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Application of Optimization in the Design of Mass-Produced Steel Buildings

Application de l'optimalisation dans le calcul des bâtiments en acier fabriqués en grandes séries

Anwendung der Optimierung beim Entwurf massengefertigter Stahlbauten

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1. INTRODUCTION

Methods of mathematical programming are finding increased useage in the automated design of structures and structural elements. Due to the ease with which the objective of cost minimization can be included it is natural that they should find particularly extensive application in the design of mass-produced structures. Furthermore, since they can be used to describe the design process with great clarity they add considerable understanding to the general design problem.

For purposes of applying optimization techniques many design problems can be stated in general form [1]:

Find $(\bar{x})$
such that
$f(\bar{x}) \rightarrow$ minimum
and

$$g_j(\bar{x}) \geq 0 \qquad j = 1,\ldots,M \qquad (1)$$

$$H_k(\bar{x}) = 0 \qquad k = 1,\ldots,L \qquad (2)$$

where $\bar{x}$ is a vector representing all design variables subject to the designer's decision, $f(\bar{x})$ is the objective or cost function, $g_j(x)$ are the limits placed on the design and $H_k(\bar{x})$ are the appropriate analysis expressions. It is, of course, impossible today to completely describe a design by $(\bar{x})$. Many decisions must be made prior to entering the optimization phase. These decisions will usually include such items as material, system and configuration selection and these quantities selected will be referred to here as design parameters. For practical application in structural design today all design information except member proportions must be treated as design parameters. Those quantities included in $(\bar{x})$ will be referred to as design variables. The function $f(\bar{x})$ is quite a simple function either on a cost or weight basis. The primary limitation on the use of a cost based objective is the lack of really reliable cost knowledge, but probably it will be generated as its rational usefulness becomes apparent. The inequality limitations or constraints, $g_j(\bar{x})$, are available from design codes and engineering practice. For typical structures they include limits on stress, deflection, buckling, and a variety of arbitrary size limits which may be imposed by architectural or functional considerations. The equality limitations $H_k(\bar{x})$ are given by the appropriate structural analysis expressions. One of the primary advantages in the use of optimization techniques is the fact that since the results of the analysis are processed automatically it becomes unnecessary to evaluate large amounts of computer output.

The expressions given above state the design task in the form of a mathematical programming problem. Since either $f(\bar{x})$ or $g_j(\bar{x})$ may be nonlinear the problem generally can be attacked using methods of nonlinear programming. A variety of methods have been studied and a thorough literature review will not be attempted. However, a brief summary and evaluation of some of the methods is appropriate.

2. LINEAR PROGRAMMING

If all of the constraints and the objective are linear functions of the design variables the problem is referred to as a linear programming problem. This problem has been studied exhaustively and well-developed methods are available for solution (ref. 2,3). Applications in the structural engineering field are available both in analysis and design. The plastic analysis problem which seeks to find the collapse load for a structure is conveniently solved by this approach. The analysis of large transmission towers which were actually tested have been reported (ref. 4) using this approach. It is found to be an efficient method for use where ultimate strength is the only consideration. The plastic design problem has also been approached in this way (ref. 5). While the literature in this area is quite extensive it does not seem to have reached the stage where practical application can be attained. In an extensive study of the design of truss structures for ultimate loads the linear programming formulation was used (ref. 6). For realistic designs these problems became very large and serious numerical difficulties were encountered with standard solution algorithms. In fact, a much more efficient solution was found by restating the problem in nonlinear form.

Linear programming has also been used to solve the nonlinear problem by successive linearizations (ref. 7). The attraction of this approach lay with the fact that computer programs for the solution of linear programming problems were readily available. However, attempts to treat these programs as a "black box" were not always successful and pitfalls were discovered that appear to limit this approach to users having considerable experience in optimization.

3. NONLINEAR PROGRAMMING APPLICATIONS

Nonlinear programming techniques were used by Schmit and his coworkers in extensive studies on the design of minimum weight structures for aerospace applications. Of particular interest are studies of the design of statically indeterminate systems and structures subject to buckling limitations (ref. 8,9,10). The constraints in these studies were usually derived from a fundamental analysis of structural failure modes rather than code type limitations typically used in civil structures. However, the insight into the behavior of structural designs is very useful.

Nonlinear solution techniques can be usefully divided into two groups, feasible direction methods and penalty function techniques. The work of Schmit mentioned above used methods which could be characterized as feasible direction methods. In general, optimization proceeds as follows: A design is found in the satisfactory, or feasible region. Design changes are made in the direction of steepest descent of the objective function. For the typical structures problem the objective function is simply defined and its gradient is easily calculated. Design changes in the negative gradient direction will eventually lead to a design where one or more of the constraints are active and further attempts to improve the design in this manner will cause the design to become unsatisfactory. Thus, further design improvements must be made in a different way. At this point a feasible direction of redesign is sought. It is defined as a direction such that the constraints become less active and the value of the objective does not increase.

