Prediction of the behaviour of concrete frames

Prediction of the Behaviour of Concrete Frames

Prédiction du comportement des cadres en béton

Die Voraussage des Verhaltens von Betonrahmen

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INTRODUCTION

A great deal of attention has been devoted to the effects of creep and shrinkage on columns having determinate end conditions. However, because of the very nature of the problem, very little direct information has been obtained on the behaviour and interaction of columns as part of indeterminant frames. This research paper describes a method for predicting the behaviour of reinforced concrete frames.

The method of analysis includes the non linear response of reinforced concrete sections under short term loading as well as the time dependent effects of creep, shrinkage and changes in concrete strength. Utilizing the digital computer this analysis could be used directly as a check for special structural cases. However, the main purpose of this study was to provide a means of assessing and perhaps altering the various Building Code design provisions for column design and moment distributions in frames. With this objective an analysis was derived to predict the behaviour and failure for short term loading to the behaviour for predetermined load versus time history including either prediction of the time of sustained load failure or the remaining capacity after a period of sustained loading.

In order to provide data to verify the numerical method a limited experimental program was implemented.

EXPERIMENTAL PROGRAM

Test Specimen: The test specimen chosen was a single bay fixed base frame. While being a relatively simple specimen to fabricate this choice, with three degrees of indeterminacy, provided ample opportunity for redistribution of bending moments due to inelastic behaviour. Therefore, the test data would provide a good check for the accuracy of the analytical predictions. Also additional information on the formation of collapse mechanisms in reinforced concrete frames would be obtained. To reduce the effects of scale and to

minimize the relative magnitude of fabrication and experimental tolerances a fairly large model was used.

As shown in Figure 1, the outside dimensions were 274.32 cm high by 294.64 cm wide. The cross-section was 20.32 cm square with a 1.91 cm diameter deformed reinforcing bars in each corner under 2.54 cm of concrete cover. The reinforcement had a sharply defined yield stress of 4150 ±35 kg/cm². The bottom of each column was composed of a 20.32 cm by 20.32 cm by 20.32 cm H section. The reinforcing bars were welded to the web of the H section prior to pouring the concrete and the bottom of the H section was welded to a base plate when the frame was placed in the test position. The base plate was stiffened and prestressed to the laboratory floor to provide the fixed end condition. Square ties made from 0.64 cm diameter plain bars were spaced at 15.24 cm and 7.62 cm respectively in the columns and beam. To avoid discontinuities the reinforcement was continuous throughout the frame. Because of difficulties encountered in a preliminary test, Frame R1, the corners of the frame were made quite stiff by the addition of diagonal ties. The length within the top portion of the H section was similarly stiffened.

A concrete mix design having predetermined creep, shrinkage and stress-strain properties was used. The following proportions by weight were used to obtain a 28 day cylinder strength of 316 kg/cm²: Portland Cement Type 1 -- 14.0%; Water -- 9.1%; Fine Aggregate(washed sand) -- 46.6%; Coarse Aggregate (0.95 cm maximum size crushed limestone) -- 30.3%. The frame was cast in a steel form and moist cured for 7 days before being placed in the test position. Companion cylinders and prisms with and without reinforcement were also poured.

Test Apparatus: Separate sets of apparatus were set up for short term loading and for sustained loading. In both cases loads were controlled using load cells employing electric resistance foil strain gauges. Dial gauges were mounted to provide deflection and base rotation data. Strains on the concrete were obtained using a demountable mechanical dial indicator with a 20.32 cm gauge length. Equipment to provide a horizontal load, H, on the left column at mid-height of the beam and a vertical load, V, at the centre of the span was required for both set ups. In each case the load was transferred to the test frame through a spherical seat.

Short Term Tests: Steel Wide Flange columns, prestressed to the floor, were fitted with cross beams to jack against. The jacks were mounted on gear driven mechanical slides to maintain the positions of the loads as the frame deflected.

Sustained Load Tests: The apparatus for the sustained load tests was located in a polyethylene covered frame inside which the temperature and relative humidity were maintained at 24°C ± 1°C and 50% ± 2% respectively. Instead of using jacks, loads were maintained by the use of springs and rods as shown in Figure 1. The coil springs reduced the decrease in load due to the long term deflection of the frame. As the loads did decrease they were adjusted to keep within 2% of the specified values. The vertical load followed the deflection of the frame by adjustment of the horizontal mechanical slide incorporated into the spring and rod assembly.

