Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 13 (1973)

Artikel: An overall ductility factor for coupled shear walls

Autor: Gluck, J.

DOI: https://doi.org/10.5169/seals-13750

An Overall Ductility Factor for Coupled Shear Walls

Un coefficient général de déformabilité pour les parois de cisaillement couplées

Ein globaler Duktilitätsfaktor für verbundene Schubwände

J. GLUCK
D.Sc. Senior Lecturer
Department of Structures
Faculty of Civil Engineering
Technion, Israel Institute of Technology
Haifa, Israel

Introduction

From studying the response of structures to strong earthquake motions it was concluded that the ability of the structure to dissipate energy by plastic deformations is very important, since an elastic analysis according to codes for earthquake resistant design is covering only moderate earthquake effects. To resist strong earthquakes the structure has to dissipate energy mainly by plastic deformations, since in modern structures the reserve of energy in non-structural elements is negligible. A common used characteristic to measure the ability of the structure to dissipate energy is the overall ductility factor, defined as the ratio between the maximum displacement at ultimate stage and the same displacement when at yield. In coupled shear walls the overall ductility factor is a direct function of the rotational ductility factor of the coupling beams defined as the ratio of the rotations at support section at ultimate, and yield. Current researches [1], [2] have shown that standard reinforced deep sprandels have a rotational ductility factor of 4 and with special diagonal reinforcing it may reach the value of 12.

For evaluation of the overall ductility factor the laminar method of analysis will be used. In this technique the coupling beams formed by vertically arranged uniform openings in a wall are replaced by infinitesimal elastic laminas of an equivalent stiffness. The displacement at yield will be involved with an elastic analysis of the coupled shear wall, a problem well covered in the literature [3], [4], while the displacementat ultimate stage requires an elasto-plastic analysis not yet completely solved. The first object of this paper is to present a solution for this problem. Approximative solutions for the elasto-plastic problem have been presented [5], [6] for the particular case where ultimate stage is reached when a collapse mechanism is formed by appearance of plastic hinges at both ends of all coupling beams and one plastic hinge develops at the base of each shear wall. In the most frequent cases the coupling beams may not supply the rotational ductility factor required by the above mentioned collapse mechanism. In this case ultimate stage is assumed to be reached when plastic hinges develop at ends of the coupling beams only over part of the height while in the remaining part they behave elastically. In the present paper this general case will be considered. Charts for evaluation of the overall ductility factor are presented for an upper triangle loading which is very often used to simulate the dynamic effect of earthquake motion.

Elastic Displacement

A prototype of a coupled shear wall is presented in Fig. 1 and its equivalent laminar model in Fig. 2. Consideration of equilibrium and compatibility condition yields the well-known differential equation of the elastic problem:

$$d^2Q/d\xi^2 - \beta^2 Q = \gamma H^2 M_o \tag{1}$$

where Q = unknown axial force function acting in the shear wall, M_o = cantilever moment produced by external load, and

$$\xi = x/H \tag{2}$$

$$\beta^2 = H^2 \, (\ell^2/I_o + 1/A_1 + 1/A_2)\, 12I^*/(hc^3) \tag{3}$$

$$\gamma = 12I^*/(hc^3 I_o) \tag{4}$$

in which H = height of the structure, ℓ = span between shear wall center lines, c = clear span of coupling beam, I^* = reduced moment of inertia of coupling beam allowing for shear distortion, I_o = sum of moments of inertia of shear wall 1 and 2, A_1, A_2 = cross setion areas of respectively shear wall 1 and 2, and h = height of story.

