II

Simple Design Procedures for Concrete Columns

Méthodes simples de calcul pour colonnes en béton

Einfache Berechnungsverfahren für Betonstützen

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1. INTRODUCTION

As load factors are reduced and higher strength concretes and steels are introduced, column design becomes more critical especially with the advent of very high buildings or in very high bridge piers. This Theme Report reviews the practical methods available for the design of short and long columns. Frame stability is discussed briefly and column design recommendations are given.

2. DESIGN METHODS AND LOADINGS

The proportioning of reinforced concrete members has been based either on design for allowable stresses at working loads or on the satisfaction of several limit states, particularly the limit state of collapse or ultimate strength. For reinforced concrete columns the allowable stress procedure has been shown to be inadequate, and for almost 40 years the column design rules in the ACI Building Code[1] have been based in part on ultimate strength principles. In this report, only ultimate strength design procedures will be discussed.

The limit states affecting the design of concrete columns and frames are:

1. Limit state of serviceability. This is generally not a serious limitation. The lateral deflections of frames should be small to prevent non-structural damage or discomfort to occupants. Differential vertical deflections due to thermal effects or elastic and creep shortening of the shear-core and the columns may cause problems in tall buildings[2].

2. Limit state of collapse.

3. Limit state of instability.

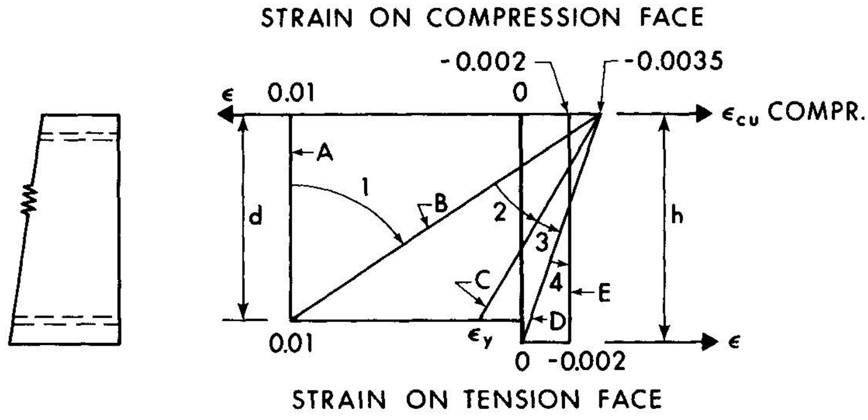
The last two of these limit states are the normal design cases and will be discussed in the balance of this paper.

3. SHORT COLUMNS

3.1 Design of Cross-Sections

3.1.1 Cross-Sectional Strength for Uniaxial Bending

In the major national building codes for reinforced concrete, the calculation of the strength of a column cross-section is based directly on equilibrium and compatibility of stresses and strains. Thus, for example, the CEB Recommendations[3] consider the variation of strain distributions shown in Figure 1 when computing the strength of a cross-section. Tensile steel strains may have any value up to 0.01 and concrete crushing strains range from 0.0035 with significant bending to 0.002 for pure axial loads.



(a) Section (b) Strain Distribution

A - Pure Tension Failure
C - Balanced Failure
E - Pure Compression Failure

Range 1. Tension strain = +0.010, Compression strain varies from +0.010 to -0.0035.
Ranges 2 and 3. Compression strain = -0.0035, Tension strain varies from +0.010 to 0.
Range 4. Strains on both faces approach -0.002.

Fig. 1 Variation in Strains in Cross-Section

By sequentially considering various strain distributions in Figure 1 and relating stress to strain, the load-moment interaction diagram in Figure 2 is obtained. The letters in Figure 2 refer to the particular strain distributions labelled in Figure 1. Various relationships between concrete stress and strain can be used in this calculation.

The most common assumption is the rectangular stress block combined with a limiting compression strain of 0.003 to 0.0035 used by the ACI[1], CEB[3] and other codes. Strengths based on this stress distribution have been compared to tests by a number of authors[4,5]. The limiting strain concept is discussed in Theme Paper I[6].

Although calculations based on the strain compatibility solution are tedious, extensive handbooks of interaction diagrams or tables are widely available for use in design. Alternately a number of computer programs are available to solve for interaction diagrams or to directly solve for column sizes.

Further study of the usable value of the limiting compression strain in columns is required to properly make use of high strength steels with yield strains in excess of 0.003. Tests under short time loads have shown that this limit is reasonable, particularly in view of the possible loss of the concrete cover at strains of 0.002 to 0.003[7,8]. With sustained loads or loads applied incrementally during construction, however, larger strains may be utilizable.

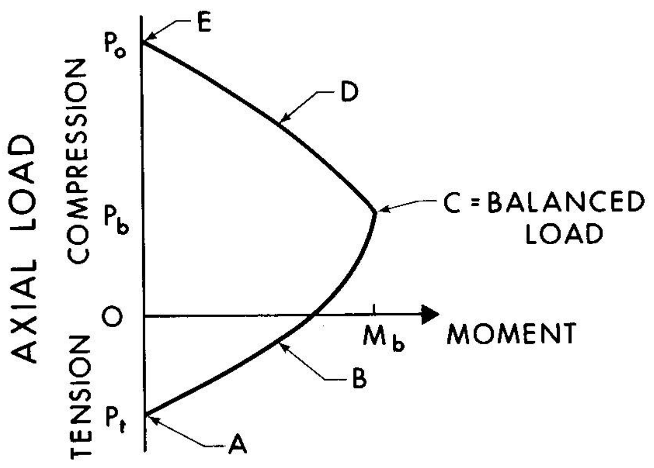


Fig. 2 Interaction Diagram

The stress-block and limiting compression strain for structural lightweight concrete needs more study. Sometimes high strength lightweight concrete made from expanded clay or shale aggregate may fail suddenly at strains less than 0.003. The effects of this on column design are discussed in Reference 9.

3.1.2 Cross-Sectional Strength for Biaxial Bending

The design of columns subjected to axial load and moments about two axes is much more complex than for axial load and uniaxial bending. The interaction curve in Figure 2 can be expanded into a three-dimensional interaction surface as shown in Figure 3. Horizontal sections (planes corresponding to constant values of P/P_o) through such a surface have the shapes shown in Figure 4. The outer limits of these curves can be approximated by the equations[10]:

$$\left(\frac{Mx}{Mox}\right)^n + \left(\frac{My}{Moy}\right)^n = 1 \tag{1}$$

or

$$e_{ox} = e_x \left[1 + (e_y/e_x)^n \left(\frac{e_{ox}}{e_{oy}}\right)^n\right]^{1/n} \tag{2}$$

where $e_{ox} = M_{ox}/P$. Equation 2 can be used to convert a given combination of e_x and e_y to the equivalent uniaxial eccentricity for the given load level P/P_o.

