ for } t = 0 \tag{2.3a}$$

$$\kappa_{2\varphi} = (k_{A2} \cdot k_{\varphi 2} \cdot \varphi) \cdot \kappa_{cg} \qquad - \text{ effect of creep} \tag{2.3b}$$

$$\kappa_{2s} = k_{s2} \cdot \frac{|\varepsilon_{cs}|}{d} \qquad - \text{ effect of shrinkage} \tag{2.3c}$$

Mean curvature κ_m:

The mean curvature κ_m is given in a general manner (clause 16.2.2 of the Model Code) by

$$\kappa_m = \frac{M}{EI_m} = \frac{\varepsilon_{sm} - \varepsilon_{cm}}{d} \tag{2.4}$$

Referring to the model presented previously, one obtains (see fig. 2.2):

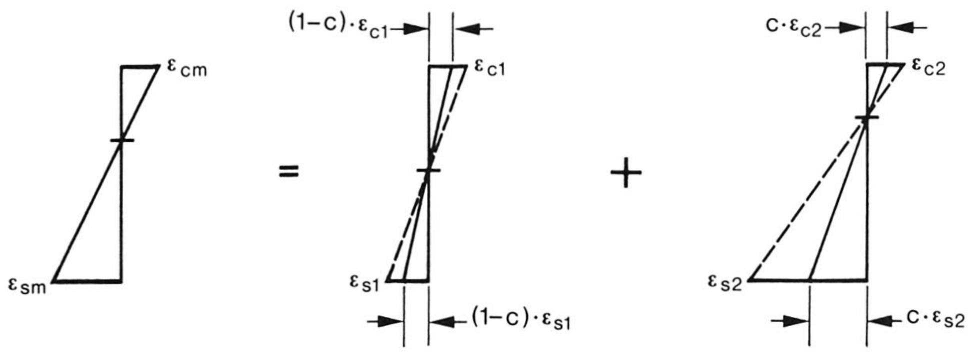


Fig. 2.2 Mean curvature

$$\varepsilon_{sm} = (1-c) \cdot \varepsilon_{s1} + c \cdot \varepsilon_{s2} \tag{2.5a}$$

$$\varepsilon_{cm} = (1-c) \cdot \varepsilon_{c1} + c \cdot \varepsilon_{c2} \tag{2.5b}$$

and the mean curvature (eq. 2.4) is expressed by:

$$\boxed{\kappa_m = (1-c) \cdot \kappa_1 + c \cdot \kappa_2} \tag{2.6}$$

Figure 2.3 shows an example of evolution of a curvature through the different states, at times t = 0 and t.

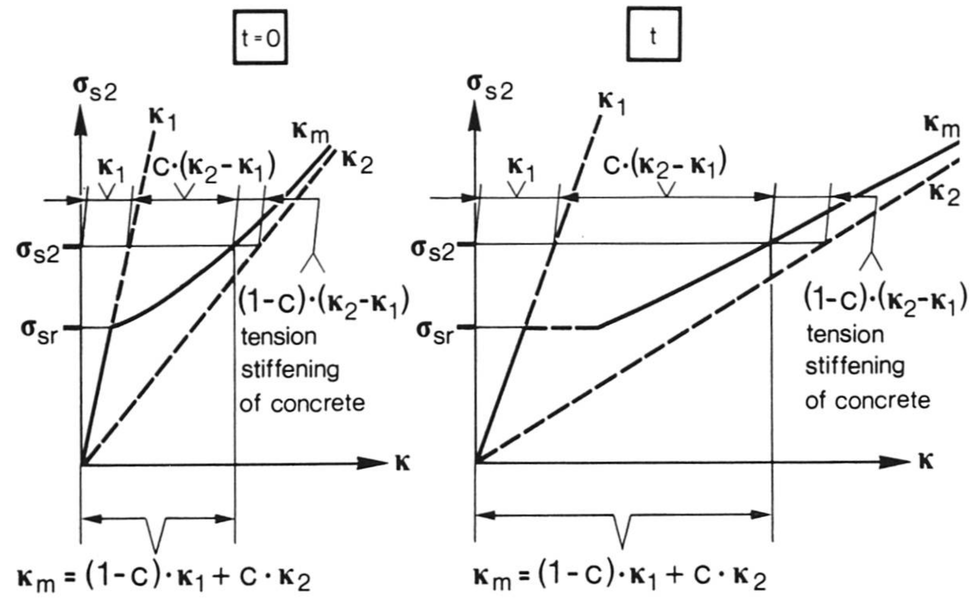


Fig. 2.3 Curvatures

2.2 Indications on the establishment of the k coefficients

The establishment of the k coefficients is shown for a section with one axis of symmetry (fig. 2.4).

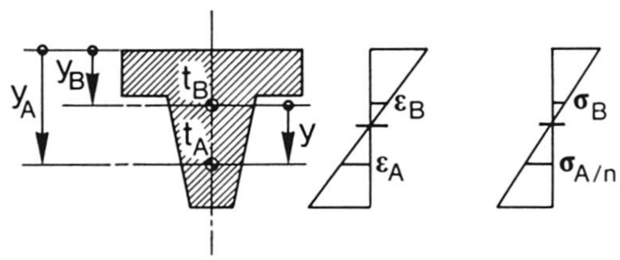


Fig. 2.4 Section

The following notation is used:

t_B - centre of gravity of the active section of concrete (in state I: the entire section of concrete; in state II: the section of concrete in compression);

t_A - centre of gravity of the entire reinforcement;

d - effective height;

x - position of the neutral axis (x_1 in state I and x_2 in state II);

y - oriented distance between the centres of gravity of the active section of concrete (t_B) and the entire reinforcement (t_A);

A_B - active section of concrete;

A_A - entire section of reinforcement;

I_c - moment of inertia of the section of concrete with respect to its centre of gravity;

I_x - moment of inertia of the idealized active section (taking into account the reinforcement) with respect to the neutral axis;

I_B - moment of inertia of the active section of concrete with respect to its centre of gravity;

I_A - moment of inertia of the entire reinforcement with respect to its centre of gravity;

I_{Bx} - moment of inertia of the active section of concrete with respect to the neutral axis;

I_{Ax} - moment of inertia of the entire reinforcement with respect to the neutral axis;

σ_A, σ_{AO}, $\sigma_{A\varphi}$ - stresses at the centre of gravity of the entire reinforcement in general, at times t = 0 and t (taking creep into account) respectively;

σ_B, σ_{BO}, $\sigma_{B\varphi}$ - stresses at the centre of gravity of the active section of concrete;

ε_A, ε_{AO}, $\varepsilon_{A\varphi}$ - strains at the centre of gravity of the entire reinforcement;

ε_B, ε_{BO}, $\varepsilon_{B\varphi}$ - strains at the centre of gravity of the active section of concrete.

