Buckling strength and design guide of welded, linearly tapered columns

BUCKLING STRENGTH AND DESIGN GUIDE OF WELDED, LINEARLY TAPERED COLUMNS

George C. Lee
Professor and Acting Chairman
Department of Civil Engineering
State University of New York at Buffalo
Buffalo, U.S.A.

ABSTRACT

This paper presents the results of both experimental and analytical research regarding the buckling strength and design guide for centrally loaded tapered columns. The columns considered are H-shaped sections with a linear variation in the cross-sectional depth and are fabricated by welding only on one side of the web.

The specific contents of this paper are :

1) Analytical elastic buckling solutions of tapered columns.

2) Residual stresses measured in tapered column specimens welded from both sheared and flame cut plate elements.

3) Analytical inelastic buckling solutions of tapered columns by considering the residual stresses.

4) Formulation of design guide including effective length factors for centrally loaded tapered columns.

1. INTRODUCTION

The results presented in this paper are a part of a comprehensive research program on the behavior of tapered structural members sponsored by the Naval Facilities Engineering Command, American Institute of Steel Construction, American Iron and Steel Institute, the Metal Building Manufacturer's Association, and the State Univeristy of New York at Buffalo. The study has primarily been concerned with linear, web tapered I-shapes as illustrated in Figure 1.

At first the research program was devoted to residual stress measurements in welded tapered I-shapes (Lee and Ketter, 1972). Next, an extensive analytical program was launched to determine the elastic stability (axial, lateral, and lateral-torsioned buckling) loads for such members in order to develop design recommendations (Lee et al, 1972). The philosophy of these recommendations was not to develop "new" formulas but to determine modification factors to the current American Institute of Steel Construction specifications. As a consequence, the proposed design formulas for inelastic behavior were based on Johnson's parabola and not on any analytical or experimental* data for tapered members.

Thus it is imperative to investigate the analytical behavior of inelastic columns. This paper describes the buckling strength of axial loaded tapered columns with emphasis on inelastic behavior.

2. RESIDUAL STRESSES IN FABRICATED TAPERED SHAPES

Since residual stresses have a profound effect on non-slender columns, it is necessary to discuss the results presented by Lee and Ketter. The program was divided into two series, Figure 2. Series A dealt with 20 fabricated sections from shear cut plates. Series B was broadened to include shear cut and flame cut plates, some of which were welded together to form 20 fabricated sections. By determining the residual stresses in the plates prior to welding and after welding, the effect of the welding process can be evaluated. In both series the flange to web weld was on one side only by an automatic welding process. The members were fabricated such that one flange was horizontal and the other was sloping in the longitudinal direction. The residual stresses were determined using the section method with an 8-inch gauge length. Measurements were recorded at four equally spaced panels along the length of the members (the first and last panels were 18 inches from their respective ends). In Series A the web sections in each panel were measured parallel to the horizontal flange. In Series B they were measured along a radius originating at the point of intersection of the two flanges, i.e. along an arc from the horizontal flange to the sloping flange. In Figure 3, representative residual stress measurements are illustrated. The shear cut and flame cut patterns are alike in three ways : (1) the respective patterns are similar to welded prismatic I-shapes, (2) they are unsymmetrical about the weak axis due to the one-side welding procedure, and (3) the flange pattern is independent of the depth of the member while the web has a variation with depth. The effect of the two cutting procedures appears only at the flange tips ; for the shear cut plates the flange tips are in compression and for the flame cut plates the tips are in tension. Comparing the unwelded plates with the fabricated member in Series B, the effect of welding increases the stress at the tips for shear cut plates and decreases the stress at the tips for flame cut plates. Also the effect of one-side welding causes the flanges to bow laterally.

---

*Concurrently with the analytical program, experiments were performed on tapered members to determine their bending strength (Prawel et al). The 15 beams tested do not supply positive proof for the inelastic bending formula (although all beams had greater strength than the design formulas predicted.) The author does not know of any published information regarding inelastic tapered column tests.

Two typical residual stress patterns derived from average values of all measurements (Figure 4). The distinguishing feature between residual stresses in tapered and prismatic members is reflected in the assumed patterns by allowing the value of stress in the central portion of the web to vary with the depth of the member. Both patterns are doubly-symmetric with tension yield stress at the flange-web connection and one-half compressive yield stress in the flange. The difference between the two patterns is the tension stress at the flange tips in the flame cut pattern.

3. BUCKLING STRENGTH

The elastic buckling strength of tapered columns was investigated using the Rayleigh-Ritz procedure. A ten term power series was assumed for the displacements and using the principle of virtual displacements, a homogeneous set of equations were derived. The non-trivial solution to these equations yielded the elastic axial buckling load. Since only the web depth varied with length, the weak axis buckling load was considered to be unaffected by the taper. However, the strong axis buckling load varies with the taper ratio. These results are reported by Lee and Al.