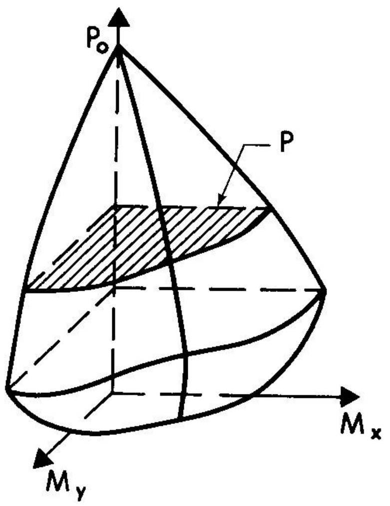


Fig. 3 Interaction Surface

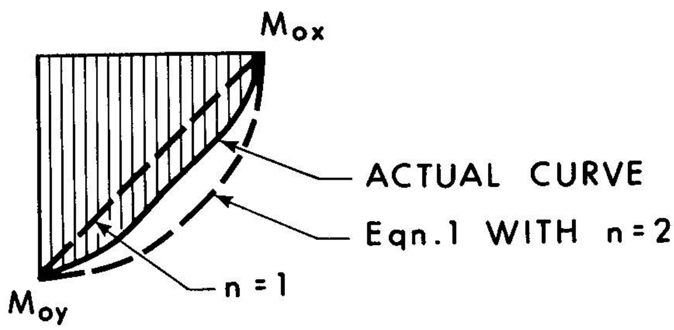


Fig. 4 Horizontal Section Through Interaction Surface

For high ratios of P/P_o these curves are essentially circular or elliptical corresponding to n=2. As the axial load ratio decreases, n decreases sometimes dropping as low as 1.1 near the balanced load. As a result, the use of n=2 will lead to unsafe results for many cases.

Although a number of authors[10-14] have proposed methods of designing biaxially loaded columns, none of the methods is widely used at present. The CEB[14] has suggested two alternate design approaches:

1. Charts giving horizontal sections through interaction diagrams similar to Figure 4 have been published.

2. Equation (3) is given to convert the actual eccentricities e_x and e_y into an equivalent uniaxial eccentricity e_{ox}:

$$e_{ox} = e_x + \frac{\alpha\, e_y\, x}{y} \tag{3}$$

provided that: $\frac{e_x}{x} \geq \frac{e_y}{y}$

where x and y are the dimensions of the column cross-section, α is a factor which is a function of the arrangement of reinforcement, cover, yield strength, the mechanical steel ratio, $\omega = p_t\sigma_y/f'_c$, and the axial load ratio P/P_o.

Based on work done by Montoya[12,14] and Parme[11], conservative values of α can be given as follows:

$$\text{for } \frac{P}{A_c f'_c} \leq 0.4: \quad \alpha = \left(0.5 + \frac{P}{A_c f'_c}\right)\left(\frac{\sigma_y + 40000}{100{,}000}\right) \geq 0.6 \tag{4a}$$

$$\text{for } \frac{P}{A_c f'_c} \geq 0.4: \quad \alpha = 1.3 - \frac{P}{A_c f'_c}\left(\frac{\sigma_y + 40000}{100{,}000}\right) \geq 0.5 \tag{4b}$$

Although the phenomenon of how a short column fails under biaxial load is adequately understood, simplified design techniques need much more study.

3.2 Minimum Eccentricity

Due to misalignment, unforeseen loading cases, or variations in the concrete quality within columns, actual columns are seldom axially loaded and seldom have exactly the eccentricity computed in a structural analysis. As shown in Figure 5 a small unintentional eccentricity causes a significant reduction in the strength of an axially loaded column regardless of the sign of the eccentricity. The same eccentricity has much less effect on the strength of a column with a large applied eccentricity. For this reason the ACI code requires that columns be designed for the actual eccentricity including slenderness effects but not less than a specified minimum eccentricity given as 0.1h for tied columns and 0.05h for spiral columns. Ellingwood and Ang[15] have shown that this leads to wide variations in safety for columns having actual eccentricities less than, equal to and greater than the specified minimum eccentricity. The CEB[3], on the other hand, requires designers to consider an additional unintentional eccentricity of h/30 but not less than 2 cm. for slender columns.

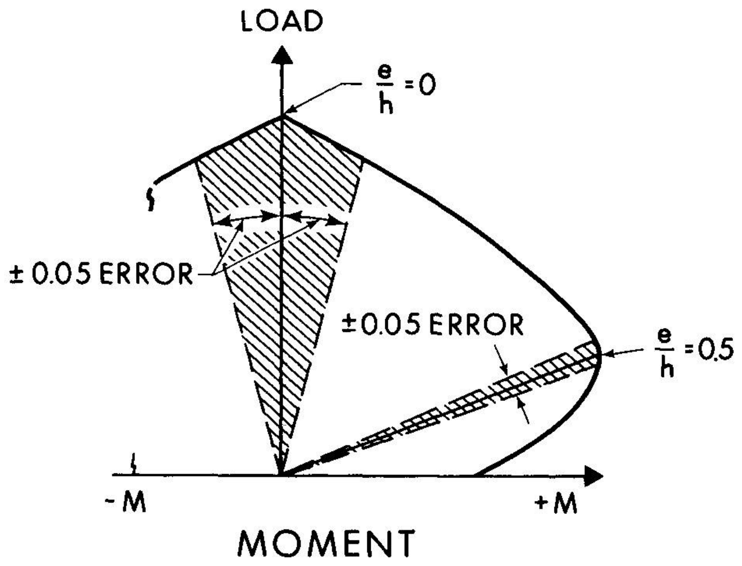


Fig. 5 Effect of Error in Eccentricity

The effect of random variations in material strengths and dimensions on the strength of a cross-section decreases as the eccentricity increases[16]. This effect could be approximated using an additional rather than a minimum eccentricity. The application of unintentional eccentricities about both axes of a column needs more study. An equivalent uniaxial eccentricity could be derived using Eqn. 3 to avoid designing all columns as biaxially loaded columns, however.

Tests of short columns with sinusuidally displaced reinforcement have shown that the reduction in strength can be accounted for by a change in the interaction diagram and the effective eccentricity at the critical section[17]. The effects of such a shift will be small except for very small or possibly very slender columns. Baker[18] has shown analytically that the effect of continuity in building frames will reduce the moments due to lack of straightness to about 0.67 Pe_u where e_u is the maximum eccentricity due to crookedness.

Recent studies of construction and analysis errors affecting columns in buildings suggest that the major causes of unintentional eccentricities were horizontal misalignment or lack of plumbness of columns[19]. Crookedness, misplacement of reinforcement and inhomogeneity were found to be less important. The data suggested that both absolute minimum and an absolute maximum unintentional eccentricities existed. Based on these studies Eqn. 5 was proposed for an additional unintentional eccentricity:

For braced columns:

$$e_u = 0.40 \text{ in.} + 0.03h \leq 2 \text{ in.} \quad (5a)$$

$$e_u = 1 \text{ cm.} + 0.03h \leq 5 \text{ cm.} \quad (5b)$$

For unbraced columns:

$$e_u = 0.80 \text{ in.} + 0.03h \leq 2 \text{ in.} \quad (5c)$$

$$e_u = 2 \text{ cm.} + 0.03h \leq 5 \text{ cm.} \quad (5d)$$

In summary it is recommended that all columns be designed for their actual eccentricity plus the additional eccentricity given by Eqn. (5) applied about one axis only.

