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ULTIMATE STRENGTH OF WIDE FLANGE AND BOX COLUMNS

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ABSTRACT

The purpose of this study is to investigate the ultimate strength of elastic-plastic steel wide-flange columns subjected to axial load and symmetrical end moments. The presence of residual stresses arising from fabrication processes leads to moment-thrust-curvature relationships which are untractable analytically. The latter is determined numerically. The integration procedure employs a numerical marching technique. The results are presented in the form of critical load-slenderness ratio relationship obtained for welding-type and cooling-type (parabolic) patterns of residual stress distribution. The effects of initial curvature on the strength of the columns are also compared with that of straight columns.

1. INTRODUCTION

The strength of steel columns is influenced by such unavoidable factors as residual stresses, initial imperfection and eccentricity of loading. Residual stresses are present as the result of uneven cooling after the hot rolling for rolled sections or of welding for built-up sections.

The strength of columns was studied earlier by Karman (10), Chawalla (5), Jezek (9), Shanley (16) and many other investigators. More recently Horne (8) presented a criterion of stability for columns which was utilized later by other investigators (7,11,15) to treat wide flange and box columns with initial imperfection but free of residual stresses. Chen and Santathadaporn (4) studied the strength of eccentrically loaded rectangular columns, formulating the governing equation in term of curvature rather than deflection. The influence of residual stresses on the buckling strength of concentrically loaded steel columns was discussed by Osgood (14) and Beedle and Tall (2). The combined effect of residual stresses and initial imperfection on the strength of concentrically loaded aluminum alloy and steel columns was studied by Batterman and Johnston (1) who employed a numerical incremental scheme to obtain the complete load deflection curves. Recently, Sherman (17) studied the strength of eccentrically loaded straight steel box column with linearly varying residual stresses across the width of the component plates. The latter also made use of Horne's criterion of stability. The reduction of strength due to the presence of residual stresses was found to be as high as 40%. Chen (3) studied the strength of beam-columns using the moment-thrust-curvature relationships of wide flange sections with linearly varying residual stresses.

The purpose of this study is to investigate the influence, on the strength of wide flange and box columns, of the eccentricity of end loading, the initial crookedness and residual stresses arising from the cooling of hot rolled wide flange sections and from the welding of built-up sections. These residual stress patterns as reported in Refs. (2,12) can be idealized more closely by the distributions shown in Fig. 1, where the welding pattern consisting of a series of straight lines represents that encountered in welded built-up shapes while the pattern with parabolic curves represents that in hot-rolled wide flange shapes. The ultimate strength is determined by numerically integrating the governing differential equation and applying Horne's criterion of stability. The moment-thrust-curvature relationships are also determined numerically for the two residual stress patterns considered. Only columns which fail by bending about the strong axis will be considered. It is of interest to mention that the results of this analysis for the special case of the concentrically loaded perfectly straight columns lie in between the values predicted by the tangent modulus theory and the reduced modulus theory.

2. MOMENT-THRUST-CURVATURE RELATIONSHIPS

In order to compute the moment-thrust-curvature relationships numerically, a wide flange section is divided into finite grid elements. The coordinate system is chosen to pass through the centroid of the section. Under the action of bending moment and axial thrust, the strain at element i with residual stress can be expressed in the nondimensional form,

$$\frac{\epsilon_i}{\epsilon_y} = \frac{\epsilon_o}{\epsilon_y} + \varphi\frac{Y_i}{d} + \frac{\epsilon_{ri}}{\epsilon_y} \qquad (1)$$

where ϵ_i = total strain at element i, positive for tensile strain; ϵ_o = strain at the centroid of the section; φ = curvature nondimensionalized by the curvature at initial yielding for pure bending moment, $\Phi_y = \epsilon_y/d$; Y_i = distance of the center of element i from the centroidal axis; ϵ_{ri} = residual strain at element i; ϵ_y = strain at yield point; and d = half depth of the section.

Assuming an elastic-perfectly plastic stress-strain relationship for the steel, the strain in Eq. 1 is related to the stress by

$$\frac{\sigma_i}{\sigma_y} = \frac{\epsilon_i}{\epsilon_y} \quad \text{for} \quad \frac{|\epsilon_i|}{\epsilon_y} < 1 \qquad (2a)$$

$$\frac{\sigma_i}{\sigma_y} = \pm 1 \quad \text{for} \quad \frac{|\epsilon_i|}{\epsilon_y} \geq 1 \qquad (2b)$$

in which σ_i and σ_y = normal stress at element i and the yield stress respectively, σ_i being positive for tensile stress.

