Assessment of concrete strength by means of non-destructive methods

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Evaluation de la résistance du béton au moyen de méthodes non destructives

Ermittlung der Betonfestigkeit mit Hilfe zerstörungsfreier Prüfmethoden

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SUMMARY

The purpose of this paper is to present a method for assessing hardened concrete strength by means of non-destructive testing methods. According to the Bayesian approach, the mean and the variance of the concrete compressive strength as well as its characteristic value are considered as random variables. The information of prior distribution supplied by non-destructive testing methods may be up-dated by means of data obtained by core tests. One example of the procedure is reported.

RESUME

L'exposé présente une méthode pour évaluer la résistance du béton au moyen de méthodes d'essai non destructives. D'après la méthode de Bayes, la moyenne et la variance de la résistance à la compression du béton aussi bien que sa valeur caractéristique sont considérées en tant que variables aléatoires. L'information de la distribution initiale obtenue moyennant des méthodes d'essai non destructif peut être mise à jour sur la base de données obtenues à l'aide d'essais sur des carottes. Un exemple du procédé employé est présenté.

ZUSAMMENFASSUNG

Zweck dieser Ausführungen ist die Veranschaulichung einer Prüfmethode zur Ermittlung der Betonfestigkeit durch Anwendung nicht-destruktiver Verfahren. Laut Bayes'Methode sind der Mittelwert und die Varianz der Druckfestigkeit von Beton sowie dessen Eigenwert als stochastische Variablen zu betrachten. Die Informationen aus der herkömmlichen Verteilung, ergänzt durch nicht-destruktive Prüfmethoden, können wirksam durch die Daten aktualisiert werden, die mittels Kernproben ermittelt werden.

NOTATIONS.

$n, \overline{x}, s^2$	=	size, mean and variance of the core sample respectively;
$n', \overline{x}', s'^2$	=	parameters of the prior joint density function;
$n'', \overline{x}'', s''^2$	=	parameters of the posterior joint density function;
$L(m, \sigma / x_1 \ldots x_n)$	=	likelihood function;
$\mu_{(\,)}, s^2_{(\,)}$	=	mean and variance of the variable indicated by the suffix ( );
M, Σ, Ψ^*	=	random variables denoting mean value, standard deviation and characteristic value of the strength;
m, σ	=	values of M and Σ respectively;
$\overline{R}_c, \delta, R_{ck}$	=	deterministic mean, standard deviation and characteristic value of the compressive strength.

1. INTRODUCTION.

Non-destructive methods to assess the mechanical properties of concrete in reinforced concrete structures are generally used when one or more compliance controls with specifications (Model--Code FIP/CEB, D.M. 26.3.1980) result negative or in case of a change of destination or damage in the structure caused by an earthquake(*). The mechanical parameter currently used to measure the properties of the material is the "characteristic compressive strength" R_{ck} defined by the following relation:

$$R_{ck} = \overline{R}_c - k\delta \tag{1}$$

where $\overline{R}_c$ = mean value and δ = standard deviation.

An investigation carried out by means of non-destructive methods should therefore provide the values of the above statistical parameters, considering that they must relate to concrete of "homogenous mixture". On the basis of experiments carried out in the past by other researchers besides the authors, reliable assessment of the casts'homogeneity can be obtained by limiting the coefficient of variation $C_v = \delta / R_c$ by 10%. For this purpose, the empirical correlations (Swedish SS 137352, Czechoslovack CSN 732411-76, Rumanian C.30-67, RILEM 1973 Standards) existing between compressive strength and non-destructive parameters can be used, which relate to the following methods:

- ultrasonic pulse test;
- surface hardness test (rebound hammer);
- penetration resistance test (Windsor Probe System);
- pull-out test (Lok Test, Capo Test, PI Test, Chabowski's method);
- combined methods.