FIGURE 1   Sustained Load Test Apparatus and Concrete Frame.
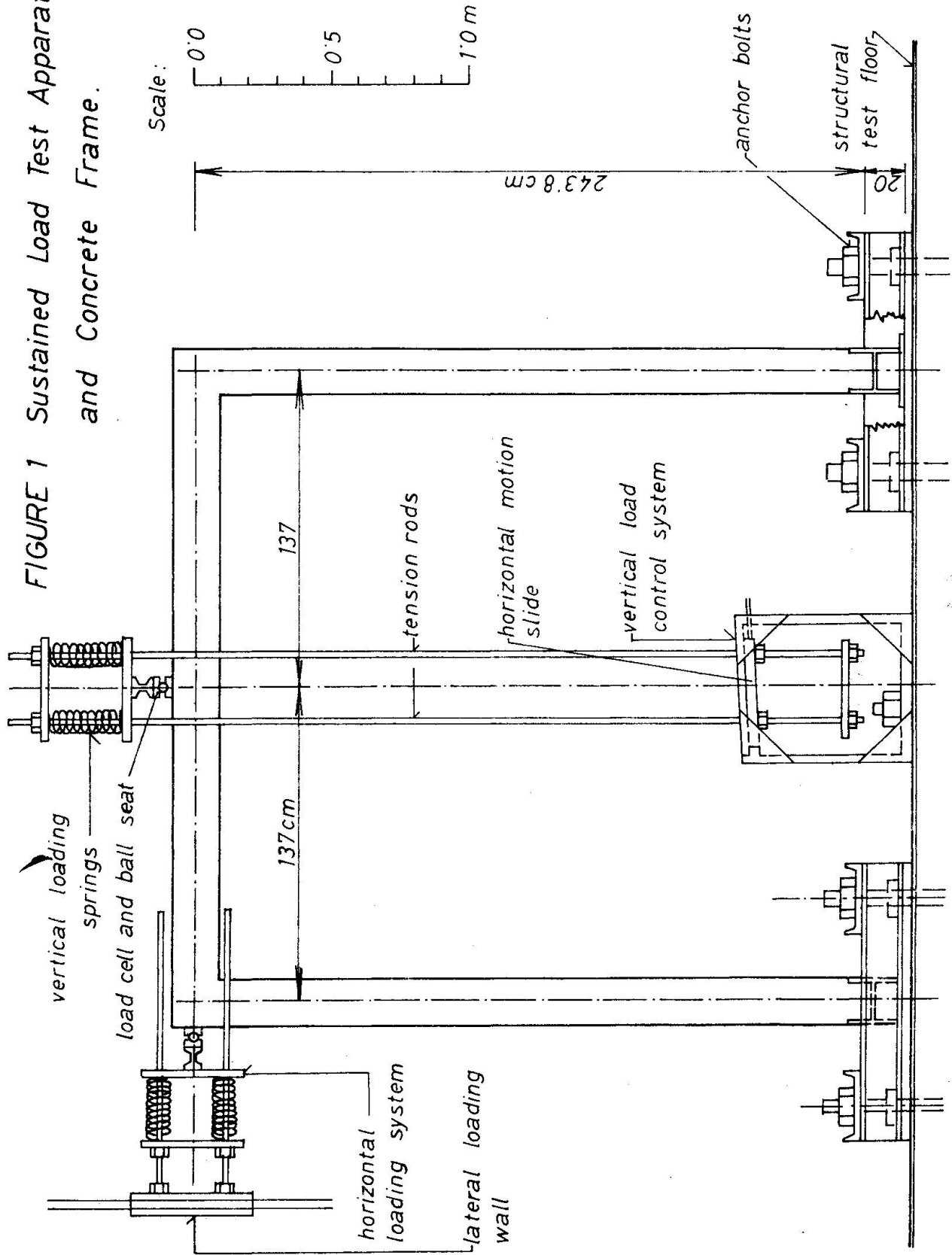
ANALYTICAL METHOD

The basis for the analytical method was intended to be generally applicable to the prediction of behaviour of beam and column combinations. However, it was necessary to evaluate the acceptibility of these fundamental processes in terms of the accuracy with which they would be utilized to predict the actual behaviour of an indeterminant framework. Therefore, the mathematical model was specifically tailored to provide theoretical predictions for comparison with data from the experimental program. Comparison with these tests provided a meaningful evaluation because the following influencing factors were included: indeterminacy – permitting redistribution of bending moments; secondary bending moments – due to deflection and axial deformations; large variations in bending moment – thereby emphasising the non linear moment-curvature characteristic; different axial loads in each member – thereby including the effect of axial loads on member stiffness; the time dependent effects of creep, shrinkage and change in concrete strength.