The method of Zoutendijk (ref. 11) is probably the most popular also the most efficient. However, it requires the calculation of the normals to the constraints and for structures problems it is frequently necessary to resort to finite difference techniques due to the effort involved in obtaining an analytic statement. In fact in many cases the constraints cannot even be stated explicity.

The design of a simply supported prestressed concrete single tee beam for minimum cost is reported in reference 12. A modification of Rosen's method (ref. 13) to give a feasible directions method was used for optimization and the constraints came from the American concrete Institute, "Building Code". The design of a steel frame was reported by Brown and Ang (ref. 14). They also used Rosen's method and presented a thorough description of the method. The design of grillages was studied by Moses and Onoda (ref. 15) using Zoutendijk's method among others. This latter paper provided a very interesting study of optimization methods and the basic design problem but it did not attempt to produce results to practical problems. The Brown and Ang study attacks a more realistic problem using practical considerations but for such solutions to be useful they would have to deal with larger structures. The prestressed beam problem did lead to results which clearly could be considered useful. Tight deflection constraints are imposed as is common in practice. The solution is useful because it can be expected that structural cost is reduced with a sbustantial reduction in design effort. Also of interest is the study of a potential design specification change to systematically evaluate the cost saving which could result from this liberalization. Such a capability could be useful to evaluate the cost effectiveness of a proposed research investment. For instance, the cost for optimum structures with and without the proposed specification liberalization could be compared to evaluate the advisability of the research investment.

In all of these examples normals to the constraints were calculated analytically. While the procedure was successful the formulating and programming effort is a distinct disadvantage of the method.

The penalty function method converts the constrained minimization problem described by equations (1) and (2) into an unconstrained problem, the minimum of which is the same as the constrained problem. The basic approach is to add some quantity to the objective function either when the constraints are approached or violated. One method, suggested by Fiacco and McCormack (ref. 16) is to form a new function

$$\phi(x,r_k) = F(\bar{x}) + r_k \sum_j \frac{1}{g_j(\bar{x})}$$

where r_k is a positive constant and the other quantities are as previously defined. For a given value of r the function $\phi(\bar{x},r_k)$ will be nearly the same as $F(\bar{x})$ if the design is far from any of the constraints limitations. As the constraints are approached the second term on the right side of the equation approaches infinity. If the value of r_k is large the effect of the constraint is felt at a considerable distance. Experience shows that optimization proceeds efficiently by beginning with a large value for r_k and finding the minimum of $\phi(\bar{x},r_k)$. For successively smaller values of r_k the unconstrained minimization problem is solved using the minimum from the previous r_k as a starting point. As r_k approaches zero the unconstrained minimum approaches the optimum of the constrained problem. If a very small value of r_k is used in the beginning the resulting function will be difficult to minimize. Therefore, the sequential approach adds substantially in efficiency.

The penalty function method, or perhaps more commonly, the Sequence of Unconstrained Minimization Technique (SUMT) is receiving considerable attention at the present time. It is attractive because it provides a possibility for treating the optimization method as a "black box" of which the engineer has (or needs) only a general knowledge. A variety of unconstrained minimization programs are becoming generally available from computing centers and they can usually be quite easily applied. It is probably advisable for the novice user to select a rather simple method such as that of Powell (ref. 17) which only requires the value of the function and avoids the calculation of gradients.

A weakness of the method given in equation (3) is the fact that the starting point must be in the satisfactory region. It is possible to formulate penalty functions where this is not the case but they approach the optimum from the unsatisfactory side and experience indicates that the interior function of equation (3) is most efficiently solved.

Extensive applications of the method have been studied by Kavlie and Moe (ref. 18). Providing that the dimensionality of the problem is not too large excellent results are obtained. The results of reference 15, which reported relative minima in the design space, were studied and it was found that with gradual changes in r_k the method converged to the global optimum if it was substantially different from other relative minima.

A practical application of particular interest in mass-produced steel structures reported in reference 19, was the selection of a minimum area cross section for a welded plate girder. The limitations of the AISC design specification (ref. 20) including lateral buckling were applied. The penalty function technique using the Powell method for searching the unconstrained function was used. The problem was programmed in about two weeks of continuous work by a rather inexperienced engineer. The result is a program of a highly modular character. If code changes occur it is only necessary that new code limitations be added.

4. HYBRID DESIGN METHODS

In some applications it is impractical or impossible to make use of the above methods due to excessive computer costs. Likewise, traditional design methods are inadequate because a unique criteria for design is lacking. In these cases it is useful to formulate a solution technique which makes use of both direct design methods and systematic search techniques. The most common direct design methods are based on the concept of full utilization and usually are referred to as fully stressed designs. In this approach the design is sought in which the forces or stresses are at their limit in some load condition for as many limitations as there are design variables. This type of design is located at a vertex point in the design space.