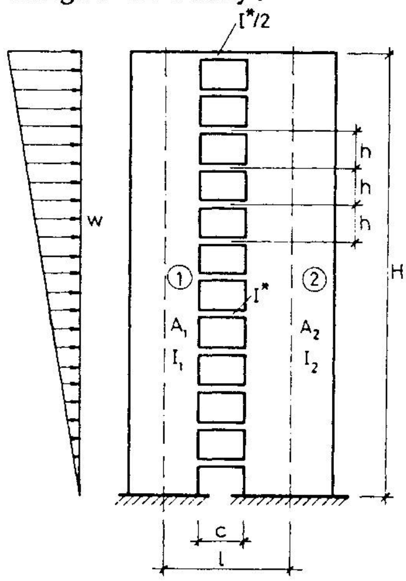

Fig. 1 Prototype of coupled shear wall.

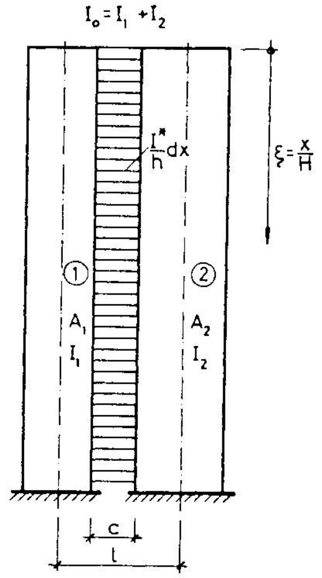

Fig. 2 Laminar model of coupled shear wall.

The solution of Eq. 1 for the normal force function Q for an upper triangle load pattern having the following moment variation

$$M_o = W\,H\,(\xi^2 - \xi^3/3) \tag{5}$$

where W = sum of lateral load, and satisfying the boundary conditions of the structure may be written in the form

$$Q = \gamma\, W\, H^3 \bar{Q}$$

where

$$\bar{Q} = C\,\mathrm{sh}\beta\xi + D\,\mathrm{ch}\beta\xi - [\xi^3/3 - \xi^2 + 2(\xi-1)/\beta^2]/\beta^2 \tag{6}$$

in which

$$C = [(2/\beta^2 - 1)/\mathrm{ch}\beta + 2(\mathrm{th}\beta)/\beta]/\beta^3 \tag{7}$$

$$D = -\,2/\beta^4 \tag{8}$$

The elastic displacement at the top of the wall is given by

$$EI_o y_{e\,max} = WH^3\left\{-1/15+\eta\{[C\ sh\beta+D(ch\beta-1)]+(1/5+2/\beta^2)/3\}/\beta^2-F\right\} \quad (9)$$

where

$$\eta = \gamma H^2 \ell \quad (10)$$

$$F = \eta[C\ ch\beta+D\ sh\beta+(1/\beta+1/4)/\beta]/\beta-1/4 \quad (11)$$

Elasto-Plastic Displacement

The general case where the rotational ductility factor of the laminas does not enable an ultimate stage with full laminar plastification is considered. In this case the ultimate stage will be reached when at the upper and lower zones or the lower zone only the laminas will behave elastically, while in the middle zone the laminas will have formed plastic hinges at their supports (see Fig. 3). To express equilibrium and continuity it is convenient to replace in the middle zone the fixed ends of the lamina supports by hinges acted by known external moments and shear forces representing the action of the ultimate laminar shear. The analytical scheme thus obtained will be treated as an elastic system acted by lateral load and distributed shear and moments at the hinged ends of the laminas of the middle zone, as shown in Fig. 4.

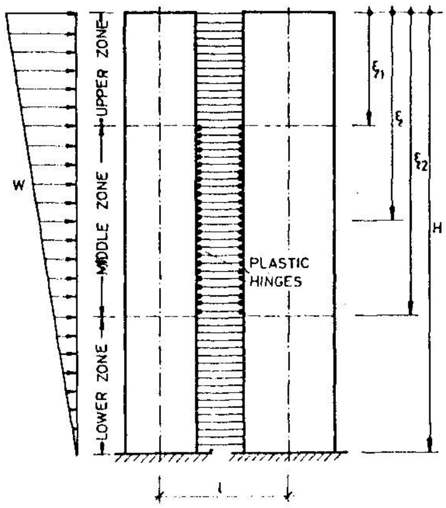

Fig. 3 Laminar model with plastified laminas in the middle zone.