The mathematical model relied on the speed and storage capacity of a digital computer to permit the frame to be divided into elements. The clear span portion of the concrete frame was divided into equal segment lengths with the cross section at the end of each length further subdivided into element strips. (Use of 25.4 segment lengths and 16 element strips per cross section yielded sufficiently accurate results). The numerical procedure employed was as follows:

1. Based on an elastic analysis an estimate of the reactions and moments at the left base was made for the specified loads.
2. From the test results the rotation and displacements of the left base was recorded.
3. For the assumed load and moment the compatible strain distribu- at the bottom of the steel base was computed. Using this curvature and the slope of the base the deformed position of the top of the H section was calculated. From equilibrium considerations the moment at the top of the H section was calculated. The strain distribution and curvature were then calculated. Using the average curvature over the length of the H section the deformed position of the top was recalculated. The iterative process was repeated until the change in curvature at the top was less than 1% or less than 1×10^{-6} radians.
4. Through an iterative process (described later) the strains in the reinforced concrete section were calculated for the load and bending moment at the top of the H section. The displaced position of the second cross section was calculated assuming the curvature to be constant over the first segment length. Using an equilibrium calculation incorporating the displacement of the second cross section the bending moment and resulting curvature were calculated for the second cross section. Then using the average curvature between section 1 and 2 the process was repeated until the change in curvature was less than 1% or 1×10^{-6} radians.
5. Starting with the moment at the upper end of the preceding element, step 4 was performed successively through all segments of the frame. At the corners and load points the appropriate changes in shears and axial forces were made.
6. Upon completion of the last segment the deformations in the H section at the right base were included in the calculation of the slope and linear displacements of the right base.

In most cases the slope and displacements of the right base were not compatible with the corresponding measured values. Therefore it was necessary to adjust the estimate of the reactions and bending moment on the left base in such a manner as to produce compatibility at the right base.

7. Based on the errors in the displacements at the right base, slope-deflection equations were used to systematically alter the estimates of the reactions and bending moment at the left base.
8. Steps 3 to 6 were repeated until the geometric errors at the right base were within acceptable limits.

The above process was repeated to obtain new values after each load increment and after selected intervals of time under sustained load. Using the method of numerical integration described above it was possible to predict the behaviour of the frame up to formation of the first plastic hinge. The time dependent effects were included in the calculation of strains at each cross section.

Distribution Of Strain On A Reinforced Concrete Section: For a specified combination of axial load and bending moment on a cross section, an iterative technique was used to vary the planar strain distribtution until the calculated resisting axial force and bending moment agreed with those specified. The following steps were used:

1. Prior to applying the initial loads the tensile stress in the concrete and the compressive stress in the reinforcement were calculated using shrinkage strains from companion reinforced prisms.
2. The cross section was divided into 16 element slices perpendicular to the plane of loading.
3. For the specified load and bending moment a trial strain distribution across the section was chosen. From this strain distribution the strains at the centroids of the reinforcement and of each element strip were calculated.
4. Using the strains at the centroids of each element the total force and bending moment were calculated by summing the contribution of each element slice and the reinforcement. The effects of the shrinkage stress were included. A fourth order polynomial equation based on a least squares fit of experimental data was used to represent the concrete compression stress-strain relationship. The concrete was assumed to crack at a tensile strain of 150×10^{-6}.
5. The calculated internal force and bending moment were compared to the corresponding external values. If either differed by more than 1% the strain distribution was altered in a manner designed to reduce the discrepancy.
Steps 3 to 5 were repeated until the internal and external forces and bending moments differed by less than 1%.
6. The calculations for creep were formulated as non linear functions of the portion of the total strain which contributed to stress. (Herein called "elastic" strain). The total strain and the "elastic" strain were stored for each element strip.
7. For sustained loading the loads and stress conditions were considered to remain constant for short increments of time. For a specified time interval the creep strain which would occur on each element was calculated and stored. (The creep calculation is described later). Additional shrinkage was computed