The axial thrust and moment are then determined from the following two equilibrium equations in nondimensionalized form,

$$p = -\frac{1}{A}\sum_{i=1}^{n} \frac{\sigma_i}{\sigma_y}\Delta A_i \qquad (3)$$

$$m = \frac{1}{Z}\sum_{i=1}^{n} \frac{\sigma_i}{\sigma_y} Y_i \Delta A_i \qquad (4)$$

where p and m = axial thrust and moment nondimensionalized by the yield load, $P_y = \sigma_y A$, and the fully plastic moment, $M_P = \sigma_y Z$, of the section respectively; A and Z = area and plastic modulus of the section respectively; ΔA_i = area of element i; and n = total number of elements.

The moment-thrust-curvature relationships are obtained from Eqs. 3 and 4, together with Eqs. 1 and 2, by specifying the residual stress distribution, hence the residual strain distribution, and systematically varying ϵ_o and φ. For simplicity, it is assumed that the residual stress is constant across the thickness and that equilibrium is maintained within each plate component. Typical curves showing moment-thrust-curvature relationships for different patterns and levels of residual stresses are shown in Fig. 2 for 8 WF 31. The moment-thrust-curvature relationships for a wide flange section is identical with those obtained for a box section provided that the cross sectional shapes of the halves of the wide flange and the box section as well as the residual stress pattern and level are identical.

3. GOVERNING EQUATIONS

The problem of simply supported columns loaded symmetrically at both ends can be represented as a cantilever column loaded as shown in Fig. 3. The free end is subjected to bending moment M and axial thrust P. Initial imperfection can be characterized by initial curvature along the length of the column. The equilibrium of moment and the curvature-displacement relationships, in terms of small deflection theory, are given by

$$M = M_f - PV \qquad (5)$$

$$\frac{d^2V}{dX^2} = \Phi + \Phi_o \qquad (6)$$

where M = moment; M_f = fixed end moment; X = distance along the length of column; V = transverse deflection; and Φ and Φ_o = bending and initial curvatures, respectively.

Introducing x and v as the nondimensionalized axial distance and transverse deflection defined by

$$x = \frac{X}{r}\sqrt{\epsilon_y} \tag{7a}$$

$$v = \frac{V}{r} \tag{7b}$$

where r = radius of gyration of the cross section, Eqs. 5 and 6 can be written in the nondimensionalized form,

$$m = m_f - \frac{Ar}{z} pv \tag{8}$$

$$\frac{d^2v}{dx^2} = (\varphi + \varphi_o) \frac{r}{d} \tag{9}$$

in which m_f = fixed end moment nondimensionalized by the fully plastic moment of the section; φ_o = initial curvature nondimensionalized by Φ_y.

With prescribed values of φ_o, for given values of m_f and p as parameters, the deflected shape of a column can be determined by integrating Eq. 9 in view of Eq. 8 and the moment-thrust-curvature relationship for a particular cross section and residual stress pattern and level. It is noted that, with this formulation, the strength of the steel is not involved and that the results can be applied to columns made of any grade of steel.

4. EQUILIBRIUM CURVES AND ENVELOPE

The integration of Eq. 9 for the general case is analytically untractable and numerical integration is necessary. The procedure is basically as follows:

(1) For a particular value of p, moment m can be determined from Eq. 8, provided displacement v is known. Knowing m, curvature φ is determined from Eqs. 3 and 4 in view of Eqs. 1 and 2. From the value of φ so obtained, together with the prescribed initial curvature, the right hand side of Eq. 9 is calculated.

(2) Dividing the column into small segments, such that curvature inside each segment may be assumed to be constant, Eq. 9 can be integrated with respect to x within the segment to yield the following relationships among the slope and displacement at both ends of the segment:

$$v'_{i+1} = k_i(\Delta x) + v'_i \tag{10}$$

$$v_{i+1} = \frac{k_i}{2}(\Delta x)^2 + v'_i(\Delta x) + v_i \tag{11}$$

where k_i = quantity on the right hand side of Eq. 9 at segment i; v_i and v'_i = deflection and slope at the left end of segment i; v_{i+1} and v'_{i+1} = deflection and slope at the right end; and Δx = length of the segment.