Using the analytical solutions for tapered columns, an elastic axial design formula was proposed. The philosophy behind the formula was not to generate a "new" formula if the present AISC column formula could be modified to account for tapered members. This has two advantages ; (1) the designer will be working with the familiar AISC formulas for prismatic columns with a modification factor included and (2) the designer can see the increase in strength of a tapered member over a prismatic member. The most likely way to incorporate tapered members into the prismatic formulas is by modifying the length of the tapered beam. Thus the buckling stress of a pin-ended prismatic column having the same smaller end cross section and length $g\ell$ is equated to the buckling stress (at the small end) of the original pin-ended tapered column, Figure 5 :

$$\frac{\pi^2 E}{(g\ell/r_o)^2} = \sigma_{taper} = \frac{P_{cr}}{A_o} \qquad (1)$$

where r_o is the radius of gyration of the smaller end. For weak axis buckling $g = 1.0$ and for strong axis buckling, it was proposed that

$$g = 1.00 - 0.375\gamma + 0.080\gamma^2 \ (1.00 - 0.0775\gamma) \qquad (2)$$

For taper ratios between zero and six, g is less than unity.

For slender columns that buckle elastically, Lee et al recommended equation (1) with an appropriate factor of safety. For non-slender columns that buckle inelastically, the "basic column curve" of the Column Research Council was recommended with the modifying factor g :

$$\sigma_{taper} = \left[1.0 \quad \frac{(g\ell/r_o)^2}{2C_c^2}\right]\sigma_y \ ; \quad \frac{g\ell}{r_o} \leqq C_c \qquad (3)$$

Where

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}}$$

The function g was derived for pin-ended columns. To include other support conditions an effective tapered length factor was developed from a four membered rectangular frame, Figure 6. The critical load was determined by using slope-deflection equations adopted to tapered members, and then the critical load was equated to the Euler buckling load for one column, i.e. ;

$$P_{cr} = \frac{\pi^2 EI_o}{(K_\gamma \ell)^2} \tag{4}$$

where $K_\gamma \ell$ is the effective tapered length and is interpreted as the length of an equivalent pin-ended prismatic column having a cross section equal to the smaller end of the tapered column. Some typical results for the effective tapered length factor are shown in Figure 7 and 8 for frames without side-sway and frames with side-sway, respectively. The quantities R_T and R_B are a measure of the restraints at the column top and bottom. To represent a pin-ended column, the top and bottom beams would have zero moments of inertia, thus $R_T = R_B = \infty$. If the top and bottom beams had an infinite moment of inertia, then $R_T = R_B = 0$.

In comparing tapered columns to prismatic columns, the effective length is seen to decrease with increasing taper ratio. Thus it is possible to have effective tapered length factors less than 0.5 when side-sway is prevented and less than 1.0 when side-sway is permitted.

For columns which are not simply supports equations (1) and (3) apply if g is replaced by K_γ :

for $K_\gamma \ell / r_o \geqq C_c$

$$\sigma_{taper} = \frac{\pi^2 E}{(K_\gamma \ell / r_o)^2} \tag{5}$$

for $K_\gamma / r_o \leqq C_c$

$$\sigma_{taper} = \left[1 - \frac{(K_\gamma \ell / r_o)^2}{2C_c^{\,2}} \right] \sigma_y \tag{6}$$

Figure 9 contains a graph of equations (5) and (6) for different yield stress levels and also includes the AISC factor of safety used for prismatic columns.

The inelastic column curve, equation (6), is not based on any analytical or experimental solution for tapered columns. It resulted from the philosophy of the design of prismatic columns and the desire to modify the prismatic formulas to account for tapered columns. The remainder of this paper will develop the procedure used to determine inelastic tapered column buckling loads and compare these results to the proposed design formula, equation 3.

The weak axis inelastic buckling load will be essentially independent of the taper, since the web has very little effect on the weak axis moment of inertia. Thus for weak axis buckling the column can be considered prismatic. The strong axis inelastic buckling load can be determined from

$$B_x(z) \frac{d^2 v}{dz^2} + Pv = 0 \tag{7}$$

where v is the strong axis deflection and B_x is the bending rigidity, e.g. $B_x(z) = EI_x(z)$ if the column is elastic. When the column starts to yield the bending rigidity becomes a complex function of z and P.

If the tangent modulus concept is used,

$$B_x(z) = \int_{Area} E_t\, y^2\, dA \tag{8}$$

where E_t is the tangent modulus. I residual stresses are present and the material is idealized as perfectly elastic-plastic, then equation (8) may be written as,

$$B_x(z) = EI_{c_x}(z) \tag{9}$$

where I_{c_x} is the moment of inertia of the elastic core at a point z along the column. Due to the complexity of equation (7), numerical procedures must be used. The particular procedure used herein is the Finite Element Method. If the column is divided into n elements such that over each element the bending rigidity is constant, then equation (7) can be transformed into the Finite Element formulation (Lee and Morrell) :

$$[K]\begin{Bmatrix} V_i \\ \theta_i \\ V_j \\ \theta_j \end{Bmatrix} - [N]\begin{Bmatrix} V_i \\ \theta_i \\ V_j \\ \theta_j \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

where

$$[K] = B_x \begin{bmatrix} 12/\ell^3 & & \underline{SYM} & \\ -6/\ell^2 & 4/\ell & & \\ -12/\ell^3 & 6/\ell^2 & 12/\ell^3 & \\ -6/\ell^2 & 2/\ell & 6/\ell^2 & 4/\ell \end{bmatrix}$$

and

$$[N] = P \begin{bmatrix} 6/5\ell & & \underline{SYM} & \\ -1/10 & 2\ell/15 & & \\ -6/5\ell & 1/10 & 6/5\ell & \\ -1/10 & -\ell/30 & 1/10 & 2\ell/15 \end{bmatrix}$$

where v_i and θ_i are the deflection and rotation degrees of freedom at node i. Since yielding will begin at the smaller end in tapered columns and progress towards the larger end, elemental lengths should be considerably smaller in the yielded portion than in the elastic portion *.