There are two problems associated with this approach. First, the optimum design may not be at a vertex. This proven in reference 1 but in practical cases for civil engineering structures experience shows that the difference between the optimum design and the nearest vertex is not of practical importance. The one exception to this generalization is the case of deflection limited designs. If it is expected that deflections will be critical considerable care should be utilized in formulating and solving the problem and in particular care should be exercised if other than the complete nonlinear programming solution is used.

The second problem is the difficulty in locating all of the vertex points if the problem is nonlinear. If a large number of constraints are present the number of vertices can become huge. Every combination of constraints must be investigated and compared to obtain the solution. An example of this approach is presented in reference 21.

It is however, possible to make use of the full utilization concept in a somewhat different way and this approach will be referred to as a hybrid method. Consider the design of a plate girder cross section. In this case four design variables are present for a symmetrical section, the width and thickness of the flange and web parts. The function of the cross section can be divided into two parts. The web plate must carry the shear force alone. It participates with the flange plates in resisting moment forces. However, since moment is much more efficiently carried by the flanges the moment contribution of the web is coincidental in its design. If then the flange width and web depth are known the determination of the other two variables can proceed by selecting the web thickness so that shear stress requirements and any side constraints are satisfied. With the web proportioned its moment capacity can be determined and then the other flange dimension selected so that bending stresses in addition to side constraints are satisfied. It is possible that, if a complex design specification is used, it may be difficult to obtain the variables described above. In many cases, however, it is a reasonable computational task. Now the problem is reduced from a four dimensional constrained minimization problem to a two dimensional unconstrained minimization. If the two dimensions, web depth and flange width are given, the other two can be calculated. Thus, the optimum cross section can be found using a two dimensional search. It has been implied that web depth and flange width occupy a special position as independent variables. In this case the other two variables can also be considered independent. In fact there is some evidence that in this particular problem the web thickness makes a more desirable independent variable than does web depth.

Another example of this approach was presented in reference 22. The design of a cold formed thin gage section was accomplished by making the plate thickness dependent on all the other variables. Thus, given all of the cross section dimensions a material thickness is determined which satisfies all the constraints. The dimensionality is reduced by one and the problem is converted to an unconstrained minimization problem.

5. CONCLUSIONS

The methods outlined above represent one of the first really fundamental changes in design procedures to appear for structural engineers. They provide a potential for attacking problems which could not be solved by traditional methods. Both the fully utilized concept and the plastic design method will become increasingly unsatisfactory with expected developments in mass produced structures. Wide useage of light gage material and higher strength steels can be expected. This will lead to more complex design specifications and increased concern with failure modes such as gross and local buckling and excessive deflections. Thus, plastic collapse represents only one, and perhaps a rather unlikely, failure mode. Since the critical failure mechanism cannot be stated apriori, particularly if the structure is subjected to multiple load conditions, the use of a replacement problem such as fully stressed design or plastic design will be unsatisfactory.

A decade of experience with optimization techniques has shown some of their strengths and weaknesses. This experience has included practical design applications. From the standpoint of the designer and current applications the following conclusions can be drawn:

1. Mathematical programming techniques can be fruitfully applied to small problems (under a dozen variables).

2. Of mathematical programming techniques the SUMT method provides the opportunity for application by relatively inexperienced users.

3. The use of hybrid techniques for reducing the dimensionality and converting constrained into unconstrained problems provides a currently useable tool.

4. Iterative techniques continue to be useful in design and can be mixed with optimization methods.

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SUMMARY

Methods of mathematical programming are finding increased usage in the automated design of structures and structural elements. Due to the ease with which the objective of cost minimization can be included, it is natural that they should find particularly extensive usage in the design of mass-produced structures. Furthermore, since they can be used to describe the design process with particular clarity they add considerable understanding to the design problem.

RESUME

Les méthodes de programmation mathématique trouvent des applications dans le calcul automatique d'ouvrages et d'éléments d'ouvrage. C'est grâce à la facilité avec laquelle la minimalisation du coût peut être réalisé que ce calcul s'applique tout particulièrement à l'étude de bâtiments fabriqués en série. De plus, depuis que ces méthodes sont employées à décrire le processus de calcul des projets, elles sont d'une aide considérable pour comprendre les problèmes de conception.

ZUSAMMENFASSUNG

Methoden mathematischer Programmierung finden vermehrte Anwendung im automatisierten Entwurf von Bauwerken und Bauelementen. Dank der Leichtigkeit, mit der die Kostenminimierung eingeschlossen werden kann, ist es natürlich, dass sie besondere ausgedehnte Anwendung beim Entwurf massengefertigter Bauten finden sollten. Ausserdem, und seitdem sie dazu verwendet werden können, den Entwurfsvorgang besonders klar hervorzuheben, leisten sie einen beträchtlichen Beitrag zum Verständnis des Entwurfsproblems.