The govering differential equation of the problem and boundary conditions will be obtained by applying the Principle of Least Work. The complementary energy will be expressed as function of the three unknown normal force functions acting along the center lines of the shear walls in the upper, middle and lower zones, denoted respectively by Q_s, Q_m and Q_i.

The complementary strain energy may be expressed in the form

$$2EU = H\,\{\int_0^{\xi_1}[hc^3(Q_s')^2/(12I^*)+(M_o-\ell Q_s)^2/I_o+(1/A_1+1/A_2)Q_s^2]d\xi\ +$$
$$\int_{\xi_1}^{\xi_2}[hc^3(Q_m')^2/(12I^*)+(M_o-\ell Q_m)^2/I_o+(1/A_1+1/A_2)Q_m^2]d\xi\ +$$
$$\int_{\xi_2}^{\xi_1}[hc^3(Q_i')^2/(12I^*)+(M_o-\ell Q_i)^2/I_o+(1/A_1+1/A_2)Q_i^2]d\xi\} \quad (12)$$

The complementary energy as given in Eq. 12 is a function of , Q_s, Q_s', Q_m, Q_m', Q_i and Q_i'. According to the principle of Least Work the first variation of the complementary energy with respect to the functions Q_s, Q_s', Q_m, Q_m', Q_i and Q_i' must vanish. The ultimate laminar shear q_u being known, the axial force function in the middle zone may be expressed in the form

$$Q_m = Q_1+q_u H(\xi-\xi_1) \quad (13)$$

After variation of Eq. 12 and substituting in it Eq. 13 and effectuating the integration by part, results

$$-\int_0^{\xi_1}(Q_s''-\beta^2 Q_s+\gamma H^2 M_o)\delta Q_s d\xi - H^2(Q_o'\delta Q_o - Q_1'\delta Q_1)$$
$$+[\beta^2(\xi_2-\xi_1)(Q_1+q_u(\xi_2-\xi_1)/2)-\gamma WH^3((\xi_2^3-\xi_1^3)/3-(\xi_2^4-\xi_1^4)/12)]\delta Q_m - H^2 q_u(\delta Q_1-\delta Q_2)-$$
$$-\int_{\xi_1}^{1}(Q_i''-\beta^2 Q_i+\gamma H^2 M_o)\delta Q_i d\xi - H^2 Q_H'\delta Q_H - H^2(Q_2'\delta Q_2 - Q_H'\delta Q_H) = 0 \quad (14)$$

where Q_o, Q_1 and Q_2, Q_H = the values of respectively Q_s and Q_i at ordinates $(0, \xi_1)$ and $(\xi_2, 1)$.

As the variations δQ_s, δQ_i and δQ_1 are arbitrary, Eq. 14 leads to the following Euler Differential equations expressing compatibility at the upper and lower zones.

$$Q_s'' - \beta^2 Q_s + \gamma H^2 M_o = 0 \qquad (0 < \xi < \xi_1) \qquad (15)$$

$$Q_i'' - \beta^2 Q_i + \gamma H^2 M_o = 0 \qquad (\xi_2 < \xi < 1) \qquad (16)$$

and the following algebraic equation expressing the compatibility condition for the middle zone:

$$\beta^2(\xi_2-\xi_1)(Q_1+q_uH(\xi_2-\xi_1)/2)-\gamma WH^3((\xi_2^3-\xi_1^3)/3 - (\xi_2^4-\xi_1^4)/12) = 0 \qquad (17)$$

The physical meaning of this condition is zero relative vertical gap between upper and lower limiting sections of the wall in the middle zone.