(3) For a specified value of m_f, the integration can be started from the fixed end where v and v' are known to be zero until m vanishes.

(4) Repeat the procedure by systematically changing p and m_f. In practical computation, the moment-curvature relationship for a particular value of p and pattern and level of residual stress such as shown in Fig. 2 was represented by a series of points and stored in the computer. The curvature corresponding to a particular value of moment was then obtained by interpolating between the points.

The m versus x relationships referred to as the equilibrium curves (7), for particular values of p and m_f, can be plotted as shown in Fig. 4. Applying Horne's criterion (8), the envelope of these curves is the boundary of the domain inside which a cantilever column with a combination of end moment, length and thrust is in stable equilibrium. It shows the relationships between the slenderness ratio and the maximum end moment that the column can carry for a given thrust. Fig. 5a illustrates a set of envelopes obtained for various values of p. They are plotted on the m - λ plane in which λ is the normalized slenderness ratio defined by

$$\lambda = \frac{1}{\pi}\frac{L}{r}\sqrt{\epsilon_y} \tag{12}$$

where L = length of simply supported columns, being twice the length X of the corresponding cantilever columns. It should be noted that the integration scheme requires no iteration and is always stable resulting in accurate predictions of the ultimate strength.

For the purpose of discussions, it is convenient to replot the results of the foregoing analysis as shown in Figs. 5b and 5c where the construction of the column curves relating the maximum load to the normalized slenderness ratio and the interaction curves relating the end moment and axial thrust are depicted respectively.

5. NUMERICAL RESULTS AND DISCUSSIONS

Parameters for Numerical Computation

It was reported by Hauck and Lee (7) that, for the study of the strength of wide flange columns, the area ratio, i.e., the ratio of the flange area to the web area, is the most appropriate parameter for describing the sectional properties. The reason for this lies in the fact that, in the non-dimensionalized formulation of the problem, the equilibrium equation and the curvature displacement relationship, Eqs. 8 and 9, involve the parameters Ar/Z and r/d respectively which are primarily functions of the area ratio. For idealized wide flange sections with thin flanges, these two parameters are given in terms of the area ratio by

$$\frac{Ar}{Z} = \frac{\sqrt{(1+R)(\frac{1}{3}+R)}}{\frac{1}{2}+R} \tag{13}$$

$$\frac{r}{d} = \sqrt{\frac{\frac{1}{3}+R}{1+R}} \tag{14}$$

in which $R = A_f/A_w$, A_f being the total flange area and A_w the web area. The moment-thrust-curvature relationship is also primarily a function of the area ratio.

The area ratios of all wide flange column sections lie between 2.9 to 3.6, and the ultimate strength analysis is insensitive to the variation of the area ratio. The numerical computations of this study are made for 8 WF 31 of which the area ratio is 3.27.

The magnitudes of the maximum compressive residual stress are varied from 0.1 σ_y to 0.5 σ_y for both the welding pattern of Fig. 1a and the cooling pattern of Fig. 1b. When initial imperfection is present, the initial shape of the column is assumed to be an arc of a circle in which the initial curvature is constant throughout the length. The degree of initial curvature is included by varying the factor φ_o in Eq. 9 in the range 0 to 0.4 for both patterns of residual stresses. The numerical results are presented in Figs. 6 to 12.

In addition to 8 WF 31, two sections which are approximately on the extreme limits of the range of the area ratio for column sections are studied; they are 8 WF 24 and 14 WF 127 whose area ratios are 2.88 and 3.40 respectively. The results, when plotted on the column curves or on the interaction curves, are almost identical to those for 8 WF 31. Therefore the results of this study on 8 WF 31 can be applied for all wide flange columns as well as box columns of similar dimensions.