3.3 Lateral Reinforcement

Lateral reinforcement provided by ties, spirals or hoops has a number of functions:

1. It provides lateral restraint for the concrete and hence tends to strengthen it.

2. It prevents or delays the buckling of the longitudinal reinforcement.

3. Closely spaced lateral reinforcement confines the core of the column increasing its ductility.

4. Lateral reinforcement will act as shear reinforcement.

Bresler and Gilbert[20] used a series of simple analyses to study the effects of ties on column strength. They concluded that when the tie spacing was equal to or less than about 1.5 times the depth of the column, the lateral restraint due to the ties changed the initial mode of failure of the column from longitudinal splitting to a conical type of failure between ties and in doing so increased the effective concrete strength. Based on this, they endorsed the existing practice that the maximum tie spacing should not exceed the least lateral dimension of the column.

To prevent buckling of the longitudinal bars after yielding Bresler and Gilbert recommended tie spacings ranging from 19.5 longitudinal bar diameters for a yield strength of 33 ksi (2300 kg/cm^2) to 13 diameters for 75 ksi (5300 kg/cm^2) steel. For columns subjected to cyclic loads causing yielding Bresler[21] recommends these values be reduced to 7 and 5 diameters, respectively.

It has long been known that lateral restraint increases the ductility of concrete. Although sufficient lateral restraint can increase the strength of concrete, this only occurs with very large strains and such a column would tend to be highly unstable. Thus lateral restraint should not be counted on to increase the strength of columns. The increase in ductility is highly desirable in seismic regions, however, where spirals or closely spaced ties are used to bind the concrete. Due to the outward deflections of the sides of the ties, the pressure on the concrete inside a tie is uneven. For this reason the ACI Building Code[1] assumes ties to be half as effective as spirals. If supplementary cross-ties are provided, hooked around the main ties to reduce these deflections, the tie effectiveness increases.

Finally, ties act as shear reinforcement. This topic is discussed more fully in Theme Paper III for this Symposium[22].

3.4 Limitations on Longitudinal Reinforcement and Column Size

Traditionally, design regulations have limited the minimum size of columns and the maximum reinforcement ratio to avoid construction problems. Fire considerations may also affect the minimum size chosen.

The minimum reinforcement ratio has generally been set so that the reinforcement will not yield under sustained loads. Such yielding has little effect on the strength of a short column but may reduce the stiffness and buckling load of a slender column. Recent studies of the stress increase with time in column reinforcement are presented in References 23 and 24. These suggest that the presence of even a little reinforcement in a column will considerably reduce the creep strains below those in an unreinforced member. The length of the construction period during which the column receives its sustained load has a significant effect also.

The ACI Code limits the total reinforcement between 1 and 8 percent of the cross-sectional area. The CEB expresses the minimum total reinforcement as:

$$p_t \geq c\,(1 + \frac{30000}{\sigma_y})\,(\frac{P}{0.85\,f'_c\,A_c})\ (N/cm^2) \qquad (6a)$$

$$p_t \geq c\,(1 + \frac{43000}{\sigma_y})\,(\frac{P}{0.85\,f'_c\,A_c})\ (psi) \qquad (6b)$$

The factor c varies from 0.004 for interior columns to 0.006 for corner columns.

Very large amounts of reinforcement require the use of either large diameter bars or bundles of bars. Tests of columns with bundled bars showed no reduction in load capacity[25]. Very large bars require mechanical or bearing splices since lap lengths become excessive.

4. SLENDER COLUMNS

4.1 Behavior of Slender Columns

A column subjected to end moments and axial loads deflects as shown in Fig. 6, leading to an increase in moment at its critical section and hence to a reduction in its axial load capacity as shown by Fig. 6(c). The factors affecting this phenomena are discussed more fully by Warner in Theme Paper I[6].

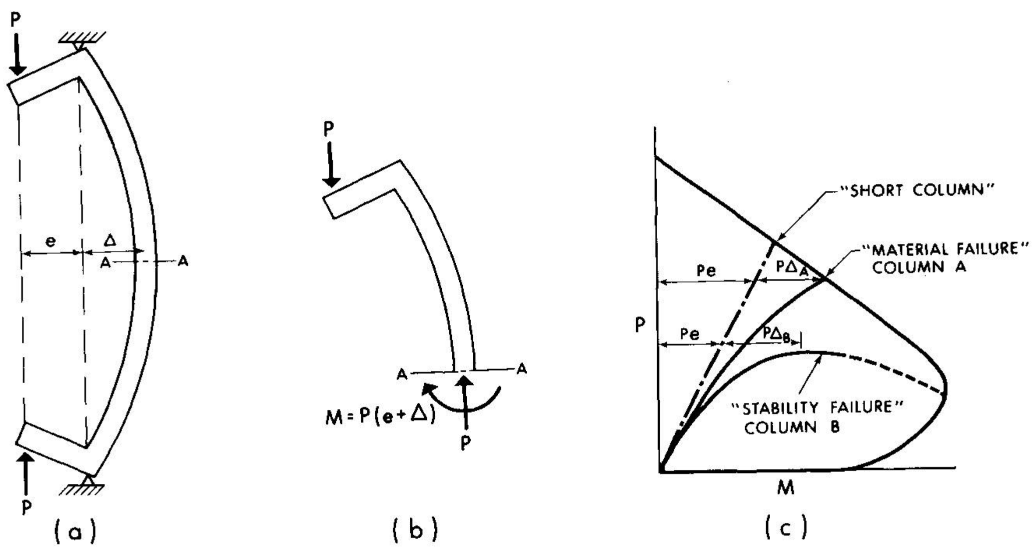


Fig. 6 Behavior of a Slender Column

When a graph of the relationship between P and M at the critical section is superimposed on the interaction diagram for the cross-section as shown in Fig. 6(c), two types of failure can be identified. A "material failure" occurs if dP/dΔ or dP/dM is positive at the time failure occurs as shown by line A. For very slender columns dP/dM may become zero or negative prior to material failure as shown by line B, giving a "stability failure". It is sometimes difficult to formulate simple design rules to cover both types of failure.

The major variables affecting the strength of a slender column include: the slenderness ratios, h/ℓ, end eccentricity, e/h, and the ratio of end moments M_1/M_2, which have a closely inter-related effect on the deflections of a column; the cross-sectional properties including the shape, the reinforcement ratio, p_t and the concrete strength, f'_c; the degree of lateral and rotational restraint at the ends of the column; and the effects of sustained loads[26].

In developing design recommendations it is customary to derive relationships for the behavior of a hinged ended column bent in single curvature (referred to in this paper as the "basic column") and then to introduce modifications to account for other curvature cases and other types of end restraint. This procedure will be followed in the following sections.

