Empirical formulation of multiple column strength curves

EMPIRICAL FORMULATION OF MULTIPLE COLUMN STRENGTH CURVES

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ABSTRACT

A procedure for the empirical formulation of a related set of steel column strength curves is presented. The formulas are developed in terms of correction functions to idealized critical load strengths. The method yields a family of column curves by altering a single coefficient. Its value, perhaps, lies more in the logic of the approach and the "way of thinking" that is introduced rather than in its feasibility for design application.

If strain hardening is neglected, the upper bound of column strength for structural steel with an assumed bilinear elastic-plastic stress-strain curve is either the yield load or the Euler Load. The proposed procedure embodies a certain rationality in that it "corrects" the upper bound strength. Two correction functions are introduced: (1) A function of column slenderness, C_L; and (2) A function of shape, column type, and fabrication process, C_S, as determined by the band of test results plotted against slenderness for a particular type of column. For a given column curve the shape function, C_S, will be a numerical constant.

For dimensionless plots the slenderness ratio, L/r, is replaced by the slenderness function, λ,

$$\lambda = \frac{L}{r}\frac{1}{\pi}\sqrt{\frac{\sigma_y}{E}} \tag{1}$$

and the Euler load,

$$P_e = \frac{P_y}{\lambda^2} \tag{2}$$

The reduction in strength from the idealized (upper bound) strength of a "perfect" structural steel column depends on a multiplicity of factors, amply discussed in other colloquium papers, and nearly always reaching a sharp peak when the Euler Load is equal to the yield load, or when λ is equal to unity.

In the proposed procedure the column strength is represented by one of the following two formulas:
for $\lambda \leqslant 1$,

$$P = (1 - C_S C_{L_1}) P_y \tag{3}$$

for $\lambda \geqslant 1$,

$$P = (1 - C_S C_{L_2}) P_e \tag{4}$$

Alternatively, introducing Eq. 2, Eq. 4 may be written,
for $\lambda > 1$,

$$P = \frac{1}{\lambda^2}(1 - C_S C_{L_2}) P_y \tag{5}$$

Thus the idealized strength of a structural steel column is used as a first approximation, corrected downward by C_S, a function of shape, fabrication process, plate thickness etc.,

$$C_S = 1 - \frac{P_{cy}}{P_y}$$, where P_{cy} is the column strength when $\lambda = 1$.

The shape function, C_S, is a single numerical coefficient for any given column strength curve, determined as indicated above. In some cases one curve may be indicated for a certain range of thickness, such as flange thickness of a rolled W shape, and another curve for a different range of thickness. In this case C_S may be made a variable, thus providing direct interpolation between curves and avoiding a sudden jump in column selection.

The numerical application of the procedure will be illustrated by closely approximating the median curve "b" recently recommended* by the European Convention of Constructional Steelwork. The curve is plotted on Figure 1 and the empirical formulation is based on four control points a, b, c, and d, as shown. Location c establishes

$$C_S = 0.4013^*$$

Letting C_{L1}, for $\lambda<1$ be represented by the quadratic,

$$C_{L1} = A + B\lambda + C\lambda^2 \qquad (6)$$

Passing Eq. 6 through control points a, b, and c,

$$C_{L1} = -0.1295 + 0.5270\lambda + 0.6025\lambda^2 \qquad (7)$$

For the region $\lambda>1$, the formulation for C_{L2} is taken as

$$C_{L2} = D + \frac{E}{\lambda} + \frac{F}{\lambda^2} \qquad (8)$$

The plot of curve "b" for $\lambda>1$, in Fig. 1, made use of control points c and d, with the added requirement that the slope be continuous at c with the curve for $\lambda<1$. Slope continuity requires that,

$$2D + 3E + 4F = \frac{2}{C_S} - B - 2C \qquad (9)$$

which gives for curve b

$$C_L = +0.0232 + \frac{0.7018}{\lambda} + \frac{0.2750}{\lambda^2} \qquad (10)$$

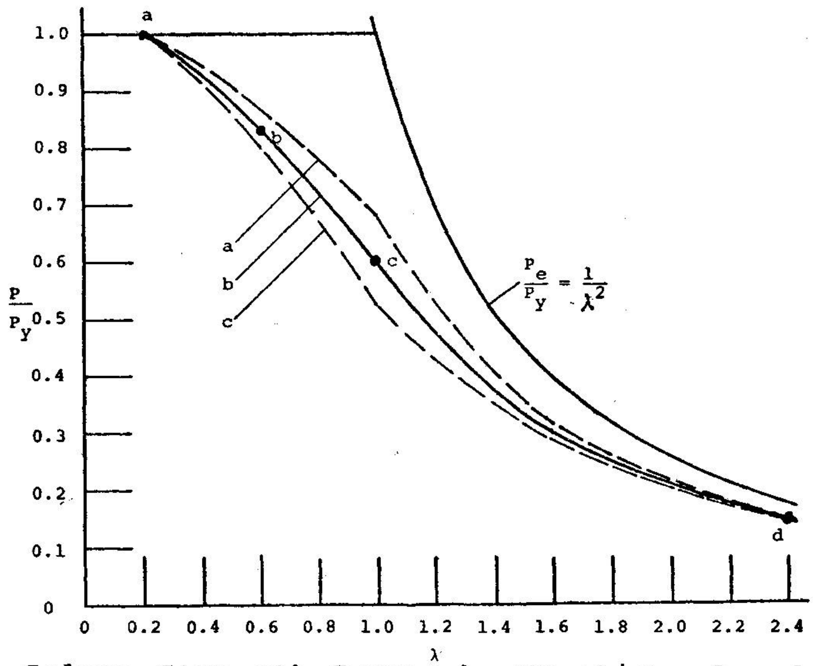

Fig. 1. Column Strength Curves by Equations 3 and 5

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*As per letter from Dr. Gerald Schulz, 12 December, 1972.

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A tabular comparison of the European Curve b values and those obtained from Equations 3, 5, 7, and 10 follows.

Values of P/P_y

λ	original curve b	approx-imation
0.2	1.0000	1.0000
0.4	0.9250	0.9287
0.6	0.8380	0.8380
0.8	0.7270	0.7280
1.0	0.5987	0.5987
1.2	0.4809	0.4718
1.4	0.3831	0.3741
1.6	0.3078	0.3014
1.8	0.2502	0.2468
2.0	0.2070	0.2055
2.2	0.1746	0.1735
2.4	0.1483	0.1482

For $\lambda<1$ the maximum deviation in the above tabulation is about 0.4%, for $\lambda>1$, about 2.4%. Better agreement for $\lambda>1$ could be obtained by permitting a very small slope discontinuity at $\lambda=1$.

Although a family of curves may be obtained simply by changing a single numerical coefficient, C_S, it may be seen by Eq. 9 that the condition for slope continuity is not independent of C_S. The result of changing C_S, alone, to obtain approximations of European Curves "a" and "c" is shown by the dashed lines on Figure 1. The differences in these curves is as much as 3%. Better agreement could be obtained, and slope discontinuities eliminated, by introducing new equations for C_L, but then the simple interrelationship would be lost. There are, of course, many other empirical equations that might be introduced within the overall framework of the procedure.

In summary, a method for systematising the formulation of empirical column strength curves for structural steel has been presented. The procedure is based on the concept of correcting the upper bound critical load and the effects of length are separated from the effects of shape and fabrication.