Such correlations, however, are not generally valid to assess compressive strength, since they are determined by means of tests on concrete of standard mixture different from the one to be examined. First-estimate values are currently modified on the basis of a total coefficient of influence, which can be defined according to the information, in case it is available, about the mixture of in-situ concrete or on the basis of the results of tests carried out on a limited core sampling [1 - 4].

(*) The use of non-destructive methods for this purpose, for instance, is provided for by the final document required by the University of Bologna - Emilia Romagna Region - Basilicata Region Convention.

When data obtained by core tests become available, they provide additional information and uncertainty associated with estimate decreases as the core sample size grows. However, the damage of the structure and the cost of the experiments increase steadily as the independent observation $(x_1, \dots, x_n)$ grows.

Thus, when there is uncertainty, it may not be necessarily economical to obtain more information. In many situations, the size of the sample minimizing the difference between the value of information and its cost, may be determined [5].

The results provided by any set of observations on cores drilled in-sity may be dealt with according to Bayes'theorem. In the Bayesian approach the unknown parameters (the mean value $\overline{R}_c$ and the variance δ in this case) are considered as random variables. The argument of Bayes requires a *prior probability distribution* on the parameters. The distribution is assigned by the engineer on the basis of his professional assessments of all related information available. Experimental observations, which are completely summarized by the likelihood function, are used in order to modify the prior distribution of the parameters. The distribution obtained after taking the sample is called *posterior distribution.* The two density distributions are related, according to Bayes'theorem. By denoting with $\underset{\sim}{\Lambda} = (\overline{R}_c, \delta)$ the vector of the parameters, we have:

$$\begin{pmatrix}\text{Posterior proba-}\\ \text{bility of } \underset{\sim}{\Lambda}\text{, gi-}\\ \text{ven the sample}\end{pmatrix} = \begin{pmatrix}\text{normalizing}\\ \text{constant}\\ \ \end{pmatrix} \times \begin{pmatrix}\text{likelihood of}\\ \text{the sample, gi-}\\ \text{ven } \underset{\sim}{\Lambda}\end{pmatrix} \times \begin{pmatrix}\text{prior proba-}\\ \text{bility of } \underset{\sim}{\Lambda}\\ \ \end{pmatrix} \qquad (2)$$

in which the sample likelihood function $L(\underset{\sim}{\Lambda}/x)$ is the probability of the observed sample $\underset{\sim}{x} = (x_1, \dots, x_n)$, given $\underset{\sim}{\Lambda}$. The Bayesian treatment permits us to deal with uncertainty connected with the determination of the strength of in-situ concrete as well as to take a decision on the basis of the posterior distribution instead of the prior one. Information on prior distribution can be supplied by the results of a non-destructive testing method.

In the general case of a normal process where no parameter is known, the form of the joint-likelihood function of $M = \overline{R}_c$ and $\Sigma = \delta$ may be expressed as the product of a normal density and a gamma density. It can be shown, in this way, that the prior $f'_{M,\Sigma}(m, \sigma)$ and the posterior $f''_{M,\Sigma}(m, \sigma)$ joint-density functions are of the same form.

The posterior distribution will conform just to the likelihood function when prior information is vague and the sample size is large, or when the prior distribution is relatively flat compared to the sample-likelihood function.

The marginal distributions on the unknown mean value $M = \overline{R}_c$ as well as standard deviation $\Sigma = \delta$ may be found by integration and the posterior synthetic values of the above random variables M and Σ can be determined.

The characteristic value of the strength will also be treated as a random variable.

As a conclusion of the work, examples of application of the above procedure on the basis of investigations carried out in-situ by the authors is reported.