4.2 Design Solutions for the Basic Hinged End Column Under Short Time Loads

4.2.1 Moment Magnifier Method

As the basic slender column is loaded, it deflects laterally as shown in Fig. 6(a) and the moments at midheight increase as shown in Fig. 6(b). This behavior can be represented by Eqn. 7, which is derived in Theme Paper I[6]:

$$M = \frac{M_o}{1 - P/P_{cr}} \qquad (7)$$

and

$$P_{cr} = \frac{\pi^2 EI}{\ell^2} \qquad (8)$$

Eqn. (7) is used for design in the ACI Code[1] and the Soviet design regulations[27,28,29].

The major problem in presenting such an equation is the method used to define the flexural stiffness, EI, of the column cross-section. Experimental, analytical and statistical studies have shown that EI is a function of the shape and size of the cross-section expressed in terms of the gross moment of inertia, Ig; the modulus of elasticity of the concrete, E_c, the reinforcement ratio, p_t; the degree of cracking, expressed in terms of e/h and the slenderness ratio ℓ/h[30]. The following equations proposed for EI for short time loadings take these factors into account in varying degrees:

ACI Building Code - ACI 318-71[1]:

$$EI = \frac{E_c I_g}{2.5} \qquad (9a)$$

or

$$EI = \frac{E_c I_g}{5} + E_s I_s \qquad (9b)$$

Soviet Building Code[27]:

$$EI = 0.8 E_c \left[I_g \left(\frac{0.11}{0.1 + e/h} + 0.1\right) + \frac{E_s}{E_c} I_s\right] \quad (10)$$

Based on extensive computer analyses and comparisons with test results, MacGregor, Oelhafen and Hage[30] have proposed the following equations for EI to replace those in the ACI Code:

For symmetrically reinforced concrete columns with reinforcement in more than one plane:

$$EI = E_c I_g \left(\frac{0.25}{\beta} + \frac{E_s}{E_c} p_t\right) \quad (11a)$$

For unsymmetrically reinforced concrete columns with reinforcement in more than one plane and for composite columns:

$$EI = \frac{E_c I_g}{5\beta} + E_s I_s \quad (11b)$$

For walls with one layer of reinforcement:

$$EI = \frac{E_c I_g}{\beta} \left(0.5 - \frac{e}{h}\right) \geq \frac{0.1\, E_c I_g}{\beta} \quad (11c)$$

where β is a correction for sustained loads and will be discussed in Section 4.3.3.

4.2.2 Additional (Complementary) Moment Method

The basic moment magnifier equation, Eqn. (7) can be written in the form:

$$M = \frac{M_o}{1 - P/P_{cr}} = M_o + M_a \quad (12)$$

where M_a is the additional or complementary moment due to lateral deflections.

Eqn. (12) can be rewritten as:

$$M_a = \left(\frac{M_o}{1 - P/P_{cr}}\right) . P/P_{cr} = M \frac{P}{P_{cr}} \quad (13)$$

Substituting $P_{cr} = \frac{\pi^2 EI}{\ell^2}$ and $K = M/EI$ gives:

$$M_a = P \left(\frac{K\ell^2}{10}\right) \quad (14)$$

where K is the curvature at failure of the column. As a first trial, the curvature, K_b, at the balanced failure load can be estimated from the failure strain distribution using Eqn. (15). For other loads the curvatures can be related to the balanced curvature.

$$K_b = \frac{\varepsilon_{cu} + \varepsilon_y}{d} \quad (15)$$

Based on test data and this reasoning assuming $\varepsilon_{cu} = 0.003$, Aas-Jakobsen[31] derived the following expressions for the curvature. These are presented in the 1970 CEB Recommendations[3]:

$$K = \left(\frac{0.003 + \sigma_y/E_s - \frac{k\ell}{50000\ h}}{h}\right) K_1 \tag{16a}$$

where

$$K_1 = \frac{P_o - P}{P_o - P_b} \leq 1.0 \tag{16b}$$

where P_o is the capacity of the cross-section under pure axial load
P_b is the balanced load

The British Standard Code of Practice[32,33] has adopted this procedure directly, changing only the concrete strain term ε_{cu} to account for sustained loads.

The German DIN 1045[34,35] allows the use of an additional moment method for columns with $k\ell/r$ from 20 to 70:

For $0 \leq e/h \leq 0.3$

$$e_a = \left[h\left(\frac{\frac{k\ell}{r} - 20}{100}\right) \cdot \sqrt{0.10 + e/h}\right] \geq 0 \tag{17a}$$

For $0.3 \leq e/h \leq 2.50$

$$e_a = \left[h\left(\frac{k\ell/r - 20}{160}\right)\right] \geq 0 \tag{17b}$$

For $2.50 \leq e/h \leq 3.50$

$$e_a = \left[h\left(\frac{k\ell/r - 20}{160}\right)(3.50 - e/h)\right] \geq 0 \tag{17c}$$

4.2.3 Long Column Reduction Factors

For many years slender columns have been designed for an amplified axial load P/R and an amplified moment, M/R. Although easy to use for a limited number of cases, such a design procedure does not correctly reflect the behavior of slender columns since only the moment should be amplified. For this reason, such design methods have generally been abandoned during the last decade and will not be discussed further in this paper.

4.2.4 Discussion of Basic Design Procedures

It is important that a design procedure be based on an easily recognizable physical model that approaches the true behavior. Both the moment magnifier and additional moment procedures are based on the basic concept that the lateral deflections of a slender column increase the moments in the column.

Once a basic design model is selected it is necessary to present it mathematically. This involves a compromise between simplicity of application and accuracy of results. The differences between Eqns. (9a) and (17), for example, simply reflect different opinions on the acceptable degree of complexity and different boundaries on the domaine of acceptable solutions resulting from different ecconomic conditions and different building traditions. In choosing the final form of design equations the code writer should also be guided by the actual range of variables encountered in the structures to be designed.

4.3 Modifications of Basic Design Solutions

Three methods of designing the basic hinged-ended column bent in single curvature have been presented. In the following sections the moment magnifier and complementary eccentricity methods will be extended to other deflected shapes and other types of end restraint. Since both of these design procedures are closely related, the same modifications for restraints, etc. will generally be usable in both cases.

4.3.1 Effect of End Restraints

For design purposes a column in a frame can be represented by an equivalent hinged column with a length equal to the "effective length" or "buckling length", $k\ell$. The effective length will be less than the actual length of the column in a frame braced against sway and greater than the actual length in a frame free to sway laterally. If the column remains prismatic throughout the loading history (ie. no localized cracking or inelastic action, etc.) and if the relative stiffnesses of beams and columns are known and remain constant, it is possible to compute the effective length from the theory of elastic stability. For design use this can be done using alignment charts or equations which approximate the effective lengths. For unbraced frames Furlong[36] has proposed the following equations for the effective length factor:

$$k = \frac{20 - \Psi}{20} \sqrt{1 + \Psi} \qquad \text{for } \Psi < 2 \tag{18a}$$

$$k = 0.9 \sqrt{1 + \Psi} \qquad \text{for } \Psi \geq 2 \tag{18b}$$

where

$$\Psi = \sum \frac{EI_{col}}{\ell_{col}} \Big/ \sum \frac{EI_{beam}}{\ell_{beam}} \tag{19}$$

If different values of Ψ occur at the two ends the average value of k is used.