The bending rigidity in each element was determined by using Alvarez and Birnstiel's method. At each node the cross-section is divided up into a grid of "fibers".+ Each fiber m has dimensions ΔX_m and ΔY_m. The axial load is applied in increments. Within each increment the internal axial force is calculated through an iterative method until it equilibrates the applied axial load at that node. When the internal axial force is in equilibrium with the applied axial force at each node, the resulting bending rigidities are averaged and the elemental matrices are calculated and assembled for a given applied axial force. The determinate of the reduced master matrix will indicate if the column has buckled. If the determinate does not change sign from the previous increment, then another increment is applied and the process is repeated until the determinate is zero or nearly zero.

In the process of equilibrating the internal axial force, each increment in axial load is assumed to strain the elastic core remaining after the previous iteration :

$$\Delta\varepsilon_a = \frac{-\Delta P}{EA_c} \tag{11}$$

where $\Delta\varepsilon_a$ is the increment in axial strain due to the axial load ΔP and A_c is the elastic core area. The total strain is obtained by adding the preceding axial force strain and the residual strain to this increment, $\Delta\varepsilon_a$. Using the total strain, the stress and tangent modulus in each fiber can be computed from the stress-strain curve. Knowing the stress and tangent modulus, the internal axial force can be calculated and compared with the applied force ; and the bending rigidities can be determined. If the internal axial force does not equal the applied force then the process is repeated.

Using the above procedure strong axis buckling curves were obtained for a tapered column having a typical small end cross section. Three different taper ratios and two different residual stress patterns were considered, Figure 4. The results with the shear cut residual stress pattern are presented in Figure 10 and the results with the flame cut residual stress pattern are presented in Figure 11. For the prismatic column (γ = 0), the results are similar to those obtained by McFalls and Tall. Also shown on these figures are equations (1) and (3), the proposed design formulas. The tapered column solutions lie between the prismatic solutions and the design formula in the inelastic range (C_c - 117). Thus as the taper ratio is increased the design formulas become more accurate in predicting the column's behavior. This can be explained by the fact that the yielding is confined to the smaller end region, i.e., the penetration of yielding measured from the smaller end at incipient buckling is decreased as the taper is steepened.

4. SUMMARY AND CONCLUSIONS

Analytical solutions for linearly tapered I-shape columns have been described. The major emphasis of this paper was to present inelastic column solutions and compare them with the proposed design formulas (for detailed treatment of elastic tapered columns see Lee et al).

---

* Generally the column was divised into 12 elements with four elements at the smaller end having lengths equal to 1/36th of the total column length and the remaining eight elements had lengths of 1/9th the total length (for γ = 4 the total number of elements was increased to 15).

\+ At each node, each flange was divided into 400 "fibers" and the web was divided into 80 "fibers".

Typical residual stress patterns were presented for fabricated sections composed of shear cut plates or flame cut plates. A representative pattern was assumed for each and used in the analytical investigation of inelastic tapered column strength. Column curves were obtained by the solution procedure described herein using the Finite Element Method to find the critical loads. The results indicated that the effect of tapering a column moves the theoretical column curve closer to the design formula, equation (3) or (6).

ACKNOWLEDGMENT

The author is indebted to his colleagues and research collaborators, Professor Robert L. Ketter and Professor Michael L. Morrell, for their significant contributions to the research program of study in which the results presented herein are a part.

5. REFERENCES

1. Lee, G.C. and Ketter, R.L.
RESIDUAL STRESSES IN WELDED TAPERED SHAPES
Civil Engineering Research Report, SUNYAB, Feb. 1972

2. Lee, G.C., Morrell, M.L. and Ketter, R.L.
DESIGN OF TAPERED MEMBERS
Welding Research COuncil Bulletin, No. 173, June 1972

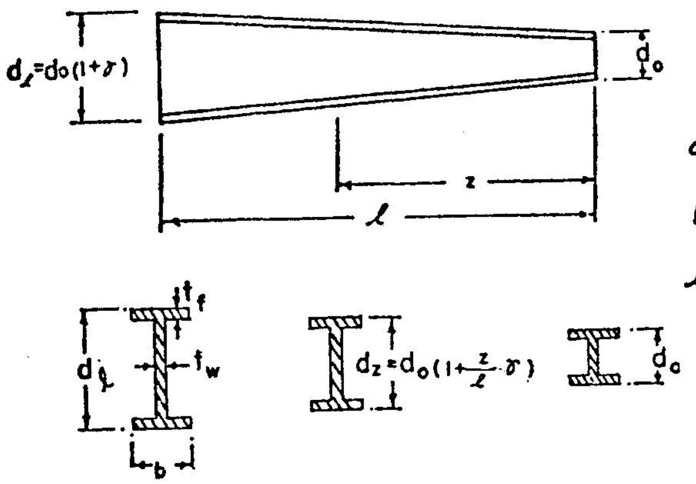
3. Prawel, S.P., Morrell, M.L. and Lee, G.C.,
BENDING AND BUCKLING STRENGTH OF TAPERED STRUCTURAL MEMBERS
to appear in J. Exp. Mech.