Coefficient of correction k_A:

Let us express the curvature at the time t = 0 (κ_0, κ_{10} for state I, κ_{20} for state II_o) by means of the basic curvature κ_c; this relation being linear for a given section

$$\kappa_0 = k_A \cdot \kappa_c$$

one obtains for k_A:

$$k_A = \frac{\kappa_0}{\kappa_c} = \frac{M/(EI_x)}{M/(EI_c)} = \frac{EI_c}{EI_x} = \frac{E_c I_c}{E_c I_{Bx} + E_s I_{Ax}} = \frac{I_c}{I_{Bx} + nI_{Ax}}$$

Note: *The neutral axis, in state I or II_o, is determined by*

$$\frac{x}{d} = \frac{\dfrac{y_B}{d} + \dfrac{nA_A}{A_B} \cdot \dfrac{y_A}{d}}{1 + \dfrac{nA_A}{A_B}}$$

Coefficient of correction k_φ:

Let us express by means of the curvature at the time t = 0 due to the permanent loads

$$\kappa_0 = k_A \cdot \kappa_{cg}$$

the increase of curvature due to creep (κ_φ, $\kappa_{1\varphi}$ for state I, $\kappa_{2\varphi}$ for state II_o):

$$\kappa_\varphi = \varphi \cdot k_\varphi \cdot \kappa_0$$

Coefficient k_φ can be developed as follows:

$$k_\varphi = \frac{1}{\varphi} \cdot \frac{\kappa_\varphi}{\kappa_0}$$

$$= \frac{1}{\varphi} \cdot \frac{\frac{1}{y} \cdot (\varepsilon_{B\varphi} - \varepsilon_{A\varphi})}{\frac{1}{y} \cdot (\varepsilon_{B0} - \varepsilon_{A0})} = \frac{1}{\varphi} \cdot \frac{\dfrac{\sigma_{B0}}{E_c} \cdot \varphi + \dfrac{\sigma_{B\varphi}}{E_c} \cdot (1 + \chi \cdot \varphi) - \dfrac{\sigma_{A\varphi}}{E_s}}{\dfrac{\sigma_{B0}}{E_c} - \dfrac{\sigma_{A0}}{E_s}}$$

Note: *χ - aging coefficient - see Model Code - appendix e.*

By introducing the condition of equivalence between the internal forces (M,N,...) and the stresses (σ,τ) in the section

$$\sigma_B \cdot A_B + \sigma_A \cdot A_A = 0$$

one finds:

$$k_\varphi = \frac{\frac{E_sA_A}{E_cA_B} + \frac{1}{\varphi} \cdot \frac{\sigma_{A\varphi}}{\sigma_{AO}} \cdot \left(1 + \frac{E_sA_A}{E_cA_B} \cdot (1 + \chi \cdot \varphi)\right)}{1 + \frac{E_sA_A}{E_cA_B}}$$

The only unknown is the ratio $\sigma_{A\varphi}/\sigma_{AO}$ which can be determined by means of the conditions of compatibility (between the concrete and steel) and of equivalence; one obtains:

$$\frac{\sigma_{A\varphi}}{\sigma_{AO}} = \varphi \cdot \frac{1 - \frac{E_sA_A}{E_cA_B} \cdot \frac{E_sI_A}{E_cI_B} \cdot (1 + \chi \cdot \varphi)}{D}$$

and therefore for coefficient k_φ:

$$k_\varphi = \frac{1}{1 + \frac{nA_A}{A_B}} \cdot \left[\frac{nA_A}{A_B} + \frac{1 - \frac{nA_A}{A_B} \cdot \frac{nI_A}{I_B} (1 + \chi \cdot \varphi)}{D} \cdot \left(1 + \frac{nA_A}{A_B} \cdot (1 + \chi \cdot \varphi)\right)\right]$$

with

$$D = 1 + \frac{nA_A}{A_B} \cdot \left(1 + \frac{A_B}{I_B} \cdot y^2\right) \cdot (1 + \chi \cdot \varphi) +$$

$$+ \frac{nI_A}{I_B} (1 + \chi \cdot \varphi) \cdot \left(1 + \frac{nA_A}{A_B} \cdot (1 + \chi \cdot \varphi)\right)$$

Coefficient of correction k_s :

Let us express the curvature due to shrinkage κ_s (κ_{s1} for state I, κ_{s2} for state II_o):

$$\kappa_s = \frac{1}{y} \cdot (\varepsilon_{Bs} - \varepsilon_{As}) = \frac{1}{y} \cdot \left(\frac{\sigma_{Bs}}{E_c} \cdot (1 + \chi \cdot \varphi) + \varepsilon_{cs} - \frac{\sigma_{As}}{E_s}\right)$$

By introducing the condition of equivalence one finds:

$$\kappa_s = \frac{\varepsilon_{cs}}{y} \cdot \left(1 - \frac{\sigma_{As}}{E_s \cdot \varepsilon_{cs}} \cdot \left(1 + \frac{E_s A_A}{E_c A_B} \cdot (1 + \chi \cdot \varphi)\right)\right)$$