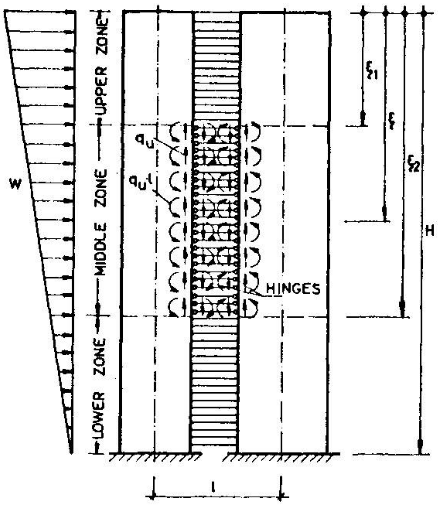

Fig. 4 Analytical model with applied loads.

Assuming a variation of the Q_s function so that $\delta Q_o=0$, and $\delta Q_1 \neq 0$ and the variation of the Q_i function so that $\delta Q_2 \neq 0$, and $\delta Q_H \neq 0$, the following boundary conditions result

$$Q_o = 0 \quad ; \quad Q_1' = 0 \qquad (18),(19)$$

$$Q_2' = 0 \quad ; \quad Q_H' = 0 \qquad (20),(21)$$

Eq. 17 together with the following equilibrium equation at limiting section between middle and lower zone

$$Q_2 = Q_1 + q_uH(\xi_2 - \xi_1) \qquad (22)$$

will serve to determine the limiting ordinates ξ_1 and ξ_2.

The solution for the upper zone represented by Eq. 15 satisfying the boundary conditions given by Eqs. 18 and 19, has the same form as that given by Eq. 6, where the coefficients C and D are replaced by C_s and D_s having the values:

$$C_s = [2(sh\beta\xi_1)/\beta^3+(\xi_1^2-2\xi_1+2/\beta^2)/\beta^2+\bar{q}_u]/(\beta ch\beta\xi_1) \qquad (23)$$

$$D_s = -2/\beta^4 \qquad (24)$$

where

$$\bar{q}_u = q_u/(\gamma WH^2) \qquad (25)$$

The normal forces in the middle zone are given by Eq. 13.

The solution for the lower zone represented by Eq. 16 satisfying the boundary conditions given by Eqs. 20 and 21 has as well the same form as that given by Eq.6, where the coefficients C and D are replaced by C_i and D_i having the values:

$$C_i=\{(2/\beta^2-1)/(\beta^2ch\beta)-[(2/\beta^2-1)ch\beta\xi_2/(\beta^2ch\beta)-(\xi_2^2-2\xi_2+2/\beta^2)/\beta^2 - \bar{q}_u]th\beta/(th\beta ch\beta\xi_2-sh\beta\xi_2)\}/\beta \qquad (26)$$

$$D_i=[(2/\beta^2-1)ch\beta\xi_2/(\beta^2ch\beta)-(\xi_2^2-2\xi_2+2/\beta^2)-\bar{q}_u]/[\beta(th\beta ch\beta\xi_2-sh\beta\xi_2)] \qquad (27)$$

In the particular case when the plastification of the coupling beam ends extend until the top of the wall, there will be only two zones; a plastic one in the upper part and an elastic one in the lower part, with the limiting section having ordinate ξ_2. The axial force in the plastified zone will be given by

$$Q_m = q_uH\xi \qquad (28)$$

and that in the elastic zone will be the same as that of the lower zone, mentioned above.

The elasto-plastic displacement at the top of the wall is given by

$$y_{p\,max} = H\int_1^{\xi_2}\phi_s d\xi + H\int_{\xi_2}^{\xi_1}\phi_m d\xi + H\int_{\xi_1}^{1}\phi_i\, d\xi \qquad (29)$$

where

$$EI_o\phi_s = H\int(M_o - Q_s\ell)d\xi + F_s \qquad (30)$$

$$EI_o\phi_m = H\int(M_o - Q_m\ell)d\xi + F_m \qquad (31)$$

$$EI_o\phi_i = H\int(M_o - Q_i\ell)d\xi + F_i \qquad (32)$$

in which F_s, F_m and F_i = constants of integration to be determined from the following boundary conditions:

1. Full restrain at support: $(\phi_i)_{\xi=1} = 0$ (33)

which leads to $F_i=\eta[(C_i ch\beta+D_i sh\beta)+(1/\beta^2+1/4)/\beta]/\beta-1/4$ (34)

2. Continuity of deflection line at $\xi=\xi_2$: $(\phi_i)_{\xi=\xi_2}=(\phi_m)_{\xi=\xi_2}$ (35)

which leads to

$$F_m=\eta\{\bar{Q}_1\xi_2+\bar{q}_u(\xi_2^2/2-\xi_1\xi_2) -(C_i ch\beta\xi_2+D_i sh\beta\xi_2)/\beta-[\xi_2^3/3-\xi_2^4/12-2(\xi_2^2-\xi_2)/\beta^2]/\beta^2\}+F_i \qquad (36)$$

3. Continuity of deflection line at $\xi=\xi_1$: $(\phi_m)_{\xi=\xi_1}=(\phi_s)_{\xi=\xi_1}$ (37)

$$F_s=\eta\{(C_s ch\beta\xi_1+D_s sh\beta\xi_1)/\beta +[\xi_1^3/3-\xi_1^4/12-2(\xi_1^2/2-\xi_1)/\beta^2]/\beta^2-\bar{Q}_1\xi_1+\bar{q}_u\xi_1^2/2\} +F_m \qquad (38)$$

where

$$\bar{Q}_1 = Q_1/(\gamma WH^3) \qquad (39)$$

Substituting the corresponding relations in Eq. 29 and integrating yields

$$EI_o y_{p\,max}=WH^3\Big\{\eta\{[C_s sh\beta\xi_1+D_s(ch\beta\xi_1-1)+C_i(sh\beta-sh\beta\xi_2)+D_i(ch\beta-ch\beta\xi_2) +(\xi_1^4-\xi_2^4)/12-(\xi_1^5-\xi_2^5)/60+1/15+(\xi_1^2-\xi_2^2-\xi_1^3/3+\xi_2^3/3+2/3)/\beta^2]/\beta^2-\bar{Q}_1(\xi_1^2-\xi_2^2)/2 -\bar{q}_u[(\xi_1^3-\xi_2^3)/6-{}_1(\xi_1^2-\xi_2^2)/2]\}-1/15-F_s\xi_1+F_m(\xi_1-\xi_2)+F_i(\xi_2-1)\Big\} \qquad (40)$$