4. Alvarez, R.J. and Birnstiel, C.,
INELASTIC ANALYSIS OF MULTI-STORY, MULTI-BAY FRAMES
ASCE J. Struct. Div., Vol. 95, Nov. 1969

5. Lee, G.C. and Morrell, M.L.
FINITE ELEMENT ANALYSIS OF SPACE FRAMES OF THIN-WALLED MEMBERS
Civil Engineering Research Report, SUNYAB, Feb. 1972

6. McFalls, R.K. and Tall, L.
A STUDY OF WELDED COLUMNS MANUFACTURED FROM FLAME-CUT PLATES
Welding Research Supplement, pp. 141-s-153-s, April 1969

TAPERED BEAM GEOMETRY



$d_o = 6''$, $d_\ell = 6'' \sim 42''$, $t_w = 0.10 \sim 0.25$

$b = 6''$, $t_f = 0.25'' \sim 0.75''$

$\ell = 96'' \sim 432''$

Fig. 1 -General tapered geometry-

SERIES A : SHEAR CUT EDGES

ℓ	d_o	d_ℓ	No. of beams	No. of panels
100"	6"	12"	2	4
97	6	16	2	4
98	10	36	3	3

25 - SHEAR CUT MEASUREMENTS

SERIES B : SHEAR AND FLAME CUT EDGES

ℓ	d_o	d_ℓ	No. of beams		No. of panels
			S.C.	F.C.	
120"	6"	6"	1	1	4
120	6	12	1	1	4
120	6	18	1	1	4
120	6	24	1	1	4
120	24	24	1	1	4