The only unknown, the ratio $\sigma_{As}/(E_s \cdot \varepsilon_{cs})$, can be determined by means of the conditions of compatibility and of equivalence; one obtains:

$$\frac{\sigma_{As}}{E_s \cdot \varepsilon_{cs}} = \frac{1 + \frac{E_s I_A}{E_c I_B} \cdot (1 + \chi \cdot \varphi)}{D}$$

and therefore for the curvature κ_s:

$$\kappa_s = \frac{\varepsilon_{cs}}{d} \cdot \frac{\frac{nA_A}{I_B} \cdot y \cdot d \cdot (1 + \chi \cdot \varphi)}{D}$$

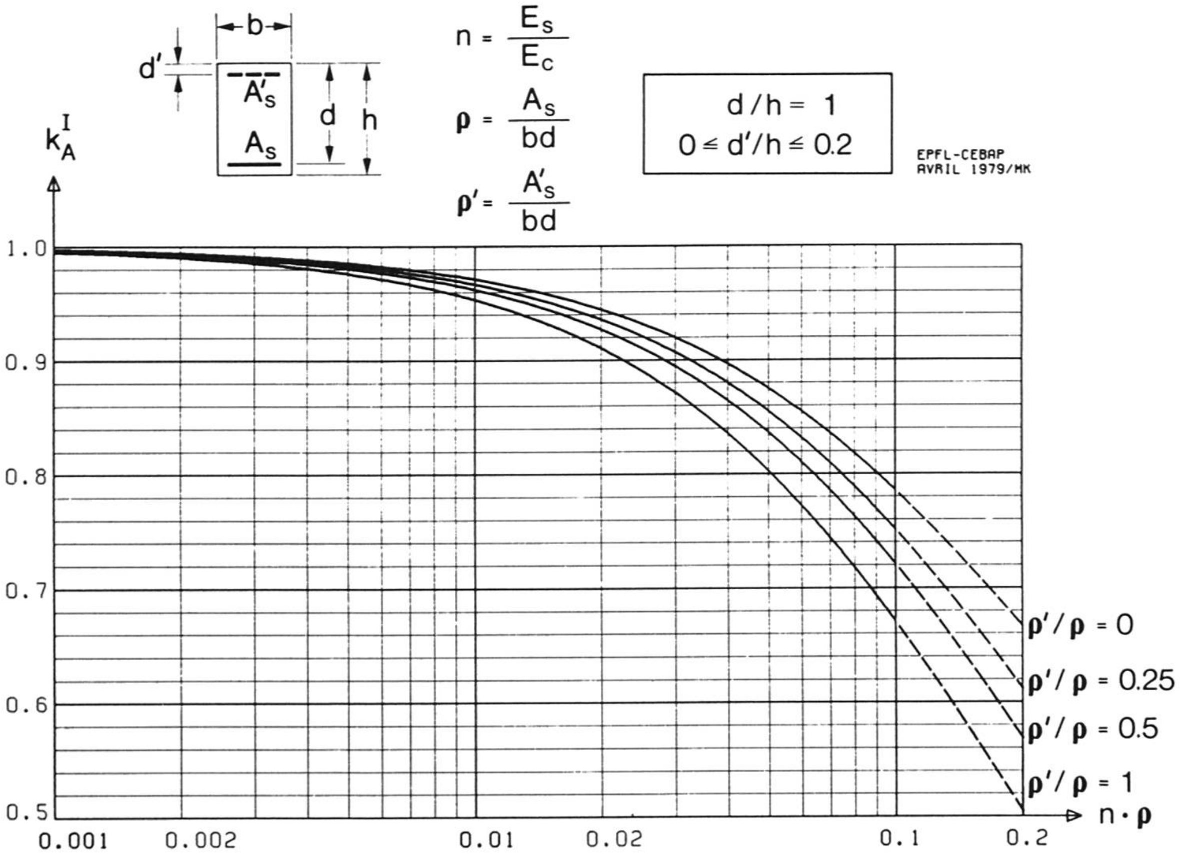
With

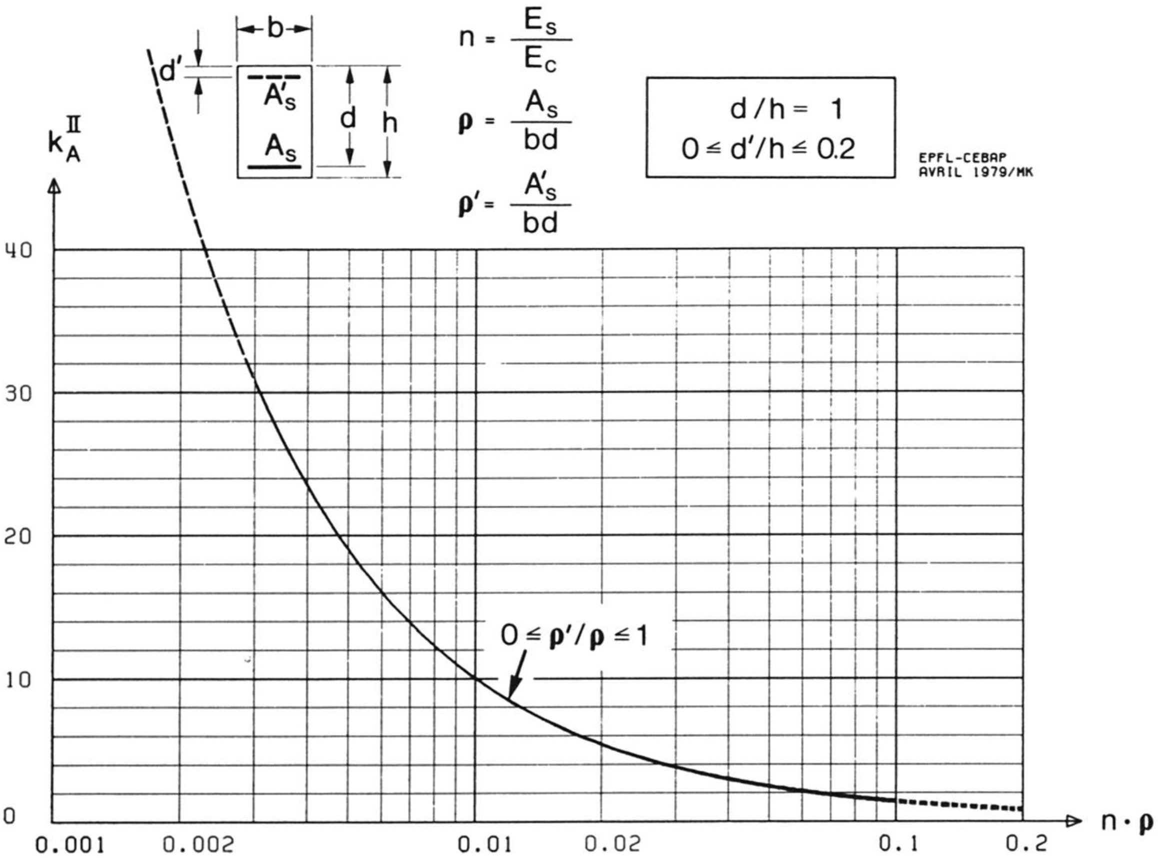
$$\kappa_s = \frac{\varepsilon_{cs}}{d} \cdot k_s$$