In actual fact, however, the degree of cracking and inelastic action does vary from point to point along the column and its restraints so that elastic values of k can not truly be applied to reinforced concrete columns. More study of this problem is required.

The reinforcement ratios in the beam and column have a significant effect on the relative stiffness and hence on the effective length[37,38]. For columns with a slenderness ratio up to $k\ell/r = 60$ it is sufficiently accurate to use an elastically computed effective length based on Ψ values evaluated considering the 0.8 times the uncracked moment of inertia of the concrete in the columns and 0.4 times the uncracked moment of inertia. For longer columns Reference 38 gives guidance.

Since it is difficult to accurately estimate the relative stiffnesses of the columns and restraints, the 1972 British Standard Code of Practice[32] gives simple upper bounds to the effective lengths using Eqns. 20:

For braced columns use the smaller of:

$$k\ell = \ell\left[0.7 + 0.05\,(\Psi_1 + \Psi_2)\right] \leq \ell \qquad (20a)$$

$$k\ell = \ell\,(0.85 + 0.05\,\Psi_{min}) \leq \ell \qquad (20b)$$

For unbraced columns use the smaller of:

$$k\ell = \ell\left[1.0 + 0.15\,(\Psi_1 + \Psi_2)\right] \qquad (20c)$$

$$k\ell = \ell\,(2.0 + 0.3\,\Psi_{min}) \qquad (20d)$$

where Ψ_1 and Ψ_2 refer to the relative stiffnesses at the two ends of the column.

For unbraced frames the BSCP recommends that the average $k\ell/r$ for all the columns in a storey be used in calculating e_a. The ACI Code accomplishes the same thing by substituting ΣP and ΣP_{cr} for all the columns in a storey into Eqn. (7) when solving an unbraced frame.

For columns resting on foundations a value of Ψ can be calculated from the moment-rotation response of the foundation.

The problem of whether a structure is laterally braced or not will be discussed in Section 6.2.3 of this paper.

4.3.2 Effect of the Ratio of Initial End Moments

The derivation of the basic moment magnifier or additional moment relationships assumed that the column was bent in symmetrical single curvature so that the maximum deflection moments could be added directly to the maximum applied moments. If the maximum applied moment occurs at one end of the column, the maximum deflection and applied moments occur at different sections and cannot be added directly. Massonnet[39], and others have shown that the design of such columns can be based on an equivalent, symmetrical, single-curvature bending moment diagram which would give rise to the same maximum moment as occurs under the actual loading. In the case of braced columns where no transverse loads are applied between the ends of the column a reduced moment, $C_m M_o$, can be used in Eqns. (7) or (12)[26]:

$$M = \frac{(C_m M_2)}{1 - P/P_{cr}} \geq M_2 \qquad (21)$$

$$C_m = 0.4 + 0.6\,M_1/M_2 \quad \text{but not less than } 0.4 \qquad (22)$$

where M_1 is the smaller initial end moment, taken as negative when the column is bent in double curvature and M_2 is the larger initial end moment, taken as positive. For columns not braced against sway $C_m = 1.0$.

The German concrete design standards[34] require that columns in braced frames be designed for the maximum eccentricity in the mid-third of the effective length. For a braced frame with average end restraints this gives essentially the same value of $(C_m M_2)$ as Eqn. (21).

The British Standard Code of Practice[32] and the CEB[40] suggest ways of adding M_o and M_a at various points along the column so that reinforcement can be varied in very long members such as piers. In the author's opinion the reinforcement should remain constant from end to end of a column since the stability of a concrete column with variable moment of inertia has not been adequately studied and was not considered in the derivation of Eqn. (16).

4.3.3 Effect of Load Duration

Sustained loads have three significant effects on the strength of columns[28,40,42,43]. First the lateral deflections are increased due to creep hence weakening the ability of the column to carry additional loads. Second, for very long columns, failure may occur due to "creep buckling" during the period of sustained load. Third, the lateral deflections cause a reduction in the rotational stiffness of the column which, if the column is in a braced frame, results in a reduction in the column end moments that tends to offset the deflection moments. For restrained columns up to $k\ell/r$ about 20 in braced frames this will tend to cause an increase in the axial load capacity of the columns[43].

For design purposes there are essentially three major methods of accounting for sustained load effects. These will be discussed in the following paragraphs.

(a) Reduced Modulus Method

The modulus of elasticity in the EI equations can be reduced to account for creep. In the derivation of Eqns. (10) and (11) it was assumed that only the concrete term would be reduced. Thus for Eqn. 10[28,29] the first term inside the square brackets is divided by $(1 + 0.5\ \phi\ M_\phi/M_o)$ while in Eqn. 11 the term β in the EI equations is given by[42]:

$$\beta = 0.9 + 0.5\ (P_\phi/P)^2 - 12\ p_t \geq 1.0 \tag{23}$$

where P_ϕ and M_ϕ refer to sustained load or moment and P and M_o refer to total load and moment (unmagnified), ϕ is the creep coefficient and p_t is the total longitudinal steel ratio. For practical design cases β will vary from 1.0 to 1.20. A flat value of $\beta = 1.20$ will simplify the design but not be excessively conservative.

The 1971 ACI Code[1], on the other hand, includes the effect of creep by dividing the entire EI equation by $(1 + M_\phi/M_o)$. This reduces both the steel and concrete terms because the load transfer from concrete to steel due to creep may cause a reduction in EI if the compression reinforcement yields prematurely. This procedure is excessively conservative[42].

The CEB Recommendations[3] and the British Standard Code of Practice[32] account for creep by increasing the compression strain in the concrete. In the CEB the increase is a function of the ratio M_ϕ/M_o and the amount of creep that is anticipated. This is simplified in the BSCP which calculates curvatures for all loading conditions using $\varepsilon_{cu} = (1.25 \times 0.003)$.

(b) Dischinger Procedure

As described in Theme Report I[6], Dischinger represented the creep deflections in slender uncracked visco-elastic columns by adding a creep eccentricity given by Eqn. (24) to the actual deflections before calculating the effects of slenderness using Eqns. (7) or (12).

$$e_c = e\left[\exp\left(\frac{\phi}{P_{cr}/P_\phi - 1}\right) - 1\right] \quad (24)$$

For design use Kordina and Warner[41] have developed Eqn. (25) for the 1972 German Code DIN 1045[34]:

$$e_c = e\left(\frac{0.8\phi}{P_{cr}/P_\phi - 1 - 0.4\phi}\right) \quad (25)$$

Oelhafen and MacGregor[42] have proposed:

$$e_c = e_\phi\ (0.6 + 1.3\ P_\phi/P) \geq e_s \quad (26)$$

where P_ϕ and e_ϕ in the various equations refer to the sustained load and its eccentricity.