The overall ductility factor of the coupled shear wall will be now

$$\mu_o = y_{p\,max}/y_{e\,max} \qquad (41)$$

The graphs shown in Fig. 5 a,b,c,d may serve to establish the value of μ_o as

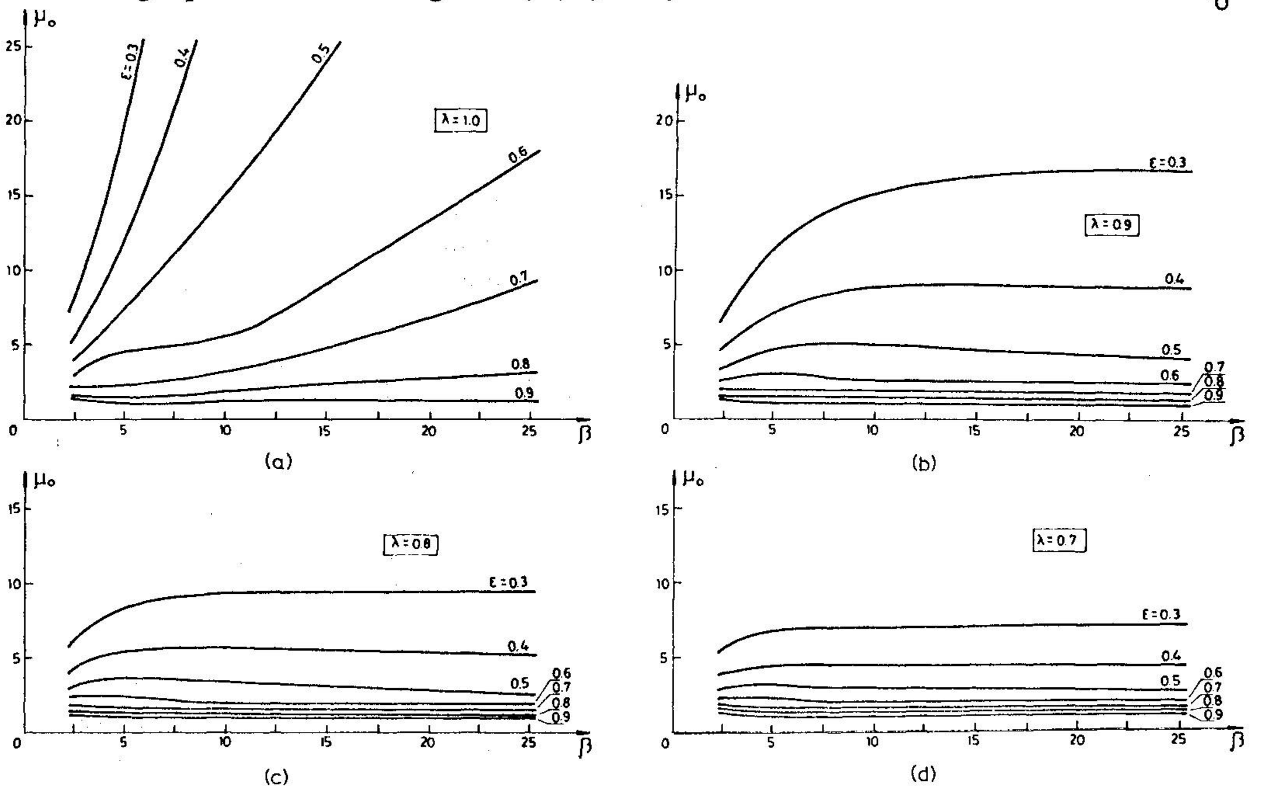

Fig. 5 Overall ductility factor.

function of β for various values of $\lambda=\eta/\beta^2$ and $\varepsilon=q_u/q_{max}$, where q_{max}= the maximum elastic laminar shear.

An important factor to be considered is the rotational ductility factor of the coupling beam end, which in fact limits the value of the ultimate laminar shear q_u to be taken in Eq. 40.

Considering the elastic wall rotations and extensional deformations it may be shown easily that the plastic laminar rotation at any height ξ as shown in Fig. 6 is given by:

$$\phi_p = \phi\ell/c - (d_1+d_2)/c - \phi_y \qquad (42)$$

where = the elastic rotation of the wall and ϕ_y = yield rotation of the coupling beam, which is related to the ultimate laminar shear as follows

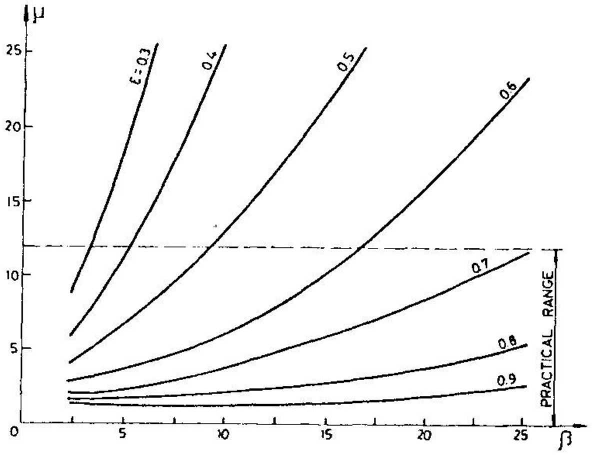

Fig. 7 Rotational ductility factor of coupling beam end.