the coefficient k_s becomes:

$$k_s = \frac{\frac{nA_A}{I_B} \cdot y \cdot d \cdot (1 + \chi \cdot \varphi)}{D}$$

Note: The coefficients k_A, k_φ, k_s are dimensionless and can be tabulated for different types of sections. We give here, as an example, the graphical representation of each of the coefficients for a rectangular section. These curves were established by means of a program developed at the Institute of Reinforced and Prestressed Concrete (IBAP) of the Swiss Federal Institute of Technology in Lausanne.


b
d'
A's
As
d
h
$n = \frac{E_s}{E_c}$
$\rho = \frac{A_s}{bd}$
$\rho' = \frac{A'_s}{bd}$
d / h = 1
0 ≤ d'/h ≤ 0.2
EPFL-CEBAP
AVRIL 1979/MK
k_A^I
1.0
0.9
0.8
0.7
0.6
0.5
0.001
0.002
0.01
0.02
0.1
0.2
n · ρ
ρ'/ρ = 0
ρ'/ρ = 0.25
ρ'/ρ = 0.5
ρ'/ρ = 1


b
d'
A's
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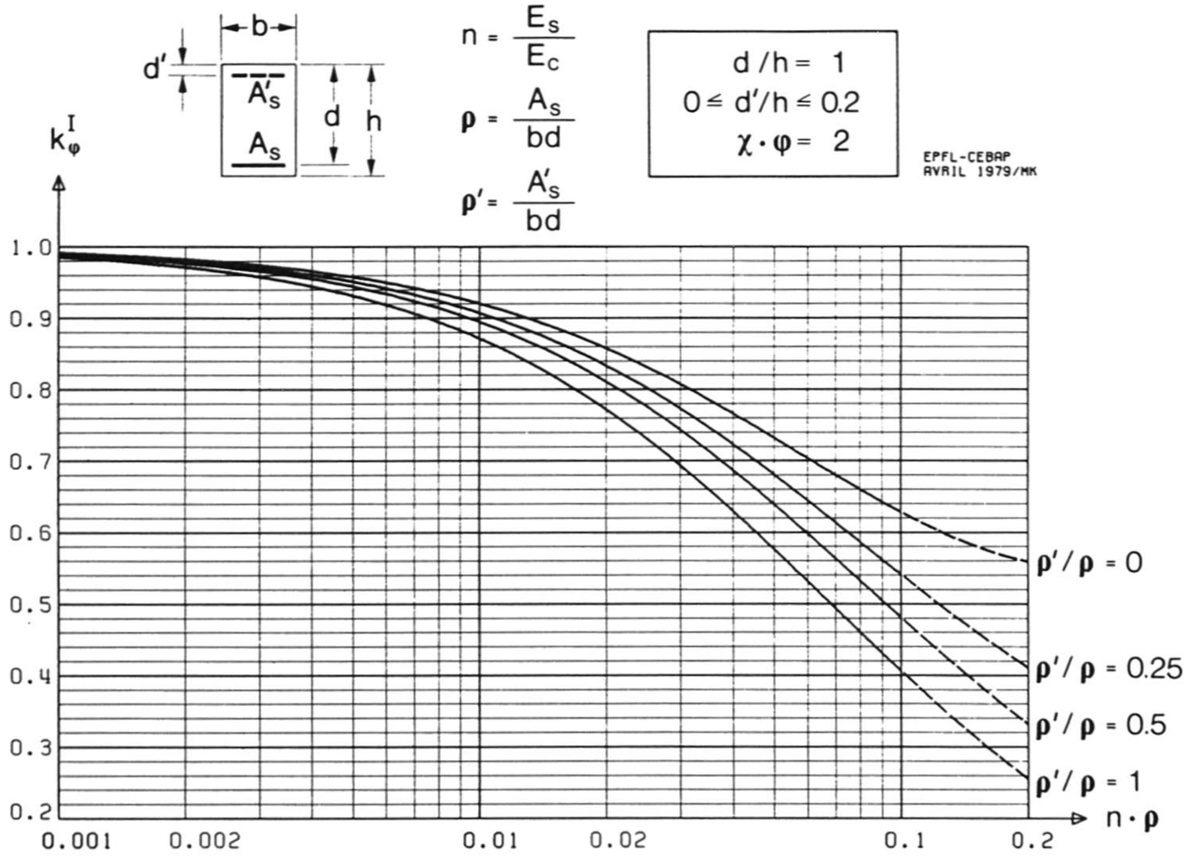
h
$n = \frac{E_s}{E_c}$
$\rho = \frac{A_s}{bd}$
$\rho' = \frac{A'_s}{bd}$
d / h = 1
0 ≤ d'/h ≤ 0.2
EPFL-CEBAP
AVRIL 1979/MK
k_A^{II}
40
30
20
10
0
0.001
0.002
0.01
0.02
0.1
0.2
n · ρ
0 ≤ ρ'/ρ ≤ 1