2. MATHEMATICAL THEORY.

When neither parameter m or σ of the normal distribution is known, the likelihood function may be written as follows [6]:

$$L(m, \sigma/x_1, x_2, \ldots, x_n) = \left\{\frac{1}{\sigma} \exp\left[-\frac{1}{2}\left(\frac{m-\bar{x}}{\sigma\sqrt{m}}\right)^2\right]\right\}\left[\frac{1}{\sigma^{n-1}} \exp\left(-\frac{n-1}{2}\frac{s^2}{\sigma^2}\right)\right] \quad (3)$$

in which $\bar{x}$ and s^2 are the sample statistics:

$$\bar{x} = \Sigma\, x_i/n \ , \qquad s^2 = \frac{1}{n-1}\, \Sigma\,(x_i - \bar{x})^2 \quad (4)$$

and n is the number of observations.

The joint conjugate prior distribution on m and σ is:

$$f'_{M,\Sigma}(m,\sigma) = \left\{\frac{1}{\sqrt{2\pi}\ \sigma/\sqrt{n'}} \exp\left[-\frac{1}{2}\left(\frac{m-\bar{x}'}{\sigma/\sqrt{n'}}\right)^2\right]\right\} \times$$

$$\times \left\{\frac{\left(\frac{n'-1}{2}\right)^{\frac{n'-2}{2}}}{\Gamma\left(\frac{n'-2}{2}\right)}\ \frac{2}{s'}\left(\frac{s'^2}{\sigma^2}\right)^{\frac{n'-1}{2}} \exp\left(-\frac{n'-1}{2}\frac{s'^2}{\sigma^2}\right)\right\} \quad (5)$$

with parameters n', $\bar{x}'$, s'^2, where n' may be interpreted as an equivalent prior sample size.

The posterior joint density function is of the same form as the prior one:

$$f''_{M,\Sigma}(m,\sigma) = N\ \ f'_{M,\Sigma}(m,\sigma)\ \ L(m, \sigma/x_1, x_2, \ldots, x_n) \quad (6)$$

in which N is a normalizing constant with parameters n'', x'', s''^2 which may be calculated in the sequential order:

$$n'' = n + n'$$

$$\bar{x}'' = (n\bar{x} + n'\bar{x}')/(n + n') \quad (7)$$

$$s''^2 = [(n-1)s^2 + (n'-1)s'^2 + n\bar{x}^2 + n'\bar{x}'^2 - n''\bar{x}''^2]/(n''-1)$$

The marginal posterior distributions of M and Σ may be obtained from Eq. 5 by integration.

The posterior mean and variance of M are:

$$\mu_M = \bar{x}''$$

$$s_M^2 = s''^2\, \frac{n''-1}{n''(n''-2)} \quad (8)$$

The posterior mean and variance of the standard deviation are:

$$\mu_\Sigma = s''\sqrt{\frac{n''-1}{2}}\ \frac{\Gamma[(n''-3)/2]}{\Gamma[(n''-2)/2]} \qquad n'' > 3 \quad (9)$$

$$s_{\Sigma}^2 = s''^2 \frac{n''-1}{n''-4} - \mu_{\Sigma}^2 \qquad n'' > 4 \tag{10}$$

From a classical point of view, the characteristic value R_{ck} of the strength may be expressed as in eq. (1), in which k = 1,64 when the strength is normal-distributed and a 5% fractile is considered. From a Bayesian point of view, the strength parameters M, Σ are treated as random variables, each of which possesses a distribution with parameters μ_M, s_M and μ_Σ, s_Σ respectively. Consequently, the characteristic value $\Psi^* = R_{ck}$ of the strength has also to be considered as a random variable with parameters μ_{Ψ^*} and s_{Ψ^*}. When the random variables M and Σ are supposed to be normal-distributed and independent, on the basis of the (1) we have:

$$\Psi^* = M - k\,\Sigma \tag{11}$$

and consequently:

$$\mu_{\Psi^*} = \mu_M - k\,\mu_\Sigma \tag{12}$$

$$s_{\Psi^*} = \sqrt{s_M^2 - k^2 s_\Sigma^2} \tag{13}$$

3. APPLICATIONS.

The example here reported refers to an investigation carried out by the authors on a building, the plan of which is reported in Fig. 1. The building is composed of 19 stories and is made up of elements cast in-situ as well as of precast elements. In this work we have reported only the results concerning beams and columns cast in-situ, which belong, according to the instructions provided by the design, to the same strength class. The homogeneity of the concrete was confirmed by the results.