Effect of Residual Stresses on Column Curves

Figs. 6 and 7 show the strength of straight columns with residual stresses of the welding and cooling patterns respectively. It can be seen from Fig. 6 that the reduction of column strength is greatest for concentrically loaded columns. Generally speaking, the larger the maximum compressive residual stress, the larger is the reduction in strength. However, in columns with very small eccentricity, higher maximum compressive residual stress results in smaller reduction in strength for the lower range of the slenderness ratio. This can be explained in terms of the penetration of yielding. In the case of low maximum compressive residual stress, when yielding starts, the stiffness decreases faster than the case with high maximum compressive residual stress. It is also noted that the strengths of concentrically loaded columns are constant over a larger range of λ for lower maximum compressive residual stresses.

For eccentrically loaded columns, the effect of residual stresses tends to decrease with increasing eccentricity. The presence of residual stresses reduces the strength appreciably for columns of medium length loaded with the same eccentricity, the reductions being smaller for the short and long columns.

Proceeding from Fig. 6 to Fig. 7, the reduction of strength is less severe for the case of cooling type residual stresses. The strength of concentrically loaded columns for the same λ is much higher for the latter than that for the welding type with the same maximum compressive residual stress level. The effect of residual stresses diminishes with increasing eccentricity more rapidly for the case of welding type residual stresses.

The different effect of the two residual stress patterns on column strength, together with the fact that the magnitude of compressive residual stresses present in welded sections is larger than those in hot rolled shapes (2,11), suggests that different consideration may be necessary in the design of welded built-up columns and hot rolled wide flange columns.

Effect of Initial Imperfection

The effect of initial curvature alone and the combined effect of residual stresses and initial curvature for both types of residual stresses can be seen in Figs. 8, 9 and 10. Initial curvature generally decreases the strength of otherwise straight columns. The effect of initial curvature is greatest in the intermediate column range. The same conclusion was reported by Batterman and Johnston (1). Comparing with Figs. 6 and 7, the reduction in strength due to initial curvature covers a wider range of λ than that due to residual stresses alone. The behaviour of eccentrically loaded columns with the presence of initial curvature alone or in combination with residual stresses show the same trends. The effect of the difference in residual stress patterns on column strength tends to be diminished by the presence of initial curvature.

Effect of Residual Stresses on Interaction Curves

The interaction curves shown in Fig. 11 may be more convenient for presenting the ultimate strength of beam-columns. The effect of different patterns and levels of residual stresses is shown in this figure. The parameter λ covers the range $0 \leq \lambda \leq 1.5$. It can be seen that the reduction of strength is significant for high p and the effect tends to diminish with increasing λ. Finally, the cooling type exhibits less effect than the welding type of residual stresses.

Buckling and Ultimate Strengths

It is well understood that the tangent modulus load is the smallest axial load at which bifurcation of the equilibrium position for a concentrically loaded straight column can occur and that the reduced modulus load is the upper bound of the bifurcation load (6). Fig. 12 shows a comparison between the ultimate strengths of initially straight columns obtained in this study and the buckling loads of concentrically loaded straight columns with welding type residual stresses (13). The effect of maximum compressive residual stresses of 0.2 σ_y and 0.4 σ_y is shown. For each residual stress level, the ultimate strength curve lies between the two buckling curves, being close to the tangent modulus curve at high p and tend to approach the reduced modulus curve as the load decreases.

6. CONCLUSIONS

The present study supplements the investigation of the influence of residual stresses, initial imperfection and eccentricity of loading on the ultimate strength of steel columns. Two types of residual stress patterns are chosen with varying magnitudes to represent the residual stress distribution present in hot rolled wide flange and welded built-up I and box columns.

It was found that eccentricity, initial imperfection and residual stresses are adverse factors which reduce the strength of columns. For low eccentricity, the reduction of column strength due to initial imperfection and residual stresses is more pronounced in the intermediate column range. However, the reduction diminishes as the eccentricity increases. It was also found that residual stresses and initial curvature exhibit similar trends in the reduction of the strength of practical columns.

The welding type residual stress causes more pronounced reduction in strength than the cooling type for the same level of maximum compressive residual stress. This fact suggests that different considerations should be given to the design of welded built-up sections and hot rolled wide flange columns.