(c) Sustained Load Eccentricity Method

Aas-Jakobsen[40] has suggested that column design for sustained loads could be based on:

$$M = P(e + e_a + e_c) \quad (27)$$

where

$$e_c = (M_\phi/M)\ \phi h/20 \quad (28)$$

Somewhat better correlation is obtained with a wide range of shapes and properties if e_c is expressed using Eqn. (29)[42]:

$$e_c = -0.40 + 2\ P_\phi/P \geq 0.40 \quad (29)$$

4.4 Comparison with Tests

The various design procedures described in this report are compared to tests of slender columns in References 26, 29, 31, 41, 30 and 42. In general the scatter with respect to test/calculated load is about 1.5 to 2 times that for test/calculated moment. The comparisons in Reference 31 are based on ratios of loads for small eccentricities and ratios of moments for large eccentricities and hence the coefficients of variation quoted in this reference cannot be compared to those reported in the other references. The design methods quoted predict the test strengths with coefficients of variation varying from about 0.15 to 0.25.

5. COMPOSITE AND PRESTRESSED COLUMNS

5.1 Composite Columns

The stress and strain compatibility-interaction diagram solution described in Section 3.1 for reinforced concrete columns is equally applicable to composite columns made up of structural steel shapes enclosed in concrete[44] or concrete filled pipe columns[45]. Basu[44] has shown that negligible errors result from ignoring residual stresses in the steel shapes when calculating interaction diagrams for the cross-section and suggest they can also be ignored in deriving slender column interaction diagrams. This requires more study.

Because Poisson's ratio is greater for steel than for concrete, the lateral expansion of a structural shape encased in concrete will exceed that of the concrete and vertical cracks may develop in the concrete shell. These cracks can be controlled by providing adequate reinforcement in the shell.

In the case of concrete filled pipe columns there will be an initial tendency for cracks to develop between the surfaces of the concrete and the steel. When the concrete stress exceeds the discontinuity limit of the concrete, the concrete tends to expand and in circular columns eventually is restrained by the steel tube[46].

In recent years Japanese engineers have used steel truss assemblies to reinforce concrete columns and beams to form "SRC" or Structural Steel Reinforced Concrete[47]. The lattice bracing in the truss webs gives these members desirable shear resistance.

The ACI Code[1] recommends the use of Eqns. (21) and (9b) for the design of slender composite columns. Basu and Somerville[48] have derived interaction diagrams for composite columns similar to those presented in structural steel design specifications.

5.2 Prestressed Concrete Columns

Columns, wall panels or piles may be prestressed to resist handling stresses or to reduce service load deflections due to lateral loads between their ends. Once again, design can be based on ultimate strength interaction diagrams derived by the stress and strain compatibility procedures[49].

The ACI Code[1] recommends the use of Eqns. (21) and (9a) for the design of slender prestressed concrete and there is good correlation between measured and calculated capacities using this procedure[26].

6. STABILITY OF SYSTEMS

6.1 Factors Affecting the Stability of an Entire Structure

Based on simple second-order analyses Rosenblueth[50] and Stevens[51] have shown that the critical load of a storey in a sway frame is approximately equal to:

$$P_{ci} = \frac{K_{\ell i} h_i}{\gamma} = \frac{H_i}{\gamma} \cdot \left(\frac{h_i}{\Delta_i}\right) \qquad (30)$$

This equation shows the strong relationship between the critical load, P_c and the lateral stiffness, K_ℓ or alternately the deflection index Δ/h or a given lateral load, H. The term γ ranges from 1.0 for flexible beams to 1.22 for rigid beams. In a tall building the lateral deflections are limited to prevent non-structural damage to partitions and to prevent motion sickness of the occupants. These limits are independent of whether the building is braced or free to sway. If the lateral deflections of these two types of buildings are similar at a given load stage, as shown in Fig. 7, their PΔ moments will be similar and as shown by Eqn. (30), their critical loads will also be similar.

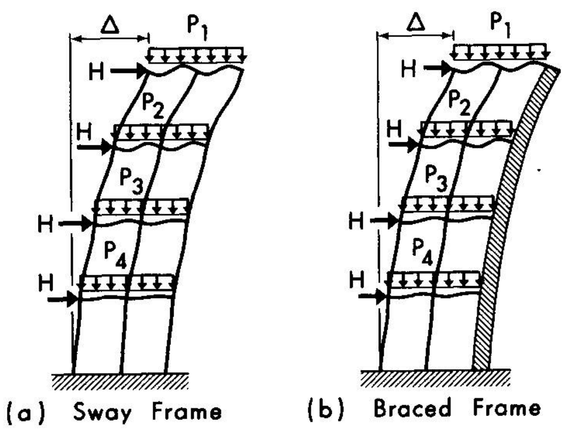


Fig. 7 Deflected Shapes of Buildings

Analyses of tall frames have suggested that the most important factors affecting their stability are:

1. The lateral stiffness of the frame. Eqn. (30) indicates that when the lateral stiffness of a frame increases, its critical load does also.

2. The number of bays. If a multi-bay frame is designed to satisfy the same drift requirement as a single bay frame, the total lateral load on the frame remains constant while the total vertical load increases. Using Eqn. (30) Stevens[51] has shown that the critical load factor, λ_c, decreases as the number of bays increases until a lower limit of λ_c is reached due to stress limitations in the frame.

3. If gravity loads control the design of both beams and columns the critical load factor decreases as the number of stories increases[51].

4. Total vertical load. A number of studies have shown that the lateral instability of a frame or storey is controlled by the total vertical load in the storey rather than the load in a single column since the entire storey must fail as a unit. An exception to this would occur if one column buckled in a no-sway mode prior to the sway instability of the storey.

5. The degree of inelastic action. Plastic hinges greatly reduce the stiffness of a frame and in doing so reduce its critical load. This is especially true if hinges form in shear walls or similar elements.

6. Rotations of the foundations of the shear walls in a tall building may cause significant lateral deflections of the structure. As a result, the lateral stiffness of the building and hence its critical load may be reduced significantly.

The stability of buildings is discussed more fully in References 50, 52, 53 and 54.

6.2 Methods of Analyzing the Stability of Structures

6.2.1 First-Order Analysis with Effective Length Factors

At the present time the most common design procedure is based on a first-order analysis of the structure followed by the proportioning of the individual columns based on their own effective lengths using the moment-magnifier or additional moment procedures. Since effective lengths are generally available for the highly idealized cases of "fully-braced" or "free to sway" frames it is necessary to have some method of differentiating between these cases. Relatively little guidance is available to designers in this regard.

Beck and König[53] have suggested that second-order effects could be ignored, or in other words, a frame could be considered as fully braced if Eqn. 31 is satisfied:

$$h_t \sqrt{\frac{P_t}{EI}} \leq 0.6 \text{ if } f > 4 \tag{31}$$

$$\leq 0.2 + 0.1f \text{ if } 1 \leq f \leq 4$$

where: p_t = total vertical load on structure.

EI = total bending stiffness of all vertical stiffening elements.

h_t = total height of the building.

f = number of stories.

Based on an elastic-plastic analysis of tall buildings Clark[55] suggested that a frame could be considered braced if the sum of the stiffnesses of the walls in any storey exceeded six times the sum of the stiffnesses of all the columns in that storey. The value six may be lower than actually required.