b
d'
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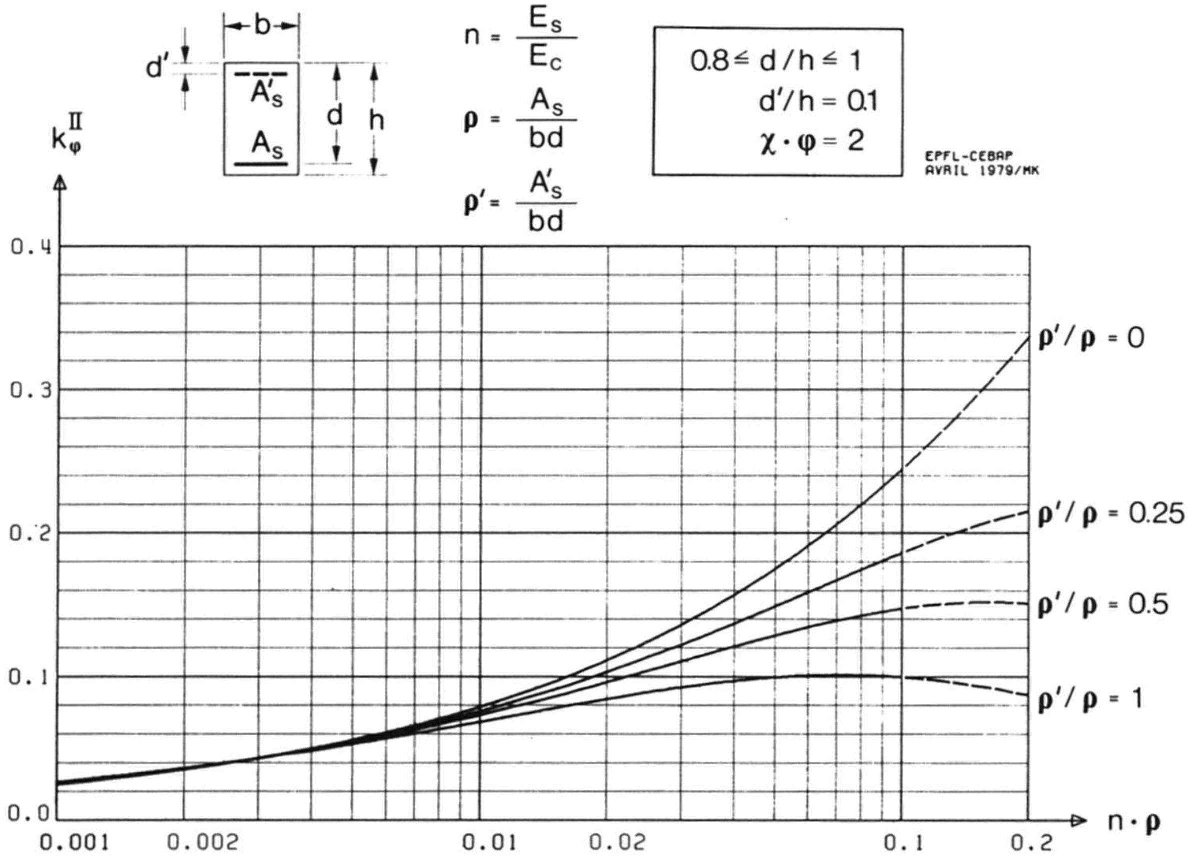
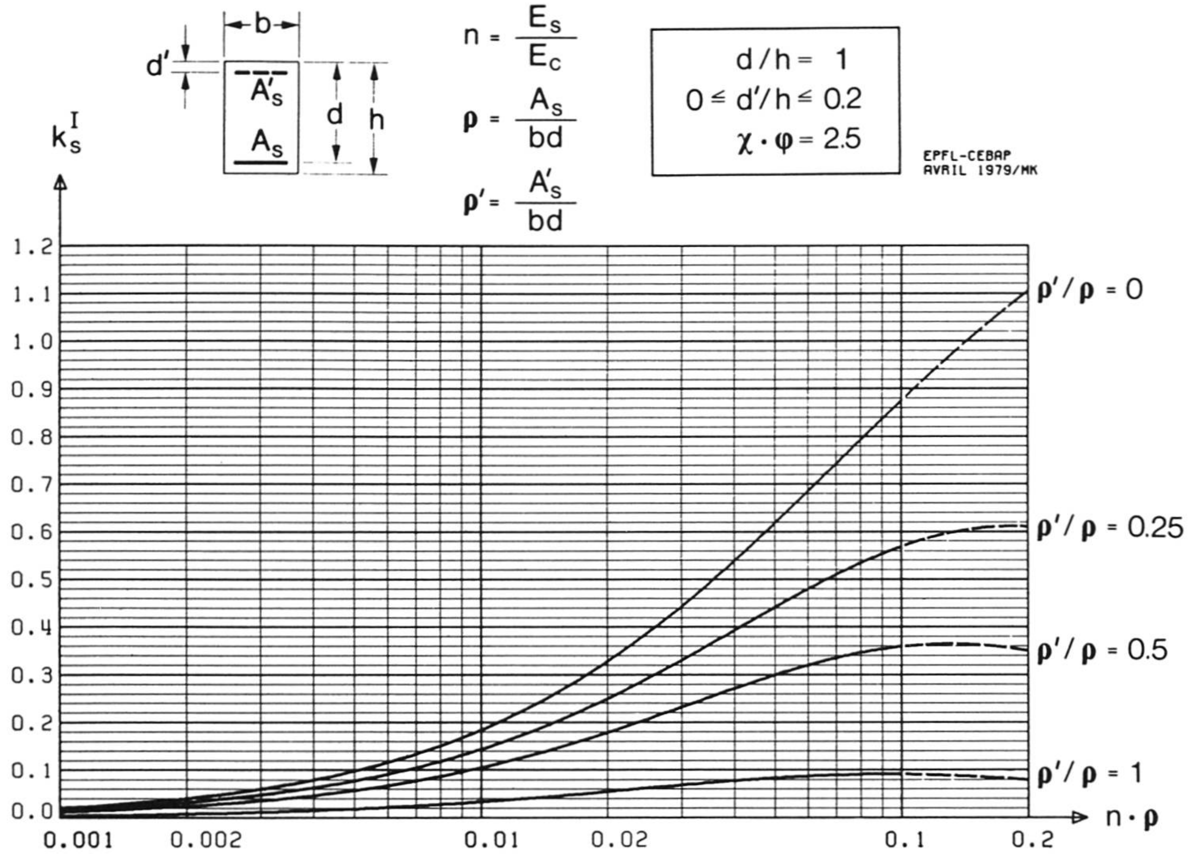
h
$n = \frac{E_s}{E_c}$
$\rho = \frac{A_s}{bd}$
$\rho' = \frac{A'_s}{bd}$
d / h = 1
0 ≤ d'/h ≤ 0.2
χ · φ = 2
EPFL-CEBAP
AVRIL 1979/MK
k_φ^I
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.001
0.002
0.01
0.02
0.1
0.2
n · ρ
ρ'/ρ = 0
ρ'/ρ = 0.25
ρ'/ρ = 0.5
ρ'/ρ = 1