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APPENDIX II.- NOTATION

The following symbols are used in this paper:

A = area of section;

A_f, A_w = total flange and web areas, respectively;

d = half depth of section;

E = Young's modulus of elasticity;

i = index number;

k = quantity on the right hand side of Eq. 9;

L = length of simply supported column;

M = bending moment;

M_f = fixed-end moment;

M_p = plastic moment;

m = M/M_p;

m_f = M_f/M_p;

n = total number of area elements;

P = thrust;

P_y = axial yield load;

p = P/P_y;

R = area ratio, A_f/A_w;

r = radius of gyration;

V = transverse deflection;

v, v' = V/r and V'/r, respectively;

X = distance along the length of cantilever column;

x = $\frac{X}{r}\sqrt{\epsilon_y}$;

Y = vertical coordinate of cross section;

Z = plastic section modulus;

ΔA = area of sectional element;

Δx = length of segment;

ϵ = total normal strain;

ϵ_o = strain at the centroid of section;

ϵ_r = residual strain;

ϵ_y = strain at yield point;

σ = normal stress;

σ_u = ultimate strength of columns;

σ_y = yield stress;

λ = $\frac{1}{\pi}\frac{L}{r}\sqrt{\epsilon_y}$;

Φ = curvature caused by bending;

Φ_o = initial curvature;

Φ_y = curvature at initial yielding for pure bending moment;

φ = Φ/Φ_y; and

φ_o = Φ_o/Φ_y.