Talwar and Cohn[56] suggested four criteria for choosing shear wall sizes in shear wall frame structures. To prevent excessive lateral deflections under service loads they propose a lateral displacement criterion:

$$P_{cr}/P \geq 700\ H/P + 1 \tag{32}$$

where P_{cr} is the critical load of the structure and H and P are the lateral and vertical service loads. So that the maximum amplification of deflections is small enough to be ignored they propose $P_{cr}/P \geq 20$ at working loads. They also propose that the critical load of the free standing wall should exceed 0.5 P_{cr} and a fourth criteria intended to prevent excessive moments due to unsymmetrical loading. The latter two criteria do not appear to adequately apply to the entire range of structures from fully braced to fully sway frames and may lead to unnecessarily stiff walls when applied to stiff frames.

In addition to difficulties in defining the degree of bracing in a frame, the design procedure based on first-order analysis and elastic effective length factors is inadequate for "braced" frames because the second-order forces in the bracing and the frames are not accounted for. It is also inadequate for unbraced frames because it usually leads to overdesign of some columns and under-design of others. Finally the method errs on the unsafe side because it does not account for the increases in beam moments which result from slenderness effects[26].

6.2.2 Subassemblage Techniques

A step by step inelastic analysis of steel building frames considering sub-assemblages consisting of a half-storey height column and one or more beams has been developed for design use by Lehigh University[57]. Following a preliminary selection of members the strength of each storey is checked by calculating a lateral load-deflection curve for each storey including the effects of inelastic action and gravity loads. Such an analysis could be applied to concrete structures with some modifications.

6.2.3 Second Order Analysis

A second-order analysis of a structure accounts for the effects of the deflections of the members and the structure itself on the forces and moments in the structure. The requirements for an "exact" second-order analysis are discussed in Reference 52.

For tall buildings designed for normal deflection limitations, an acceptable estimate of the second-order shears, moments, and forces in an elastic structure can be obtained by an iterative calculation including the "sway forces" induced by the P-Δ moments. The computation of sway forces for the combined loading case is relatively simple. The lateral and vertical loads are applied to the structure and the relative lateral displacements, Δ_i in each storey are computed by the first-order elastic analysis ignoring P-Δ terms. The additional storey shears due to the vertical loads are computed as $\Sigma P_i \cdot \Delta_i / h_i$ where ΣP_i represents the sum of the axial forces in all the columns of the ith storey, as shown in Fig. 8. At a given floor level, the sway force will be the algebraic sum of the storey shears from the columns above and below the floor. The sway forces are added to the applied lateral loads and the total forces and moments in the structure can be computed. Generally, one cycle of iteration is adequate for elastic structures of reasonable stiffness. The application of this procedure to three-dimensional buildings is discussed in References 58 and 59.

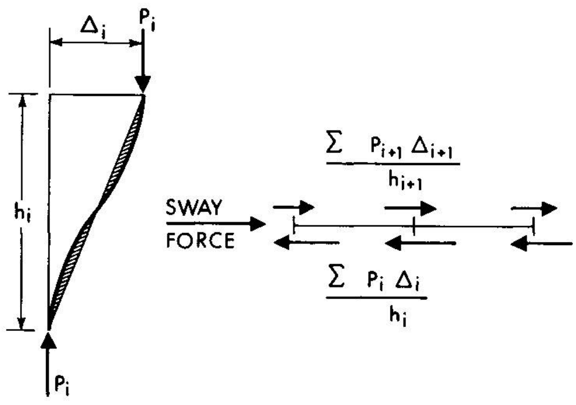


Fig. 8 Calculation of Sway Force

Alternately, the second-order deflections of each storey can be obtained directly from the first order deflections. Fey[60] and Parme[61] have both shown that the total second-order deflection; Δ_{2i}, in the ith storey of an elastic structure can be computed using Eqn. 33:

$$\Delta_{2i} = \frac{\Delta_{1i}}{1 - \dfrac{\Sigma P_i \Delta_{1i}}{H h_i}} \qquad (33)$$

where H is the shear in the storey due to the applied lateral loads and h_i is the height of the ith storey. A second-order analysis suitable for design would include:

1. A first-order analysis to determine Δ_{1i} in each storey.

2. Computation of the second-order deflection in each storey using Eqn. 33.

3. Evaluation of the sway forces as outlined previously, but using the storey deflection, Δ_{2i}.

4. Another first-order frame analysis for the frame subjected to the applied vertical and lateral loads plus the sway forces from step 3, gives second-order moments and forces.

Fig. 9 shows columns with and without lateral displacements of the ends. If translation is prevented, the buckled shape is as shown in Fig. 9(a). The moments M_t and M_b are the applied end moments while M_{rt} and M_{rb} are restraining moments caused by the rotations of the end restraints as the column deflects. Horizontal forces, H, are present if the end moments are unequal. At midheight there are secondary moments equal to the axial load times the deflections shown shaded. To account for the restraining moments M_{rt} and M_{rb} in the design of this "braced" column an effective length less than the real length is used to compute the lateral deflections.

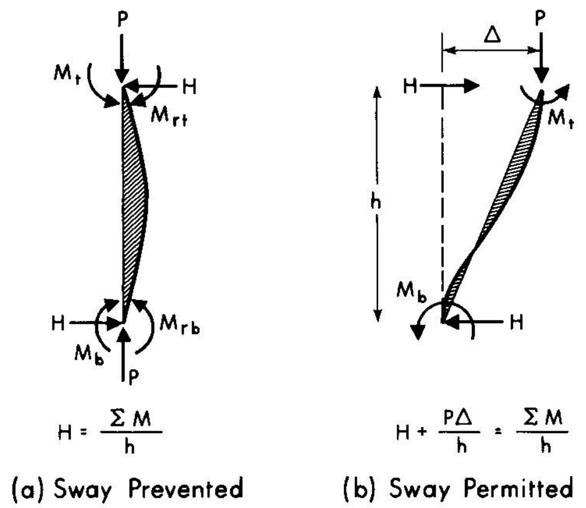


Fig. 9 Forces in Deflected Columns

If, however, the column is free to sway laterally as shown in Fig. 9(b), the moments M_t and M_b must equilibriate not only any horizontal load, H, but also a moment PΔ. The secondary moments in this column can be divided into two components, one due to the additional horizontal reaction or sway force, PΔ/h, necessary to resist the axial force in the deformed position and the second equal to the axial load times the deflections from the chord line, shown shaded. If there is no bracing the sway force PΔ/h must be provided by increasing column moments. Traditionally these have been accounted for in design by using the effective length factors for the unbraced case in designing the column.

On the other hand, if a "second-order" structural analysis is carried out including the effects of both the applied loads and the sway forces, the latter have been accounted for in the analysis and need not be considered a second time in evaluating the effective length. Under these conditions the design would be based on the effective length for a "braced" column to include the effect of the deflections of the column from the chord.

This method of analysis and column design procedure is equally applicable for structures built of cast-in-place or precast concrete, steel or mixed forms of construction. In addition it can be applied to any structural form which can be analyzed and has the advantage that the stability effects are clearly evident to the designer.