Three positions for each structural element were tested by means of the following methods:

- ultrasonic pulse test (frequency 50 KHz);
- surface hardness test (sclerometer Schmidt type N);
- penetration test (Windsor Probe System)

in accordance with the ASTM Standards.

In order to assess the strength, the results of the first two methods were combined according to the following relation [8]:

$$R_c = 7.55 \times 10^{-11} \times I_r^{1.4} \times V_L^{2.6} \tag{14}$$

where R_c is the compressive cubic strength in $N \cdot mm^{-2}$, I_r the rebound index and V_L the ultrasonic pulse velocity in $m \cdot sec^{-1}$ (Fig. 2). Such relation reproduces the constant strength curves proposed in [6] for a concrete with standard mix proportions. The estimate of the compressive cubic strength by means of the penetration test was carried out on the basis of the tables provided by the builder. The results are reported in Fig. 3. Thirty structural elements were tested in all, to each of which we

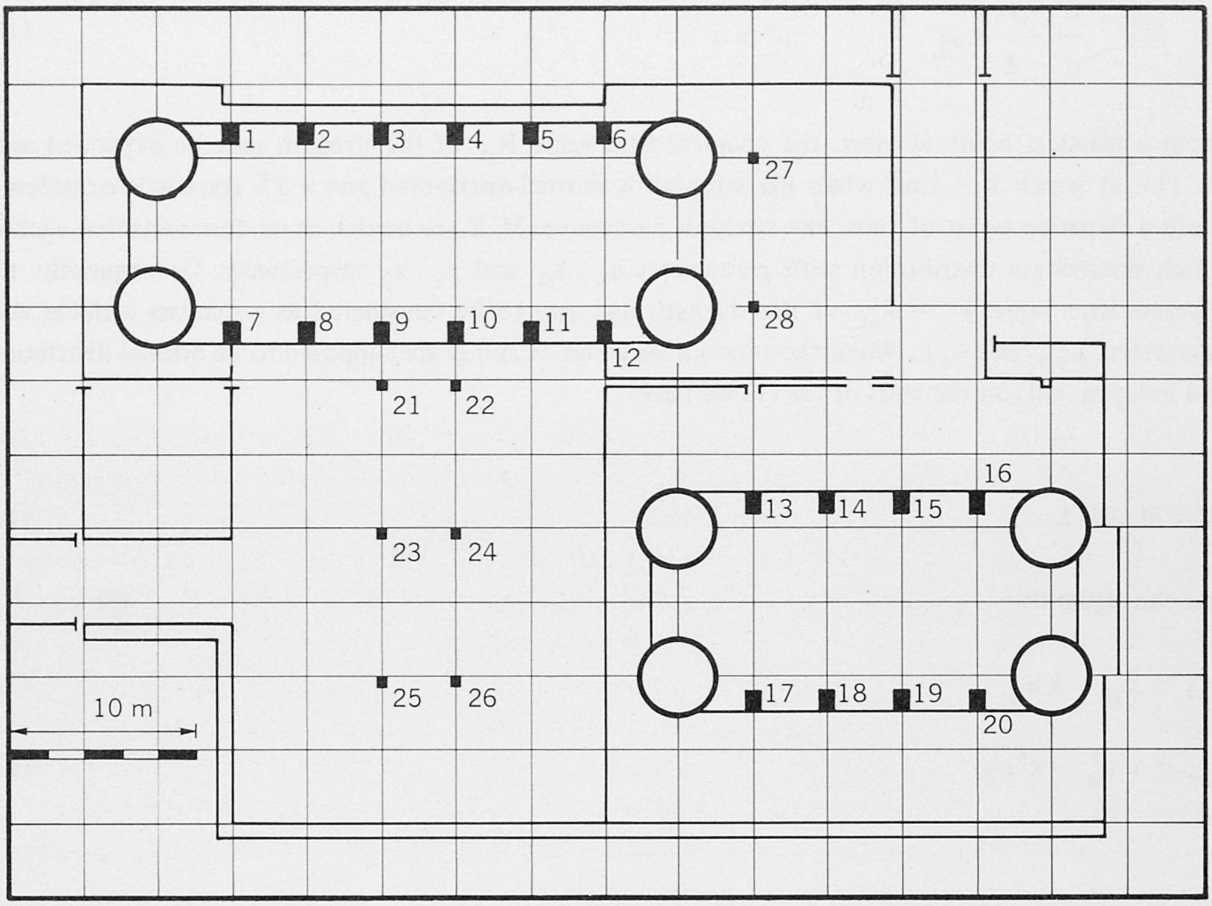

Figure 1

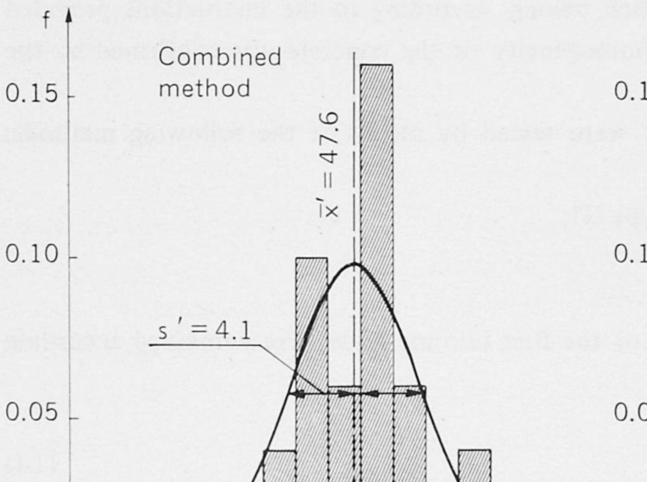

Figure 2

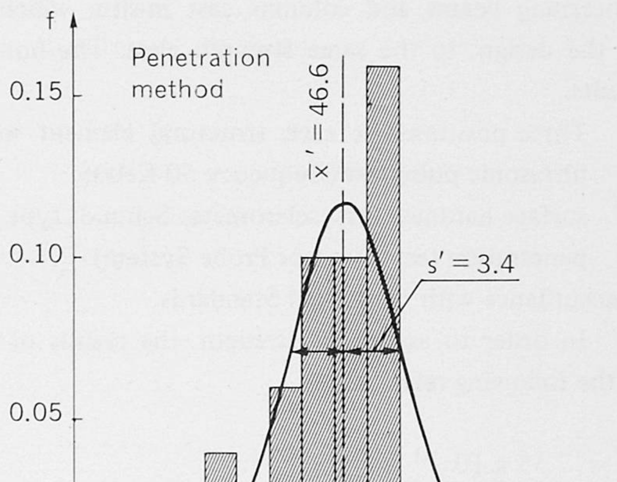

Figure 3

associated the two strength values estimated on the basis of the values of the non-destructive parameters calculated as means of the values determined in the three positions. The results are reported in Figures 2 and 3 in terms of histograms and respective frequency density curves.

By observing the synthetic parameters $\bar{x}'$ and s' relative to the two methods, almost equal mean values and more differentiated standard deviations may be noted. In particular, in the case of the penetration test, the standard deviation is lower since, in general, the method shows a lower sensitivity to the strength variations with respect to the combined method. For both methods, the values of s' are rather low, since the concrete is of good quality. In fact, as well-known, the sensitivity of the non-destructive methods decreases as the strength increases, even if in a different manner.