from equations derived from experimental shrinkage data.
8. A new total planar strain distribution was estimated in order
    to attempt to account for the effects of creep and shrinkage.
    The calculations described in steps 3 to 5 were performed with
    the only difference being that the strain contributing to stress
    on each strip element was calculated by subtracting the creep
    and shrinkage strains for that element from the total strain
    at the centroid of the element. The concrete stress-strain
    relationship was also adjusted to account for the increased age
    of the concrete.

For the next time interval the stresses on the strip elements could
be different from the previous values. These changes could result
from a change in load on the structure or from the redistribution
of load caused by the sustained load deformations. The non-linearity
of creep versus stress also could cause a redistribution of stresses
on a cross-section.

9. For the new set of "elastic" strains and taking into account the
    stress history, steps 6 to 8 were repeated for each time inter-
    val.
10. At any time the loading could be increased in increments up to
    the ultimate capacity of the cross-sections.

Creep Calculations: A modified superposition method for calculating
creep has been derived. From creep tests it was found that creep
strain for a constant stress could be accurately represented by an
equation of the form: Creep = A + B log (time) the constants A and
B were expressed as functions of "elastic" strain by third order
equations. Use of "elastic" strain as the basis for creep calcula-
tion was convenient for programming. In addition, because the con-
crete stress-strain curve was altered with increase in time, the
effect of maturing of the concrete was reflected in the creep cal-
culations. The creep calculation is described below.

If an element is loaded so that the elastic strain is EL.1 and
maintained at that load for a period of time t_o to t_1 the amount of
creep which would occur would be CR. 1. This corresponds to experi-
mental data for creep under constant stress as shown in Figure 2 (a).
If at time t_1 the load is increased so that the elastic strain is
increased to EL.2 the calculation of creep for the time interval t_1
to t_2 should take into account the previous stress history. The
uppermost curve in Figure 2 (a) represents creep under a constant
"elastic" strain EL.2. If EL.2 had existed during the time t_o to t_1
the additional creep during time t_1 to t_2 would be CR.2'. However a
lower "elastic" strain had been sustained during time t_o to t_1.
Therefore it was expected that the effect of increasing the "elastic"
strain would cause a creep strain larger than CR.2' during t_1 to t_2.
The effect of this change in elastic strain was accounted for by
adding the amount of creep which would occur for an "elastic" strain
(EL.2 - EL.1) for a period of time from t_o to (t_2 - t_1). This added
creep, CR.2' was shown in Figure 2(a). The total strain excluding
shrinkage was shown in Figure 2(b).

COMPARISON OF PREDICTED AND TEST BEHAVIOUR

Three concrete frames have been tested. The preliminary frame,
Frame Rl, was subjected to short term loading to failure. Difficul-
ties with the corners of the frames and the bases led to stiffening
of the corners and redesign of the base for subsequent tests. The
results of this test were difficult to interpret and were not
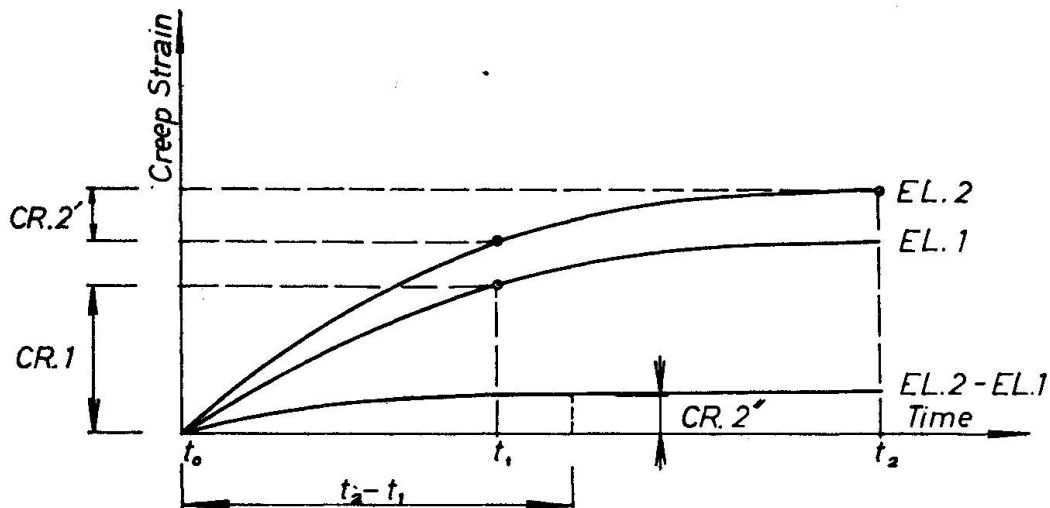included.