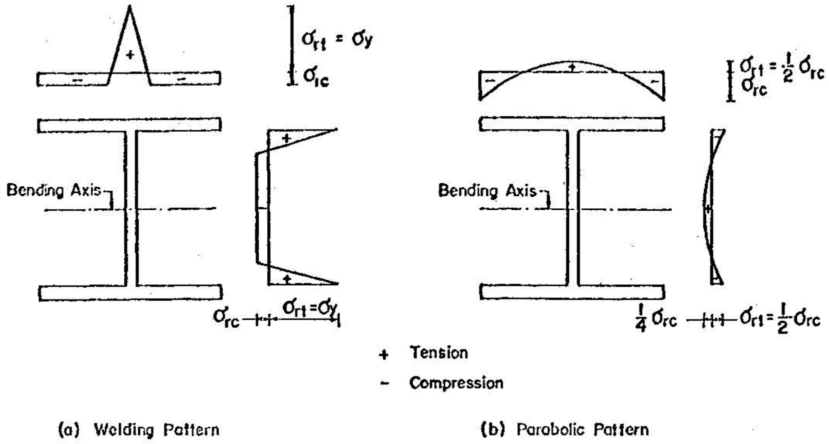

FIG. 1. - IDEALIZED RESIDUAL STRESS PATTERNS

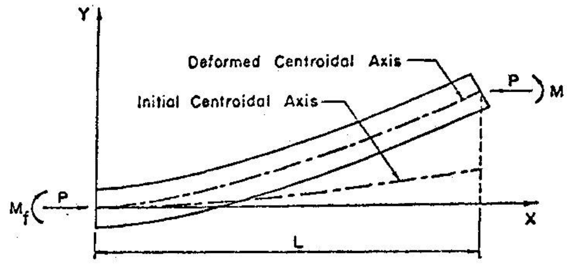

FIG. 3. - CANTILEVER COLUMN WITH INITIAL IMPERFECTION

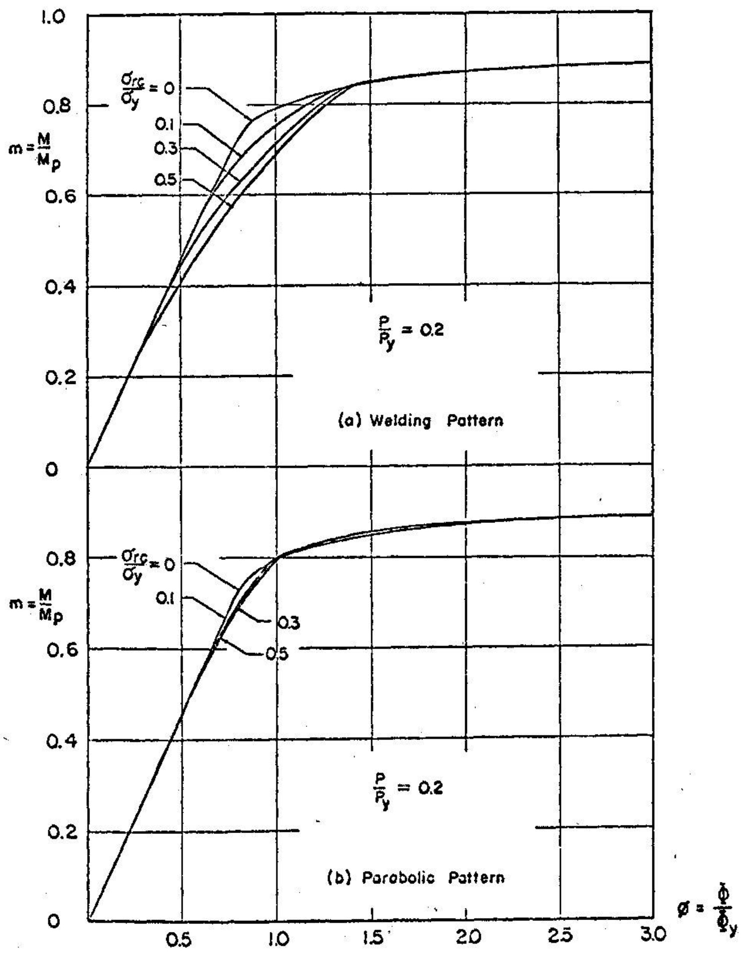

FIG. 2. - TYPICAL MOMENT-THRUST-CURVATURE RELATIONSHIPS

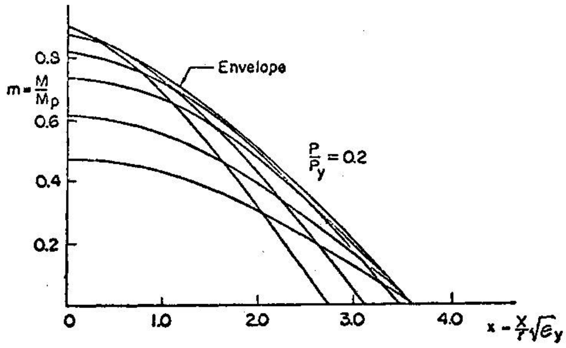

FIG. 4. - EQUILIBRIUM CURVES AND ENVELOPE

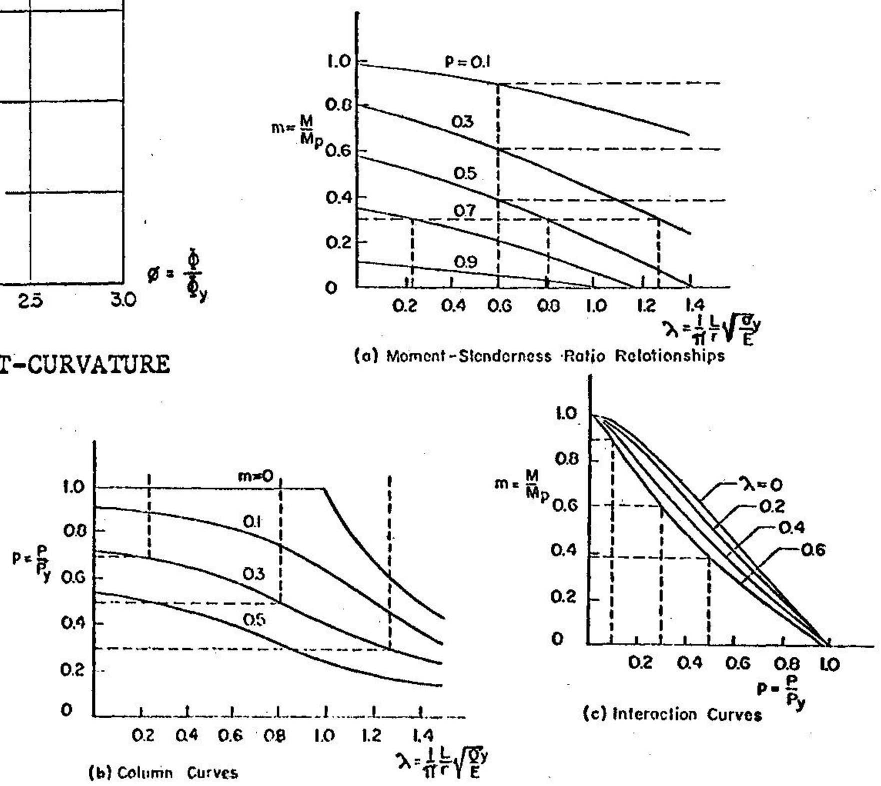