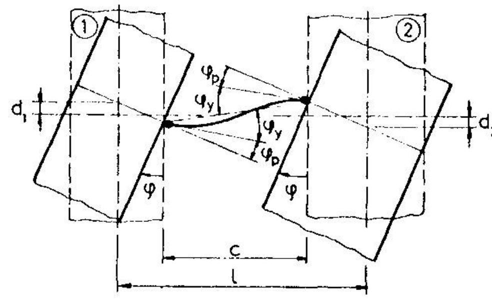

Fig. 6 Deformed position of shear wall and plastified lamina.

$$\phi_y = hc^2 q_u/(12(EI^*)) \qquad (43)$$

The rotational ductility factor at the support of the lamina will be

$$\mu = (\phi_{p\,max} + \phi_y)/\phi_y \qquad (44)$$

where $\phi_{p\,max}$= the maximum value of the plastic rotation.

The graphs given in Fig. 7 may serve to determine the rotational ductility factor as function of β for various values of q_u/q_{max}

References

1. Pauly, T., "Coupling Beams of Reinforced Concrete Shear Walls", J. of the Struc. Div., ASCE, Vol. 97, No.ST3, Proc. Paper 7984, March 1971, pp. 843-862.
2. Pauly, I., "Simulated Seismic Loading of Sprandel Beams", J. of the Struc. Div.,ASCE, Vol.97, No.ST9, Proc.Paper 8365, Sept.1971, pp. 2404-2419.
3. Beck, H., "Contribution to the Analysis of Coupled Shear Walls", ACI J., Proc. V. 59, August 1962, pp. 1055-1070.
4. Rosman, R., "Beitrag zur Statischen Berechnung Waagrecht Belasteter Querwaende bei Hochbauten", Der Bauingenieur, V. 35, No. 4, April 1960, S. 133-136.
5. Pauly, T., "Elasto-Plastic Analysis of Coupled Shear Walls", ACI J., Proc. V. 67, Nov. 1970, pp. 915-922.
6. Winokur, A. and Gluck, J., "Ultimate Strength Analysis of Coupled Shear Walls", ACI J., Proc. V. 65, Dec. 1968, pp. 1029-1036.

SUMMARY

An analytical procedure for establishing the overall ductility factor for coupled shear walls is presented. The continuum approach was applied by assuming an upper triangle lateral load pattern, very often used to simulate earthquake motion effect. Plastification of the coupling beam ends may be on part or over the whole height. The graphs presented may be used for direct evaluation of the overall ductility factor and associated with it the rotational ductility of the coupling beam end.

RESUME

Ce travail présente une méthode analytique pour déterminer le coefficient général de déformabilité des parois de cisaillement couplées. On utilise un processus d'approximation en admettant un modèle de charge latérale en triangle au sommet de la paroi, modèle qui est souvent utilisé pour simuler les effets des mouvements sismiques. La plastification des extrémités des barres de liaison peut être admise sur une partie ou sur toute la hauteur. Les diagrammes présentés peuvent être utilisés pour l'évaluation directe du coefficient général de déformabilité et, associée à ce dernier, pour la détermination de la déformabilité rotationnelle de l'extrémité de la barre de liaison.

ZUSAMMENFASSUNG

Es wird eine analytische Methode zur Bestimmung eines globalen Duktilitäts-faktors für verbundene Schubwände vorgelegt. Die Kontinuums-Näherung wurde unter Annahme einer nach oben zunehmenden dreieckigen horizontalen Last-Verteilung angewendet, die sehr oft zum Simulieren von Erdbebeneffekten verwendet wird. Plastifizierung der verbindenden Stabenden kann über einen Teil oder über die ganze Höhe auftreten. Die angegebenen graphischen Darstellungen können zur direkten Ermittlung des globalen Duktilitätsfaktors und mit ihm die Rotationsduktilität der verbindenden Stabenden verwendet werden.

Leere Seite
Blank page
Page vide