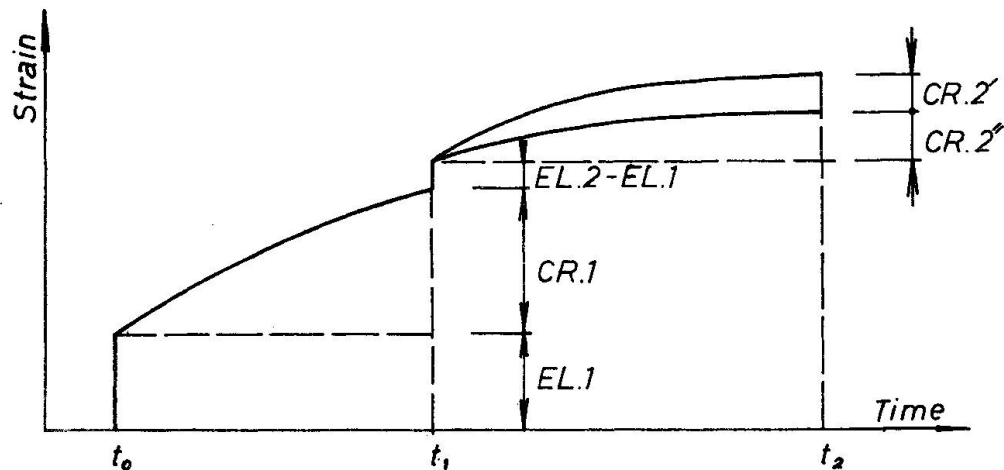FIGURE 2(a) Creep Strain for Constant Stress.

FIGURE 2(b) Total Strain Using Modified
Superposition Method.

Short Term Test: Frame R2 was loaded proportionally to failure.
Deflection and strain readings were taken at regular increments of
load.   Although it was not possible to determine accurately when the
first plastic hinge had formed, an estimate was made on the basis of
strain readings and observations of the opening up of cracks.   It
was decided that the first plastic hinge formed at the upper right
hand corner at a proportional loading slightly higher than the pre-
dicted values of H = 4082 kg. V = 8164 kg.   Formation of plastic
hinges at the centre of the beam, the right base and then the left
base resulted in an ultimate loading of H = 5220 kg. and V = 10440
kg.   The concrete strength was 316 kg/cm^2.