20 - SHEAR CUT MEASUREMENTS

20 - FLAME CUT MEASUREMENTS

Fig. 2 Summary of residual stress measurements

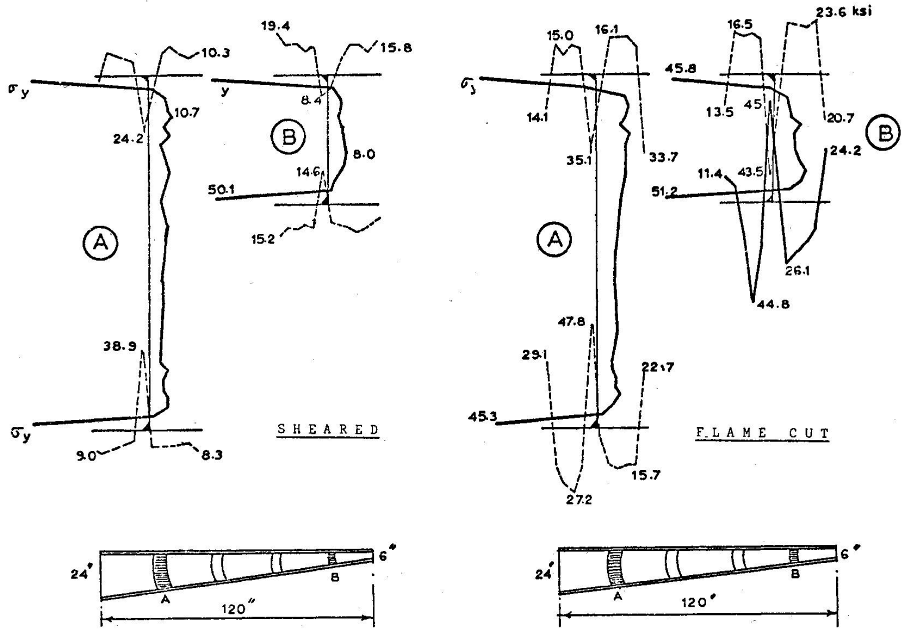


FIGURE 3 : WELDED SECTIONS

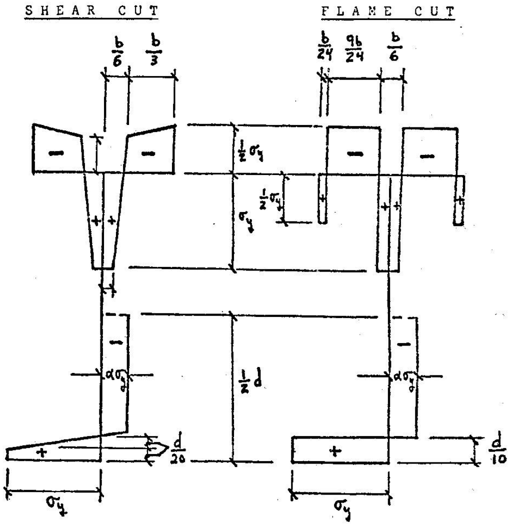


FIGURE 4 : ASSUMED RESIDUAL STRESS PATTERNS

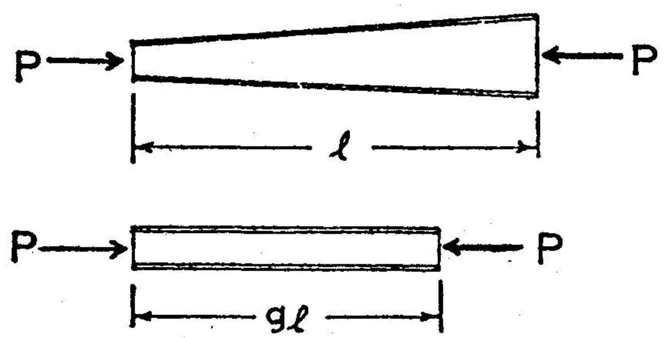


FIGURE 5 : DEFINITION OF THE LENGTH MODIFICATION g FOR COLUMN

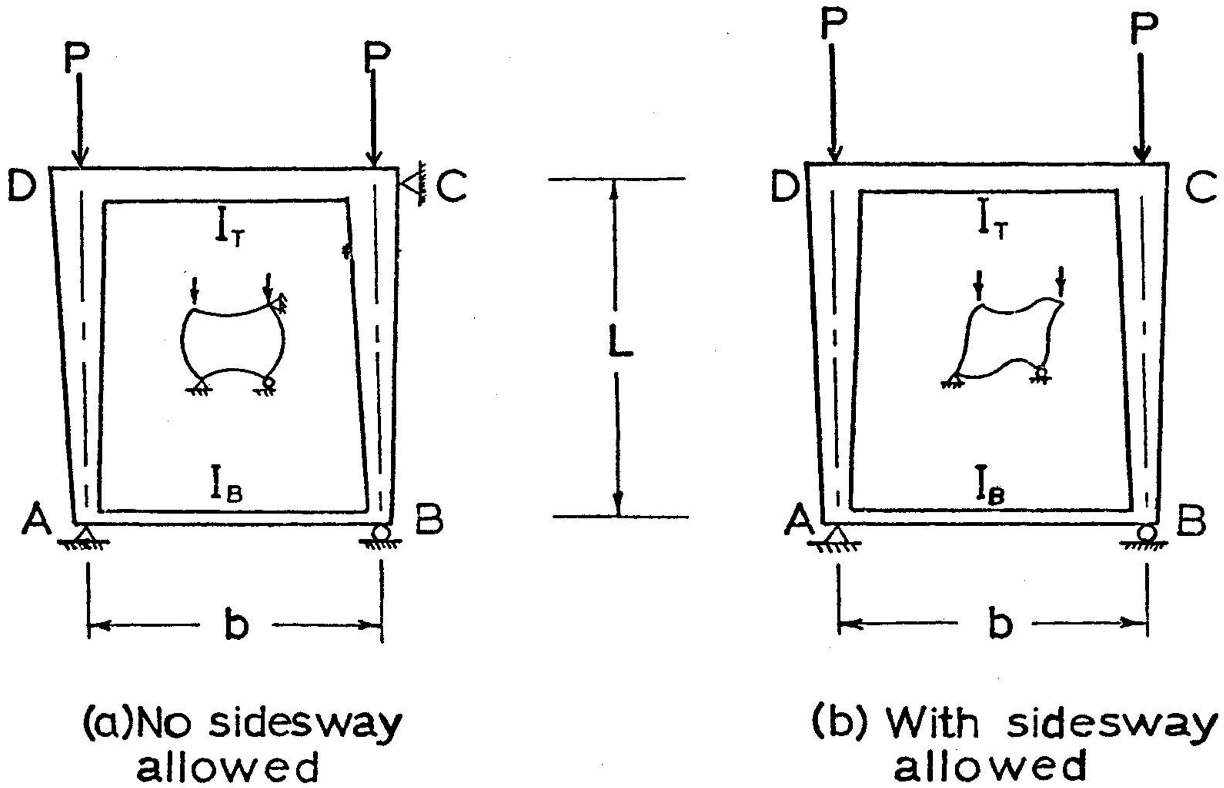
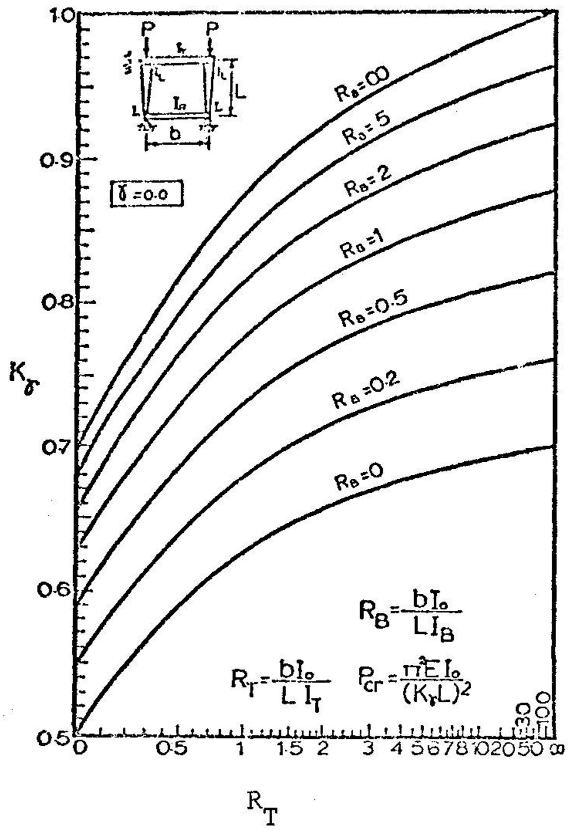
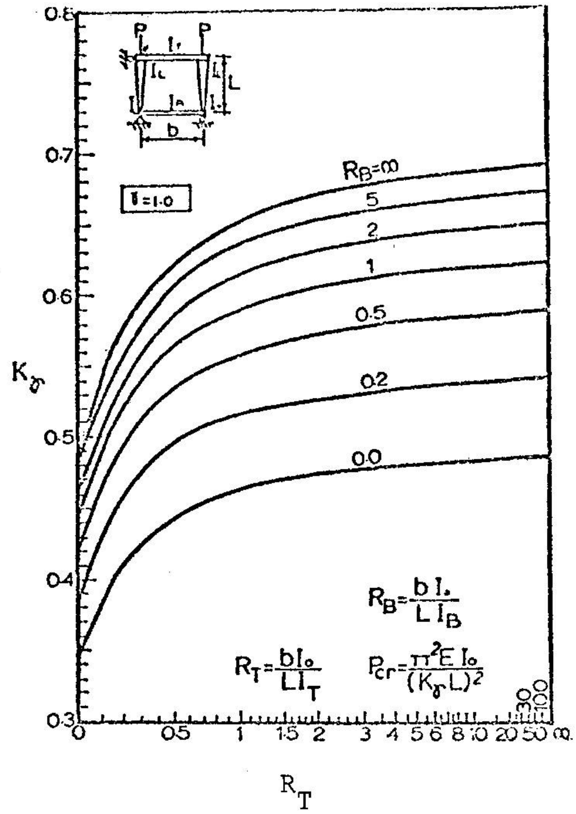