7. RECOMMENDED DESIGN PROCEDURE FOR STRUCTURES CONTAINING COMPRESSION MEMBERS

7.1 Recommended Structural Analysis

The complexity of the method used to analyze and design the columns in a structure depends on the type of structure considered. If inspection or the use of a criteria such as Eqn. 31 suggests that the structure is obviously braced, design can be based on a first order analysis. On the other hand, if lateral deflections approaching 1/500 occur at service loads a second-order analysis should be considered even if the structure has shear walls or similar bracing.

Although an inelastic second-order analysis is the best available method for determining the true deflections, moments and strength of a reinforced concrete frame, such analyses are too complex and too expensive for every day design use at the present time. It is conservative and sufficiently accurate for design purposes, however, to carry out a second-order elastic analysis for the factored ultimate horizontal and vertical loads provided that all critical sections are elastic or on the verge of yielding at the factored ultimate loads. The analytical procedure based on Eqn. 33 presented in Section 6.2.3 is recommended for this purpose.

Due to the second-order effect the increase in deflections occurs more rapidly than the increase in load factors. For this reason it is necessary to carry out this analysis using factored ultimate loads using the average member stiffnesses corresponding to this stage. In general, two second-order analyses should be carried out: one for dead load plus live load plus wind or earthquake at the appropriate load factors and the second for factored dead plus live load plus any assumed lateral drift due to construction errors.

The EI values used in a second-order analysis should reflect the type of loading and the reinforcement ratio for each member. The effective EI values proposed by Kordina[62] can be used for this purpose. Based on Kordina's equations it is possible to derive simplified estimates of EI for use in design analyses for normal reinforcement ratios. Thus, the EI values used in the analysis can be based on the initial modulus of the concrete. For the beams I should be taken as 0.4 to 0.5 I_g to account for inelastic action and cracking where I_g is the moment of inertia for the uncracked section ignoring the reinforcement. For the columns I should be taken equal to about 0.8 I_g. The effect of axial loads on the rotational stiffness of columns (i.e. on C and S) can be neglected if ℓ/h is less than 15. The effect of joint width may be important, however.

For structures with a height-to-width ratio greater than three, deflections due to axial deformations of the columns should be considered. Similarly, foundation deformations may have a significant effect on the lateral deflections.

7.2 Recommended Column Design Procedure

Once the structural analysis has been carried out, either the moment magnifier or the additional moment procedure can be used to calculate the slenderness effects for individual columns. The author prefers the use of Eqn. (21) with EI values given by Eqn. (11) and equivalent moment factors given by Eqn. (22). If the frame is braced, or if a second-order analysis has been carried out, the column effective length factors calculated with Eqns. (20a) and (20b) for the braced case can be used.

Once the maximum forces and moments on a cross-section in the column have been calculated including the effects of slenderness, an ultimate strength design procedure should be used to proportion the column.

NOTATION

A_c	=	gross area of concrete section
A_s	=	area of tension reinforcement
A_s'	=	area of compression reinforcement
C_m	=	equivalent moment factor
d	=	depth from extreme compression fibre to centroid of tension reinforcement
e	=	eccentricity
e_a	=	additional eccentricity
e_c	=	eccentricity due to creep
e_s	=	eccentricity of sustained load
E_c	=	modulus of elasticity of concrete
E_s	=	modulus of elasticity of steel
f	=	number of floors in a building
f_c'	=	28 day compression strength of a 6 by 12 in. (15 by 30 cm.) concrete test cylinder
h	=	overall depth of a concrete section
h_i	=	height of the ith storey
h_t	=	overall height of a building
H	=	lateral load
I_g	=	moment of inertia of gross (uncracked) concrete section ignoring the reinforcement
I_s	=	moment of inertia of reinforcement
k	=	effective length factor
K	=	curvature
K_b	=	curvature at balanced load
$K_{\ell i}$	=	lateral stiffness of ith storey
ℓ	=	length of a column
M	=	design moment for a column including slenderness effects
M_a	=	additional moment
M_o	=	initial end moment
M_{ox}	=	uniaxial moment capacity of column cross section about x axis

M_1 = smaller of the two column end moments, positive if column is bent in single curvature

M_2 = larger of the two column end moments, always positive

p_t = total reinforcement ratio = $(A_s + A_s')/A_c$

P = design load for a column

P_b = capacity of column cross-section at balanced failure

P_{cr} = critical load

P_o = capacity of column cross-section under pure axial load

P_t = total vertical load in a building

P_ϕ = constant sustained load

r = radius of gyration

R = long column reduction factor

α = factor to convert biaxial bending into equivalent uniaxial bending case

β = factor to account for effect of creep

Δ = lateral deflection of a column or storey

ε_{cu} = failure strain of concrete in combined bending and axial load

ε_y = yield strain of reinforcement

ϕ = creep coefficient

λ_c = load factor against elastic instability

Ψ = ratio of column stiffnesses to beam stiffnesses at one end of a column

σ_y = yield strength of steel

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SUMMARY

The factors to be considered in deriving practical design rules for column cross-sections, slender columns and overall structural stability are presented and a number of current design recommendations are compared. It is recommended that an elastic analysis including $P\Delta$ effects be used to calculate the forces and moments in columns in braced and unbraced frames. When these are known, the effect of slenderness on individual columns can be considered using moment magnifier or additional moment techniques and effective length factors for braced columns. The cross-sections should be proportioned using ultimate strength techniques.

RESUME

Le rapport présente les facteurs à prendre en considération pour l'établissement de règles simples de dimensionnement de sections de colonnes, de colonnes élancées et pour la stabilité générale d'un système. Quelques règles de dimensionnement sont comparées. On recommande une analyse élastique y compris les effets PΔ pour le calcul des forces et moments dans les colonnes en assemblages raidis et non raidis. L'effet de l'élancement des colonnes par l'agrandissement des moments de flexion ou par des moments additionnels peut alors être pris en considération en introduisant également, moyennant facteurs, l'élancement des colonnes intégrées dans le système. Le dimensionnement de la section devrait être opéré sur base de la résistance à la rupture.

ZUSAMMENFASSUNG

Die bei der Entwicklung einfacher Bemessungsverfahren für Stützenquerschnitte, schlanke Stützen und für die Stabilität ganzer Tragwerke zu berücksichtigenden Einflüsse werden diskutiert und eine Reihe üblicher Bemessungsvorschläge miteinander verglichen. Es wird empfohlen, die Stützenschnittkräfte in unverschieblichem und verschieblichem Rahmen mit Hilfe einer elastischen Berechnung zu bestimmen, welche die Formänderungen berücksichtigt. Sind diese Schnittkräfte bekannt, so kann der Einfluss der Stützenschlankheit durch Vergrössern der Biegemomente oder durch Zusatzmomente berücksichtigt werden, wobei mittels Faktoren auch die Schlankheit von im System integrierten Stützen einzuführen ist. Die Querschnittsbemessung sollte dann auf der Basis der Bruchlast erfolgen.

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