Besides the determinations by means of the non-destructive methods, four cores with size $\varnothing$ 15 x 30 cm were drawn which, in a second time, were subject to compressive tests. The strength values thus determined, transformed into cubic strength values, have provided the following synthetic parameters: $\bar{x} = 47.2\ N \cdot mm^{-2}$, $s = 1.9\ N . mm^{-2}$.

Figures 4 ÷ 9 report the distributions of the parameters μ_M, s_M, μ_Σ, s_Σ, $\mu_{\Psi*}$, $s_{\Psi*}$ with respect to n' for different values of n estimated by means of the combined method. On the basis of Figures 4 and 6, it may be noted that, as n' increases, the curves tend to the values of the prior distribution parameters $\bar{x}'$ and s' while, as n increases, they approach the values of the sample parameters $\bar{x}$ and s. Figures 5 and 7 show that the uncertainty level of the true value of the parameters M and Σ decreases as the information defined by $n'' = n + n'$ increases. Figures 10 and 11 report the comparison between the values of the parameters μ_M and μ_Σ relative to the two testing methods considered for n = 1, 4, 20. It may be noted that the influence of n and n' in the case of the penetration test is analogous to that already found in the case of the combined method.

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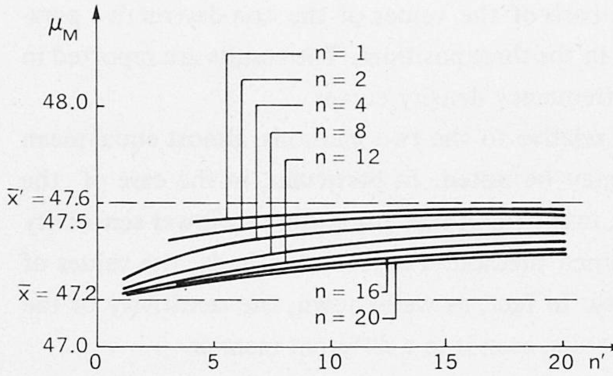