Figure 3 contains the predicted and test deflections for Frame
R2.   Two sets of horizontal deflections were compared on each column
and the vertical deflections were recorded at the mid span of the
beam.   The length of the lines representing the members of the frame
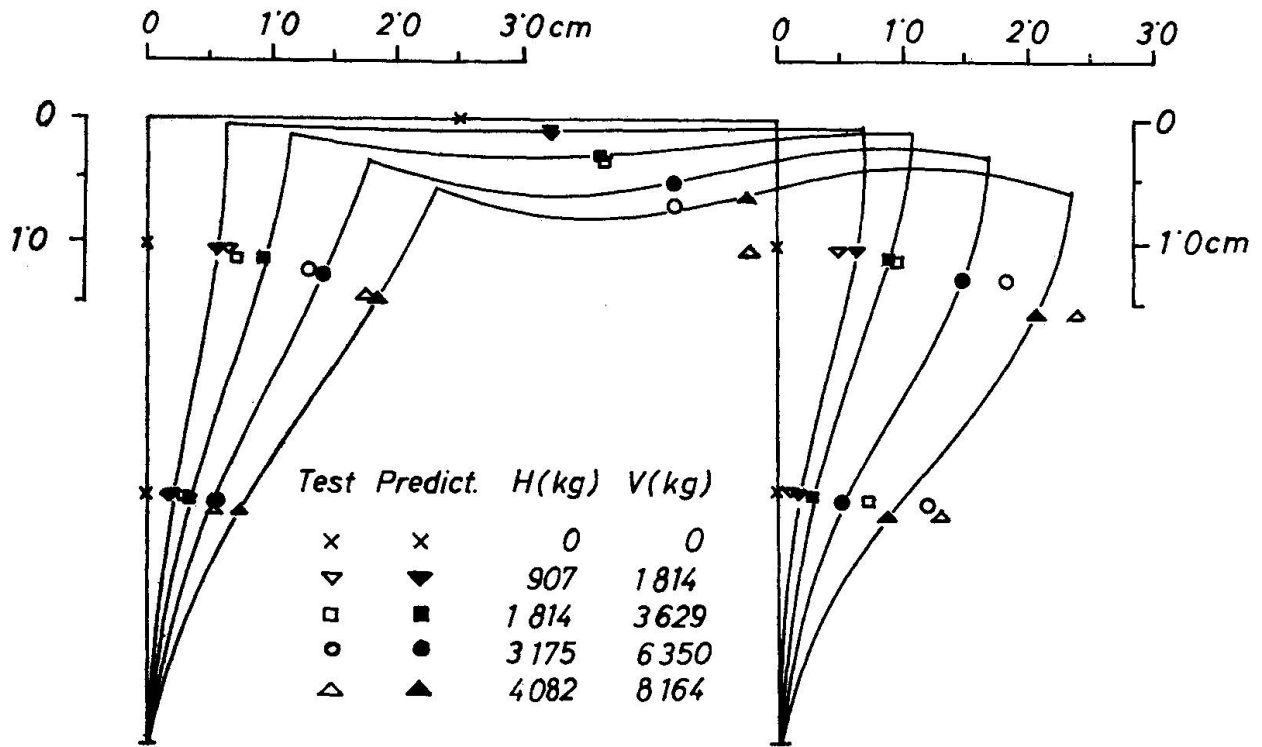were maintained to scale but the deformed shape of the frame was

Test	Predict.	H(kg)	V(kg)
✕	✕	0	0
▽	▼	907	1 814
□	■	1 814	3 629
○	●	3 175	6 350
△	▲	4 082	8 164

FIGURE 3   Predicted and Test Deflection for Frame R2.

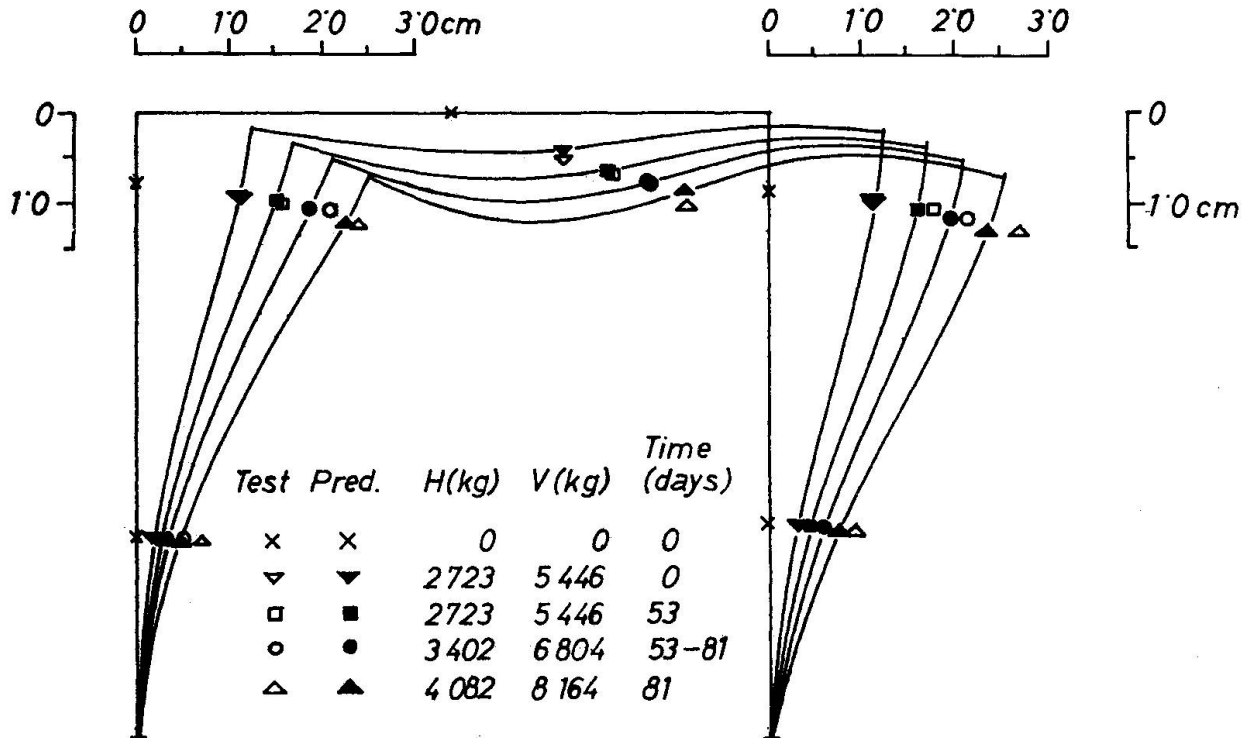| Test	Pred.	H(kg)	V(kg)	Time (days)
✕	✕	0	0	0
▽	▼	2 723	5 446	0
□	■	2 723	5 446	53
○	●	3 402	6 804	53–81
△	▲	4 082	8 164	81