FIGURE 6 : STRUCTURAL MODELS USED FOR DETERMINATION OF THE EFFECTIVE LENGTH FACTORS OF TAPERED COLUMNS IN FRAMES

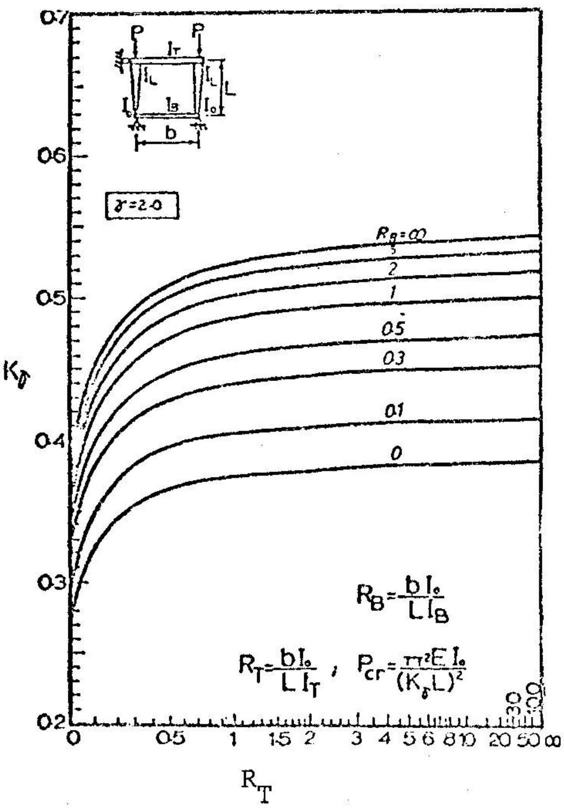
FIGURE 7



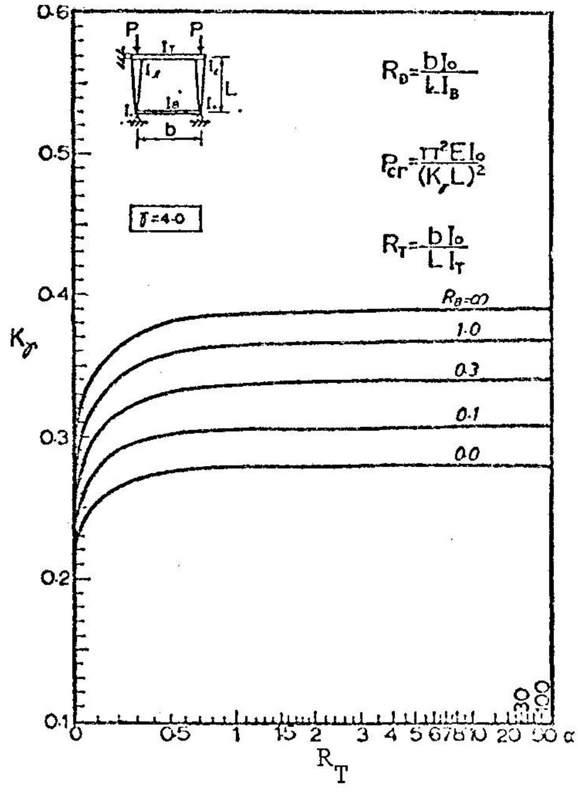
Effective length factors for tapered columns : side-sway prevented $-\gamma$ = 0