FIG. 5. - CONSTRUCTION OF COLUMN AND INTERACTION CURVES

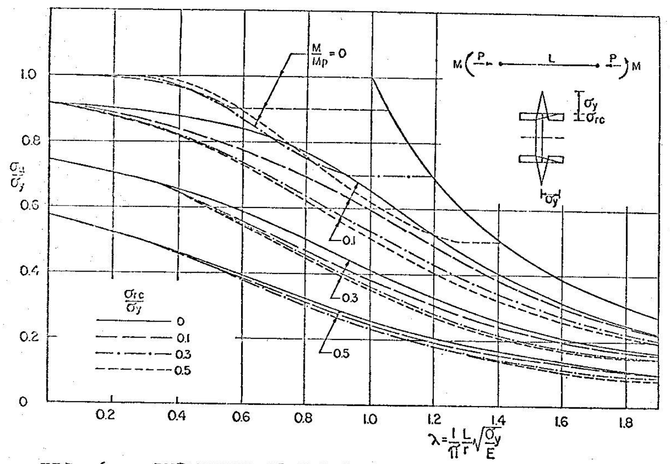

FIG. 6. - INFLUENCE OF WELDING TYPE RESIDUAL STRESSES

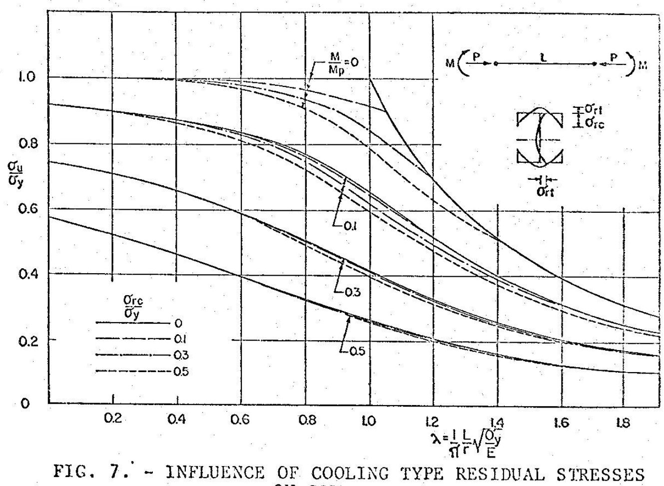

FIG. 7. - INFLUENCE OF COOLING TYPE RESIDUAL STRESSES ON COLUMN CURVES

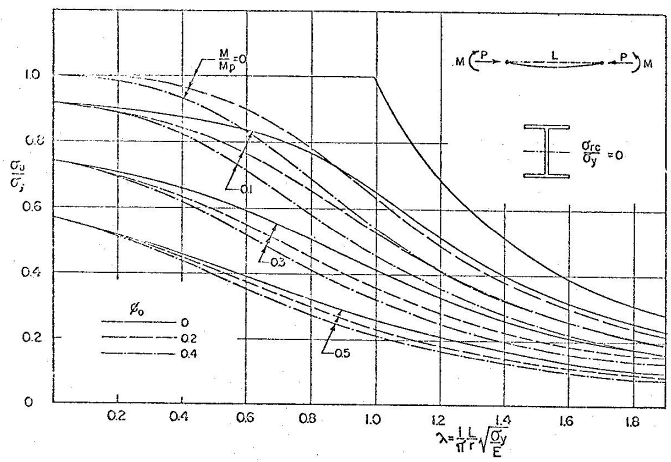

FIG. 8. - INFLUENCE OF INITIAL IMPERFECTION ON COLUMN CURVES

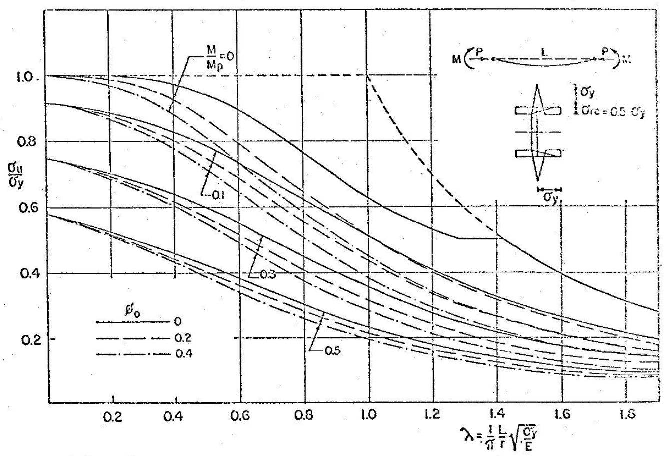