b
d'
A'_s
A_s
d
h
$n = \frac{E_s}{E_c}$
$\rho = \frac{A_s}{bd}$
$\rho' = \frac{A'_s}{bd}$
0.8 ≤ d / h ≤ 1
d'/h = 0.1
χ · φ = 2
EPFL-CEBAP
AVRIL 1979/MK
k_φ^{II}
0.4
0.3
0.2
0.1
0.0
0.001
0.002
0.01
0.02
0.1
0.2
n · ρ
ρ'/ρ = 0
ρ'/ρ = 0.25
ρ'/ρ = 0.5
ρ'/ρ = 1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d'
A'_s
A_s
d
h
k_s^I
$n = \frac{E_s}{E_c}$
$\rho = \frac{A_s}{bd}$
$\rho' = \frac{A'_s}{bd}$
d/h = 1
0 ≤ d'/h ≤ 0.2
χ · φ = 2.5
EPFL-CEBAP
AVRIL 1979/MK
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.001
0.002
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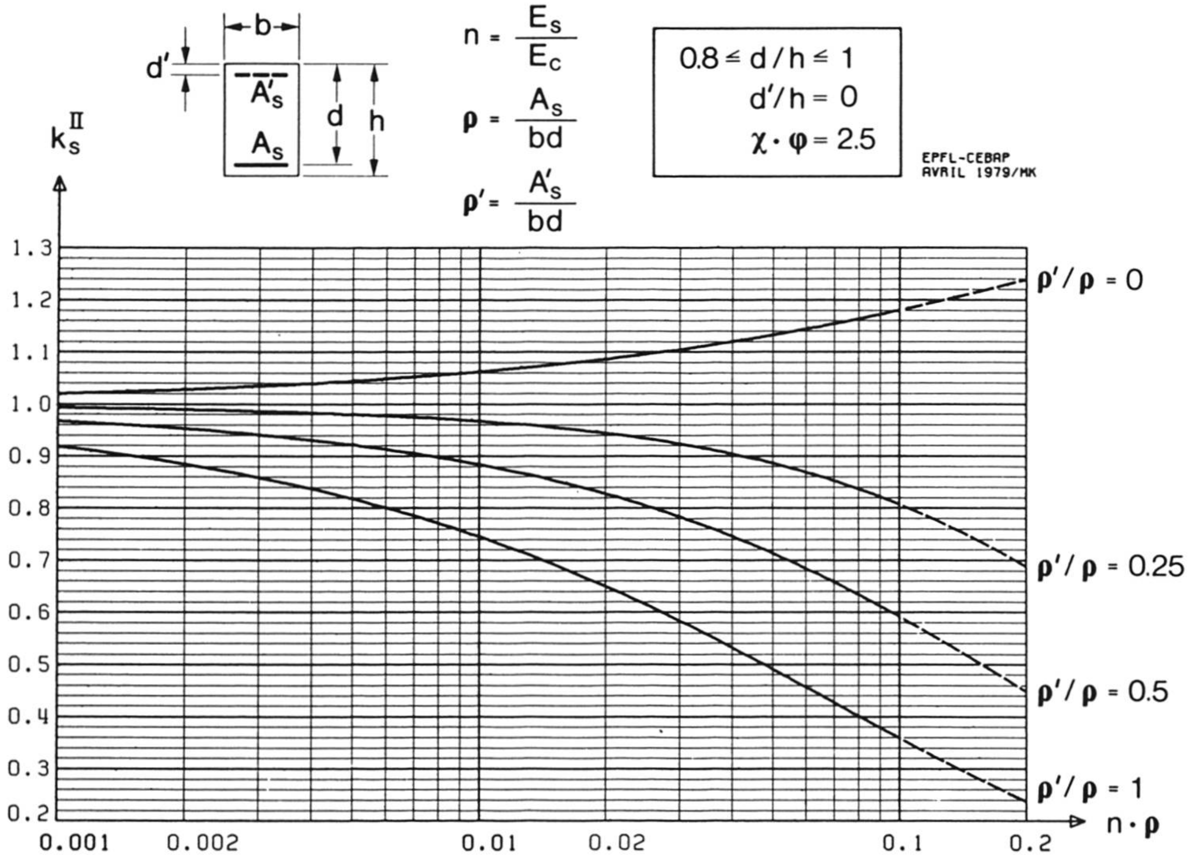
0.1
0.2
n · ρ
ρ'/ρ = 0
ρ'/ρ = 0.25
ρ'/ρ = 0.5
ρ'/ρ = 1


b
d'
A'_s
A_s
d
h
k_s^{II}
$n = \frac{E_s}{E_c}$
$\rho = \frac{A_s}{bd}$
$\rho' = \frac{A'_s}{bd}$
0.8 ≤ d/h ≤ 1
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0.1
0.2
n · ρ
ρ'/ρ = 0
ρ'/ρ = 0.25
ρ'/ρ = 0.5
ρ'/ρ = 1

3. CALCULATION OF THE DEFORMATIONS BY INTEGRATION

The deformations can be obtained by double integration of the mean curvature κ_m (eq. 2.6) along the element, respecting the end conditions.
The deformation a (displacement, rotation) of an element can be obtained by the virtual work theorem:

$$a = \int \kappa_m \cdot \overline{M} \cdot dx \tag{3.1}$$

The development of the calculation is as follows:

From the basic flexural stiffnesses (uncracked sections, effect of the reinforcement neglected), one calculates the bending moments M_c. The basic curvatures $\kappa_c = M_c/EI_c$ then enable us to calculate the curvatures κ_1 in state I and κ_2 in state II_o. The mean curvatures are defined by (eq. 2.6):

$$\kappa_m = (1-c) \cdot \kappa_1 + c \cdot \kappa_2$$

with (eq. 1.5)

$$c = 1 - \beta_1 \cdot \beta_2 \cdot \frac{\sigma_{sr}^2}{\sigma_{s2}^2}$$

For statically determinate structures, the calculation of the mean curvature ends here. For statically indeterminate structures, however, the calculation becomes iterative due to the redistribution of moments (cracking, creep). The mean flexural stiffnesses

$$EI_m = \frac{M_c}{\kappa_m}$$

make it possible to recalculate the bending moments M. The cycle previously described is then carried out again. The iteration is completed when the difference in flexural stiffnesses of two consecutive cycles is judged acceptable.

The mean stiffnesses thus obtained enable us to calculate deformation a (eq. 3.1) by numerical integration.

The calculation is tedious and requires in most cases the use of the computer. This difficulty is the reason for research for simplified calculation models.

4. PRACTICAL CALCULATION OF DEFLECTIONS : BILINEAR METHOD

In order to determine the probable deflection a, the theorem of virtual work is applied:

$$a = \int \kappa_m \cdot \overline{M} \cdot dx$$

The difficulty consists, as we saw in the previous chapter, in the calculation of the mean curvatures:

$$\kappa_m = (1-c) \cdot \kappa_1 + c \cdot \kappa_2$$

The bilinear method, which is based on the fact that the deflection-load relationship, in the serviceability state, can be approached by a bilinear function, enables us to avoid this difficulty by reducing the essential of the calculation to that of a single section, the determinant section. The latter is defined as the section where the deflection is calculated, in mid-span, or the end restraint section for a cantilever.

The simplifications made are situated at two levels, one concerning the coefficient c, the other the deflections a_1 and a_2.

4.1 Simplification of coefficient c

For a given load level, coefficient c defined by eq. 1.5

$$c = 1 - \beta_1 \cdot \beta_2 \cdot \frac{\sigma_{sr}^2}{\sigma_{s2}^2}$$

varies along the length of the element.

The following simplifications, considered acceptable,

- the stress σ_{sr} is assumed constant and equal to the stress σ_{sr} in the determinant section: σ_{sr} = constant;
- the stress σ_{s2} is assumed constant and equal to the geometrical mean of the stresses σ_{sr} and $\sigma_{s2} \equiv \sigma_{sD}$ in the determinant section:

$$\sigma_{s2} = \sqrt{\sigma_{sr} \cdot \sigma_{sD}}$$

lead to

$$c = 1 - \beta_1 \cdot \beta_2 \cdot \frac{\sigma_{sr}}{\sigma_{sD}} = \text{const. for a given load level}$$

Let us replace the ratio σ_{sr}/σ_{sD} by the ratio of the corresponding moments:

$$\frac{\sigma_{sr}}{\sigma_{sD}} = \frac{M_r}{M_D}$$

M_D being the moment calculated in the determinant section and M_r the cracking moment in the same section, as defined in the Model-Code-clause 16.2.1:

$$\boxed{M_r = W_1 \cdot (f_{ct} - \frac{N}{A_1})} \qquad (4.1)$$

with

$f_{ct} = f_{ct0,05}$ for the prevention of damage;

$\quad = f_{ctm}$ for the calculation of camber;

A_1 - idealized section, taking the reinforcement into account (state I);

W_1 - modulus of inertia of the idealized section with respect to the neutral axis (state I).

The final expression for the simplified coefficent c is:

$$\boxed{c = 1 - \beta_1 \cdot \beta_2 \cdot \frac{M_r}{M_D}} \tag{4.2}$$

4.2 Bilinear method

The probable deflection is given by:

$$\begin{aligned} a &= \int \kappa_m \cdot \overline{M} \cdot dx \\ &= \int ((1-c).\kappa_1 + c.\ \kappa_2) \cdot \overline{M} \cdot dx \end{aligned}$$

By introducing the simplifications indicated in § 4.1, the coefficient c given by eq. (4.2) becomes independent of x and can be placed in front of the integral:

$$a = (1-c). \int \kappa_1 \cdot \overline{M} \cdot dx + c \cdot \int \kappa_2 \cdot \overline{M} \cdot dx$$

As

$$a_1 = \int \kappa_1 \cdot \overline{M} \cdot dx \quad \text{- deflection in state I}$$

$$a_2 = \int \kappa_2 \cdot \overline{M} \cdot dx \quad \text{- deflection in state II}_o$$

we obtain

$$\boxed{a = (1-c) \cdot a_1 + c \cdot a_2} \tag{4.3}$$

It is obvious that this relationship is bilinear (see fig. 4.1). The deflections a_1 in state I and a_2 in state II_o represent the extremes of the probable deflection a.

These deflections a_1 and a_2 can be calculated in an exact manner from the curvatures κ_1 and κ_2 defined by eq. (2.2) and (2.3).