Effective length factors for tapered columns : side-sway presented $-\gamma$ = 1.0

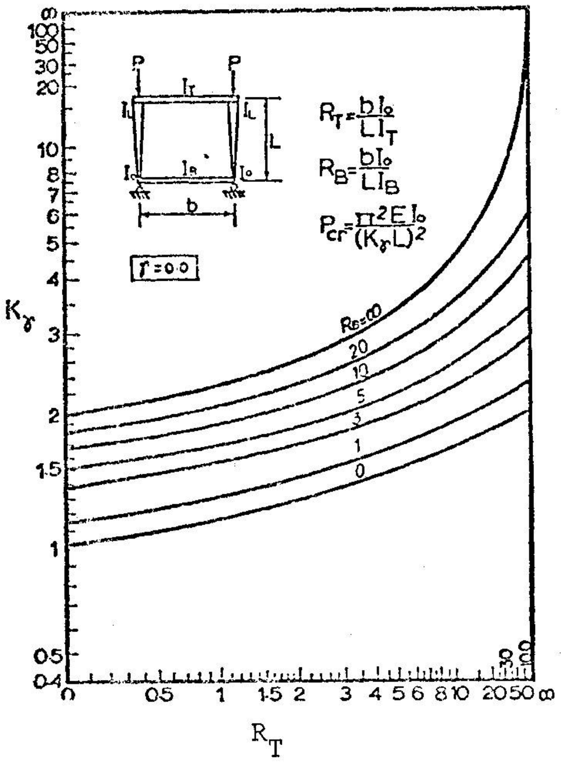


Effective length factors for tapered columns : side-sway prevented $-\gamma$ = 2.0

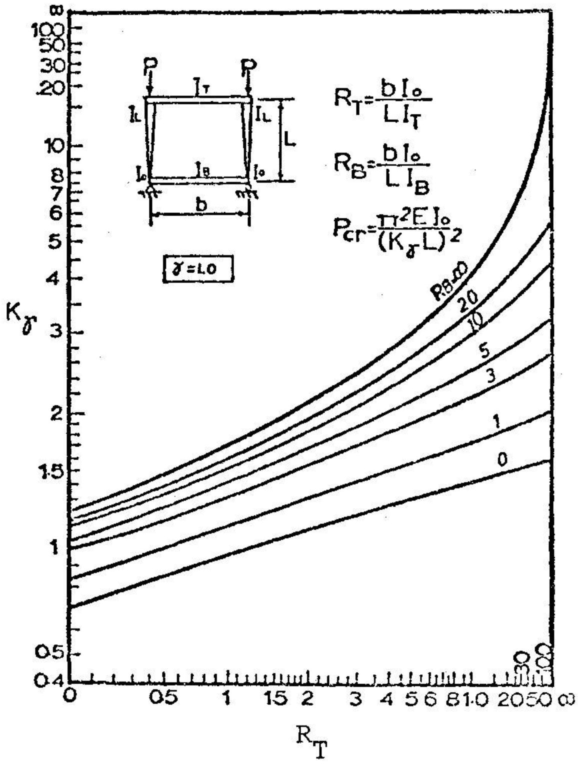


Effective length factors for tapered columns : side-sway prevented $-\gamma$ = 4.0