FIGURE. 4 Predicted and Test Deflection for Frame L1.

distorted in order to more easily compare the results.   The predicted deflected shape of the frame is quite close to the experimental values up to formation of the first plastic hinge.

FIGURE 5 Data from Strain Measurements and Predicted Values.

(a) Frame R2

(b) Frame L1

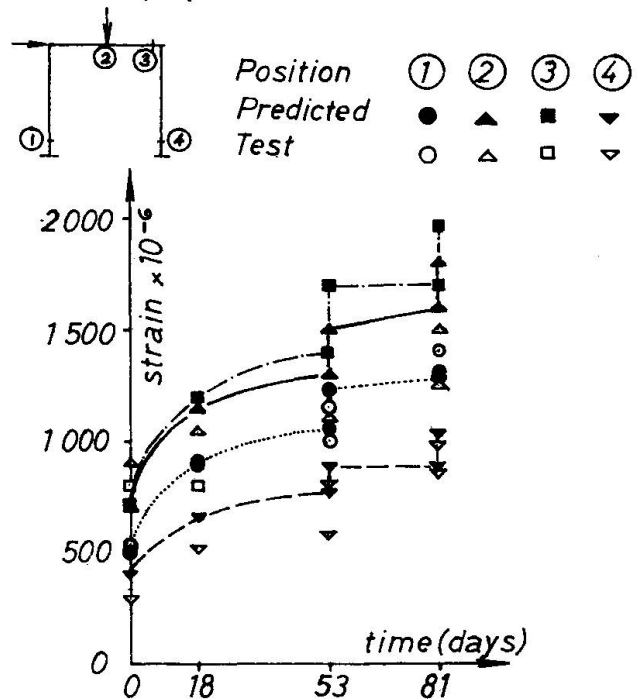| Position	①	②	③	④
Predicted	●	▲	■	▼
Test	○	△	□	▽
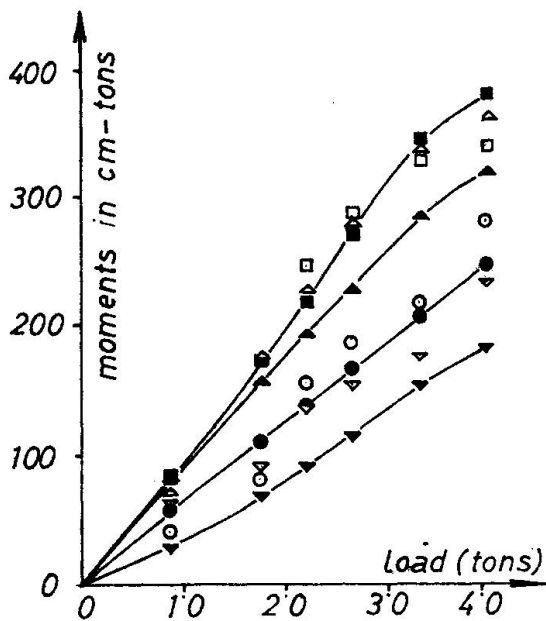
Figure 5(a) was plotted to illustrate the correlation between test and predicted bending moment calculations. The bending moments corresponding to test values were calculated from two sets of strains measured in the compression zone of the indicated regions of high bending moment. The calculated moments from experimental strains measurements were subject to fairly large errors and fluctuations because the effect of measurement errors was magnified by the fact that the compression zone was quite shallow. However the general trend of predicted and calculated bending moment were consistent and the discrepencies were acceptable when interpreted with regard to possible measurement error.