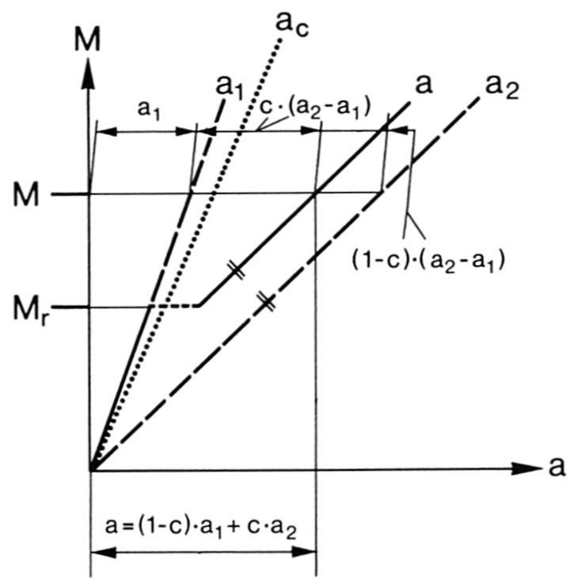

Fig. 4.1 Bilinear relationship deflection - moment

Due to the very distribution of the reinforcement and that of the virtual moment $\overline{M}$, the variation of the correction coefficients k along the element can be neglected and one obtains successively for the deflections a_1 and a_2:

$$a_1 = \int \kappa_1 \cdot \overline{M} \cdot dx = \int (\kappa_{10} + \kappa_{1\varphi} + \kappa_{1s}) \cdot \overline{M} \cdot dx$$

$$= \int (k_{A1} \cdot \kappa_c + k_{A1} \cdot k_{\varphi 1} \cdot \varphi \cdot \kappa_{cg} + k_{s1} \cdot \frac{|\varepsilon_{cs}|}{d}) \cdot \overline{M} \cdot dx$$

$$\boxed{a_1 \cong k_{A1} \cdot a_c + k_{A1} \cdot k_{\varphi 1} \cdot \varphi \cdot a_{cg} + k_{s1} \cdot \frac{|\varepsilon_{cs}|}{d} \cdot \int |\overline{M}| \cdot dx} \qquad (4.4a)$$

and similarly

$$\boxed{a_2 \cong k_{A2} \cdot a_c + k_{A2} \cdot k_{\varphi 2} \cdot \varphi \cdot a_{cg} + k_{s2} \cdot \frac{|\varepsilon_{cs}|}{d} \cdot \int |\overline{M}| \cdot dx} \qquad (4.4b)$$

Notation:

a_c - basic deflection (section of concrete only);

a_{cg} - basic deflection due to the permanent loads;

$\overline{M}$ - virtual moment in the statically indeterminate system;

k - coefficient of correction of the determinant section.

Notes: 1) This method can be improved, but at the cost of simplicity, by introducing, for example, for coefficient c:

$$c = (1 - \beta_1 \cdot \beta_2 \cdot \frac{\alpha_M \cdot M_r}{M_D}) \cdot \alpha_p$$

where

α_M - modifies the point of intersection of the two lines;

α_p - modifies the slope in state II_o.

Coefficients α_M and α_p could be determined and tabulated as functions of the end fixity and of the distribution of the reinforcement.

2) A possible redistribution of the moments (Model-Code clause 8.3.2) that will influence moment M_D must be taken into account.

Summary of the bilinear method for the calculation of deflections:

In order to calculate the probable deflection a, it is necessary to determine successively:

1. the basic deflection a_c;
2. the extremes a_1 and a_2 (eq. 4.4);
3. the cracking moment M_r (eq. 4.1);
4. the maximum moment M_D;
5. the probable deflection a (eq. 4.3).

It is obvious that all these values must be calculated for the same load.

Extension to the calculation of the deflection of slabs:

In order to determine the probable deflection a of a reinforced or prestressed concrete slab, the method described above is applied according to the following procedure:

1. Calculation of a_c by means of a classical elastic method. At this point one could introduce the effect of the anisotropy, which, however, is negligible in most cases.

2. Calculation of max m_x, max m_y in the considered field. The section in which the greatest positive moment acts is determinant (this section does not necessarily coincide with the spot where the basic deflection a_c was established): the coefficients of correction as well as the cracking moment are determined for this section.

3. Calculation of a_1, a_2 and of the probable deflection a.

Notes: 1) m_r, ρ, ρ' are introduced per unit of width.

2) Tests are at present being carried out at IBAP, which should allow verification of this procedure, especially of the influence of long term effects.

5. ESTIMATION OF DEFLECTIONS

In the majority of cases in practice, at the preliminary design stage especially, the practising engineer is only interested in a simple estimation of the probable deflection. To this effect, we propose a simplified method, based on the bilinear method.

In general, the probable deflection due to the permanent loads is given by (see § 4)

$$\boxed{a = a_{cg} \cdot (k_o + k_t \cdot \varphi)} \qquad (5.1)$$

with

a_{cg} - basic deflection due to the permanent loads, calculated with $EI = E_c I_c$;

k_o - coefficient of correction taking into account the reinforcement and cracking for the calculation of the immediate deflection:

$$k_o = k_{A1} + c \cdot (k_{A2} - k_{A1}) \qquad (5.2)$$

k_t - coefficient of correction taking into account the reinforcement, cracking and creep for the calculation of the part of the deflection due to creep:

$$k_t = k_{A1} \cdot k_{\varphi 1} + c \cdot (k_{A2} \cdot k_{\varphi 2} - k_{A1} \cdot k_{\varphi 1}). \qquad (5.3)$$

By fixing the following parameters:

$d/h = 0,9 \qquad d'/h = 0,1 \qquad \chi \cdot \varphi = 2$

coefficients k_o and k_t that are indispensable to the calculation can be tabulated as functions of the following parameters:

$n \cdot \rho \qquad \rho'/\rho \qquad c$

Knowing the basic deflection a_{cg}, the cracking moment M_r and the working moment M_D in the determinant section, one calculates the coefficient c and one estimates the percentages of reinforcement ρ and ρ' in the same section. These parameters c, ρ and ρ' enable then to read in a table the coefficients k_o and k_t from which one determines the probable deflection a.

The calculation is thereby greatly simplified, but still provides a reasonable estimation of the deflection.

CONCLUSIONS

The definition of calculation models composed of two parts, one acting in state I and the other in state II_o, enabled us to deal with the problem of deformations due to bending by means of an exact method. The successive introduction of a number of simplifications led us to a bilinear method for the calculation of deflections and then to a simplified form of this same method, allowing a rapid estimation of deflection.

For practical applications, it is indispensable to have aids to ease the calculation of the coefficients of correction k. One can refer for this to [3] or to the CEB/FIP "Cracking and Deformation" Manual, at present being prepared.

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