Figure 4

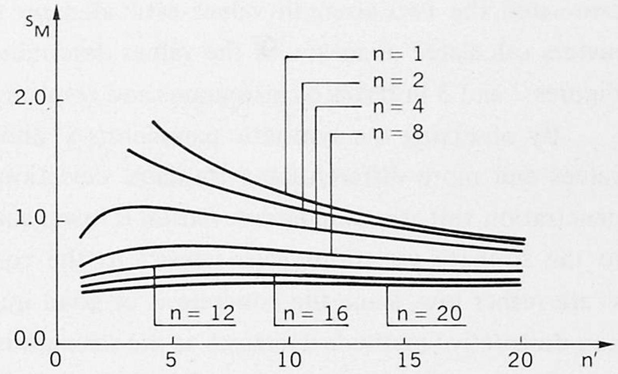

Figure 5

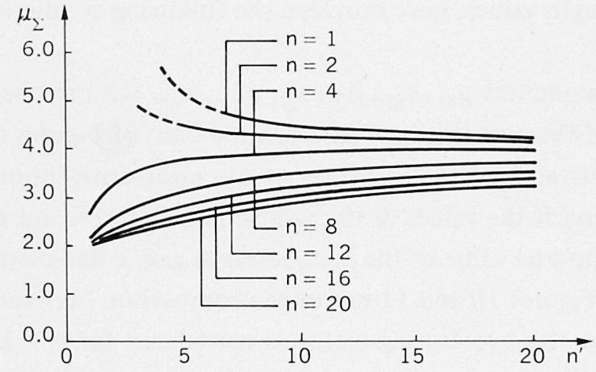

Figure 6

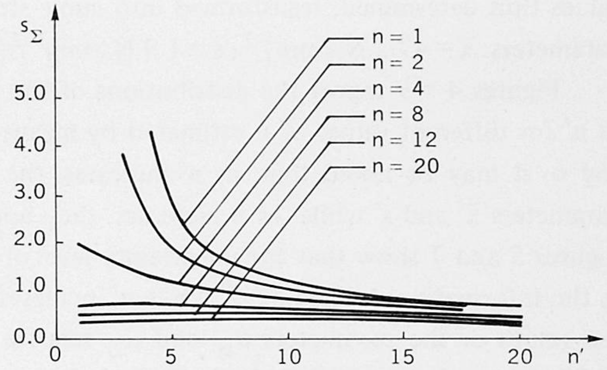

Figure 7

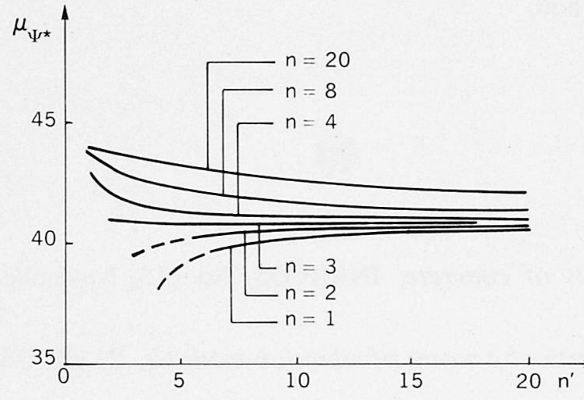

Figure 8

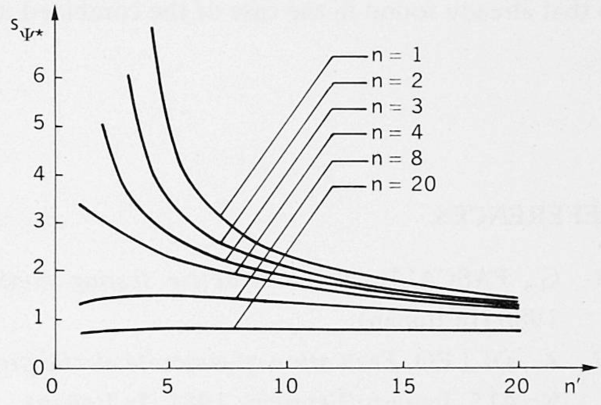

Figure 9

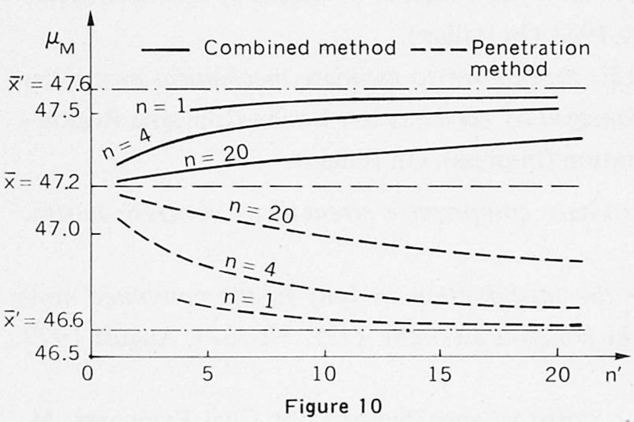

Figure 10

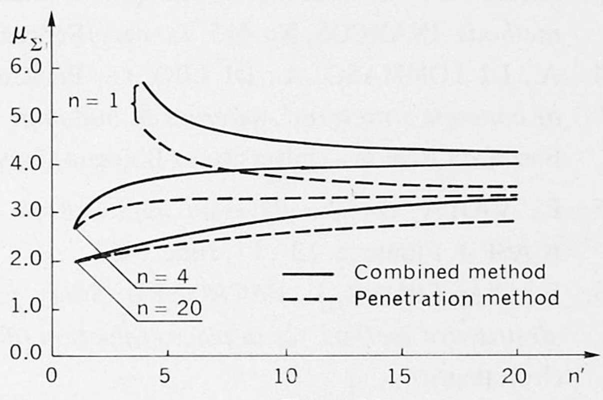

Figure 11