FIG. 9. - INFLUENCE OF INITIAL IMPERFECTION AND WELDING TYPE RESIDUAL STRESSES ON COLUMN CURVES

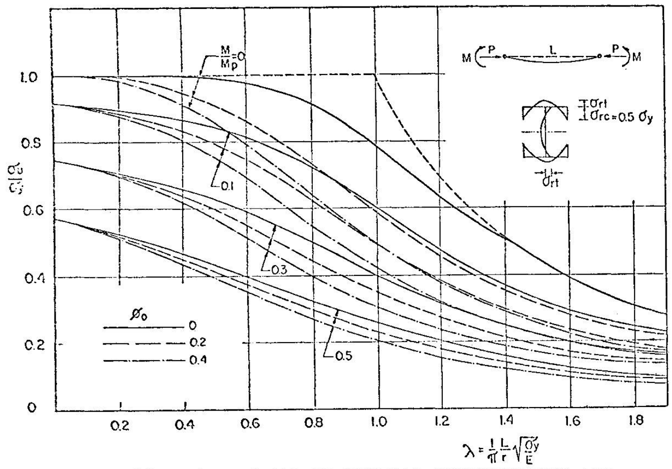

FIG. 10. - INFLUENCE OF INITIAL IMPERFECTION AND COOLING TYPE RESIDUAL STRESSES ON COLUMN CURVES

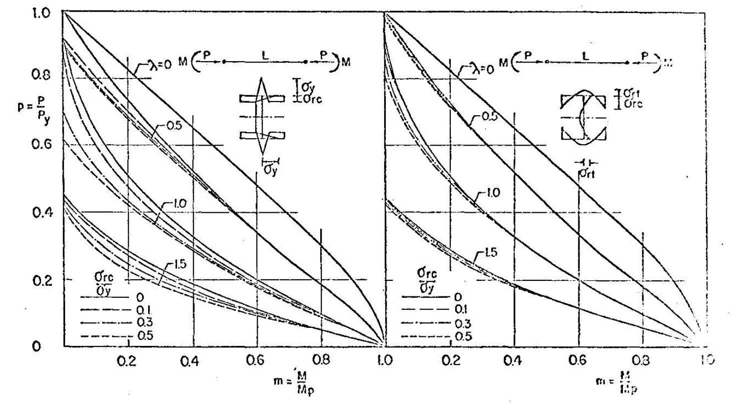

FIG. 11. - INFLUENCE OF RESIDUAL STRESSES ON INTERACTION CURVES

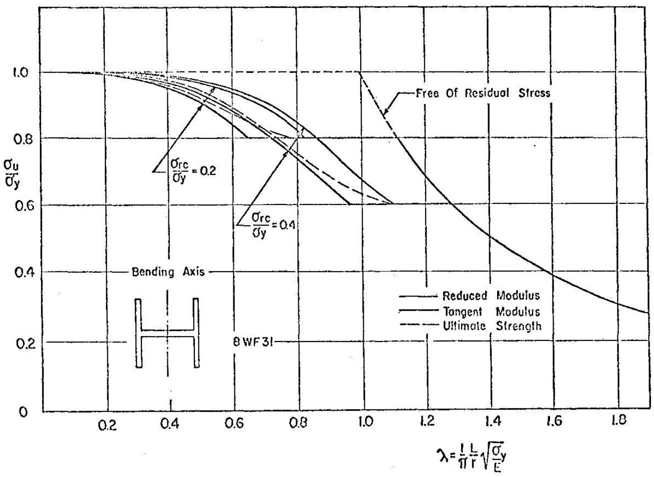

FIG. 12. - COMPARISON OF BUCKLING AND ULTIMATE STRENGTHS