Sustained Load Test: Frame L1 was initially loaded to values of H = 2723 kg. and V = 5446 kg. At the time of loading and concrete strength was 354 kg/cm². The initial load was sustained for 53 days. In the early stages the deflections increased quite rapidly but were increasing very slowly at 53 days under load. At this stage the loads were increased 25% to H = 3402 kg and V = 6804 kg and sustained at this level for an additional 28 days at which time the frame was loaded proportionally to failure. At formation of the collapse mechanism the loads were H = 5710 kg and V = 11420 kg. Because of creep deformations it was extremely difficult to pin point the formation of the first plastic hinge. However a noticeable change in frame behaviour was evident at loads H = 4540 kg and V = 9080 kg. The first plastic hinge was predicted to occur at H = 4082 kg and V = 8164 kg. At the termination of testing the concrete strength was 386 kg/cm². During the final 28 days under load the deflections increased by only 6% to 13%.

Figure 4 contains the predicted and test deflections for Frame L1. These results were illustrated in the same way as was done in Figure 3. Very good correlations between predicted and tests results was found.

Figure 5(b) was plotted to provide a comparison of predicted and measured strains for Frame L1. Because of creep the measured strains could not be used to compute bending moment. The extreme compression fibre strains were compared for regions of high bending moment. The accuracy of the experimentally derived extreme fibre strains was influenced by the same factors as mentioned in the discussion of Figure 5(a). Although this comparison was not as reliable as the comparison of deflections the same trends were obvious.

For Frame L1 there was not any marked redistribution of bending moment due to creep. This result should be expected because the axial loads were not high enough to contribute significant secondary bending moment. Also the "elastic" strains were generally within the range of strain where creep is nearly directly proportional to "elastic" strain. The bending moment at the left base was predicted to decrease 9% during the first 53 days of sustained load and 4% during the following 28 days under the second level of sustained loading. The other high bending moments changed by only about 1%.

CONCLUSION

The method of analysis has been evaluated by comparisons of the computed predicted behaviour with the test results. This comparison confirmed the validity of the frame analysis for short term loading and variable sustained loading followed by quick loading to failure. The procedures to predit failure under sustained load have been established. However lack of information about the creep of concrete at stresses near ultimate precluded the accurate predictions of behaviour as sustained stresses approach the ultimate strength of concrete. It was reasoned that in terms of assuring safety it is most important to be able to predict the remaining capacity after a period of sustained loading.

SUMMARY

The analysis includes the effects of : the non linear time dependent stress versus strain for concrete, the degree of cracking as influenced by the level of load and bending moment, and creep, shrinkage and elastic deformations to predict the behaviour of an indeterminant frame. Comparisons with test data have verified the accuracy of the analysis.

RESUME

Pour déterminer le comportement des cadres hyperstatiques, l'analyse comprend plusieurs influences:
- la relation non-linéaire (fonction du temps) entre les contraintes et les allongements,
- le degré de fissuration, influencé par la valeur de la charge et du moment, fléchissant,
- le fluage, le retrait et les déformations élastiques.

Des comparaisons avec les résultats d'essais ont confirmé l'exactitude des calculs.

ZUSAMMENFASSUNG

Diese Untersuchung umfasst die Wirkung der nichtlinearen zeitabhängigen Spannung gegenüber der Dehnung des Betons, des Rissegrades infolge Lasthöhe und Biegemoment sowie des Kriechens, Schwindens und der elastischen Verformung, um das Verhalten eines unbestimmten Rahmens vorauszusagen. Vergleiche mit Prüfdaten haben die Genauigkeit der Berechnung bestätigt.