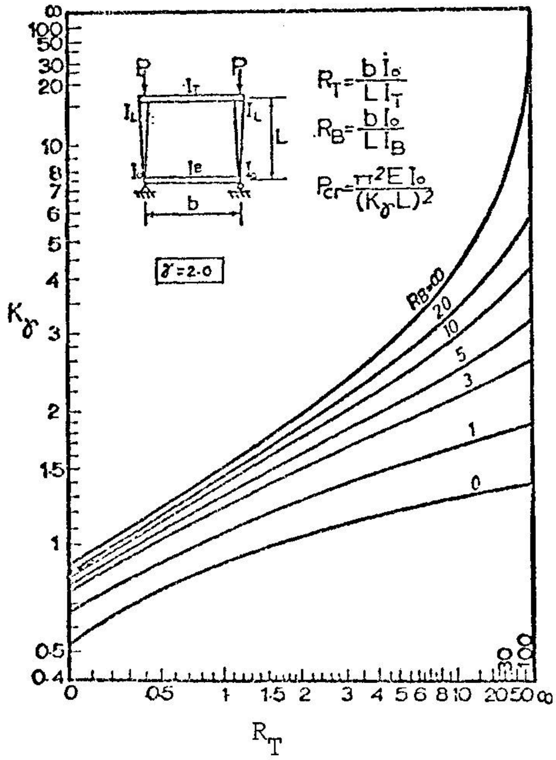
FIGURE 8



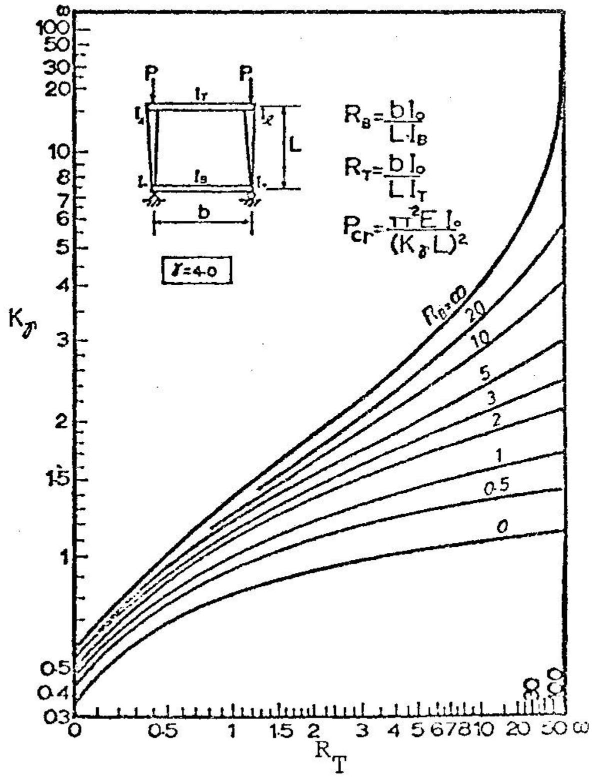
Effective length factors for tapered columns : side-sway permitted $-\gamma = 0$



Effective length factors for tapered columns : side-sway permitted $-\gamma = 1.0$



Effective length factors for tapered columns : side-sway permitted $-\gamma = 2.0$



Effective length for tapered columns : side-sway permitted $-\gamma = 4.0$

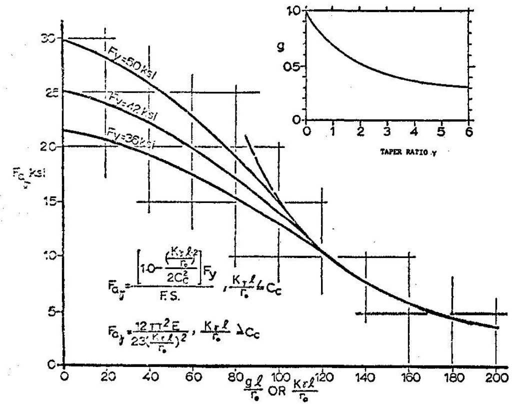


FIGURE 9 : AXIAL DESIGN CURVE FOR TAPERED COLUMNS

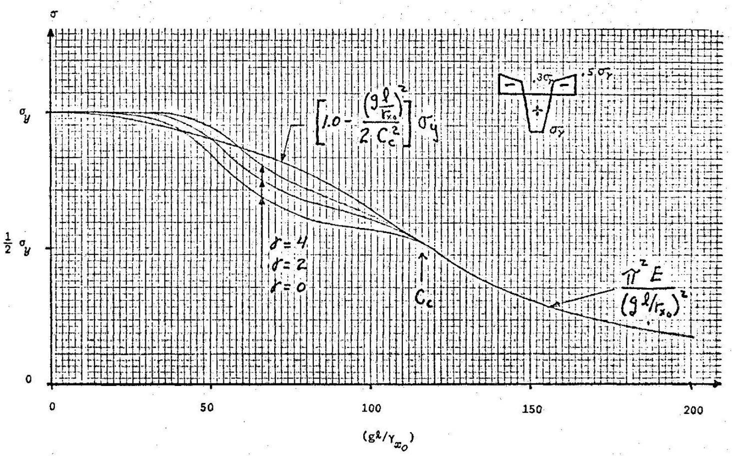


FIGURE 10 : STRONG AXIS — SHEAR CUT EDGES

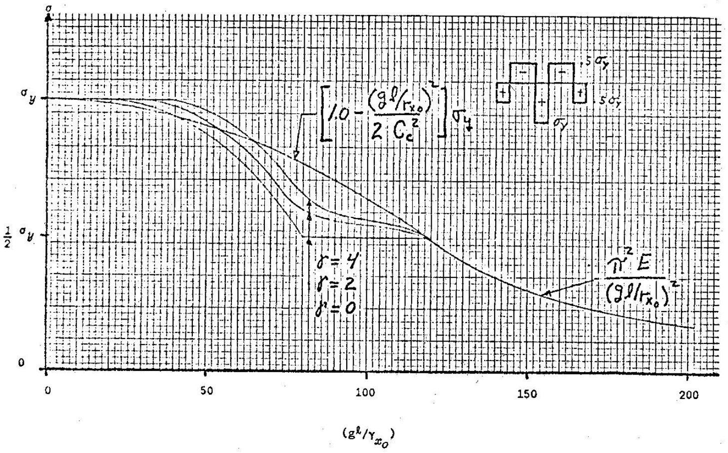


FIGURE 11 : STRONG AXIS — FLAME CUT EDGES