Analysis of a Brunelleschi-type dome including thermal loads

Autor(en): **Chiarugi, A. / Fanelli, M. / Giuseppetti, G.**

Objekttyp: **Article**

Zeitschrift: **IABSE reports = Rapports AIPC = IVBH Berichte**

Band (Jahr): **46 (1983)**

PDF erstellt am: **23.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-35847

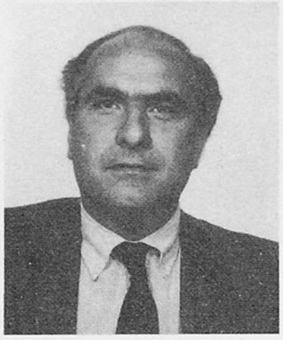
Analysis of a Brunelleschi-Type Dome Including Thermal Loads

Analyse d'une coupole tenant compte des effets thermiques

Untersuchung von Kuppeln unter Berücksichtigung thermischer Belastungen

A. CHIARUGI
Professor
Univ. of Florence
Florence, Italy

Born in 1937 in Florence. Took his Civil Engineer degree in Bologna, where he began his career as a University Professor in 1965. He has been teaching Structural Technology since 1972 at the Engineering Faculty of Florence.

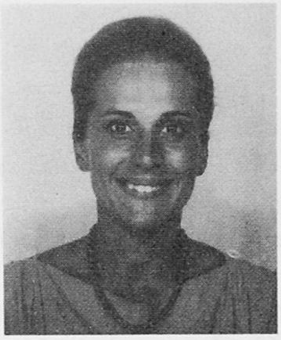


M. FANELLI
Professor
ENEL-CRIS
Milano, Italy

Born in 1931 in Florence. Civil Engineer degree cum laude in 1954. Concerned with computer analysis of structural and hydraulic problems. Head of the Theoretical Analysis Service of the CRIS of ENEL. Since 1981 is Chairman of WC VI of IABSE

G. GIUSEPPETTI
Engineer
ENEL-CRIS
Milano, Italy

Born in 1945 in Pesaro, obtained her degree in Civil Engineering (Transport) in 1969. She was engaged at ENEL in 1971 and now she is coordinating the activity of a group in the field of advanced structural analysis.



SUMMARY

This paper deals with an important historical monument: the Florence Cathedral dome. A Brunelleschi-type octogonal double-shell dome structure, was studied using a finite element model which reproduced its general geometry and various cases of load and restraint. The material was assumed to be linear-elastic, however, non-linear geometry was introduced by analysing gradual crack development as indicated by the present situation. Besides static loads with various assumed restraints, thermal loads were also considered in accordance with yearly periodic variations. The superposition of these effects gives some hints on the origin of the present cracks and the static performance of the cracked structure.

RESUME

Il s'agit d'une contribution à la connaissance d'un important monument historique: la coupole de la Cathédrale de Florence. Sur un modèle à éléments finis reproduisant sa géométrie générale, on a étudié plusieurs cas de charge et de liaison. Le matériau est censé être élastique-linéaire, mais on introduit des variations de géométrie qui représentent le développement progressif des fissures jusqu'au niveau actuel. En plus du poids propre, on a considéré des charges thermiques d'après un cycle annuel schématique. La superposition des effets donne quelques idées sur l'origine des fissures actuelles ainsi que sur la performance statique de la coupole fissurée.

ZUSAMMENFASSUNG

Der Beitrag gibt Aufschlüsse über ein wichtiges historisches Baudenkmal: die Kuppel des Doms von Florenz. Die doppelschalige, achteckige Kuppel wurde unter Beachtung der Geometrie des Bauwerks mit einem Modell aus finiten Elementen nachgebildet. Unter der Annahme linear-elastischen Materialverhaltens und der Berücksichtigung nicht-linearer Geometrie aus der Beobachtung des Rissverhaltens wurden verschiedene Lastfälle und geometrische Restriktionen erfasst. Neben den statischen Lasteneinwirkungen wurden unter Berücksichtigung verschiedener, angenommener Spannungszustände auch hypothetische thermische Belastungen, entsprechend den jährlichen Temperaturschwankungen, untersucht. Die Ueberlagerung der verschiedenen Einwirkungen liefert interessante Hinweise über die Entstehung der vorhandenen Risse und das statische Verhalten der gerissenen Kuppel.

1. FOREWORD - In order to fight the deterioration of historical monuments, all means made available by today technology should be used. In this spirit -heeding the damaged state of the Dome of Florence Cathedral- we deemed useful to attempt an analysis effected according to methods perfected for studying the most complex present structures, so as to gain a better insight of the statical conditions. We defined a numerical F.E. model of a Brunelleschi-type dome, according to an overall shape reproducing, with some simplifications and regularizations, the monument own one; elastic linear behaviour was assumed for the material. In spite of these simplifications, the model is deemed rich in overall insight potentiality: used as a "verification" tool, indeed, it allows to gain some pregnant trends of the mechanical behaviour of a structure. These, together with physical evidence desumed from direct observation of the dome, can yield useful contributions to the diagnosis of a structural damage. With the present paper we intend to illustrate, apart from yet unwarranted geometrical and mechanical refinements, only a few of the 26 cases of load and constraint analyzed, emphasizing how these results can be useful in the abovesaid sense.

2. DESCRIPTION OF THE MATHEMATICAL MODEL USED AND ITS RESULTS - In order to achieve the abovesaid aims we defined at first the geometry of an ideal octogonal, regular, double-shell symmetric dome, the vertical profiles at corners being circular. The two shells are connected not only by the corner ribs, but also by two equally spaced ribs for every panel. To this model we attributed ideal mechanical characteristics of an elastic, homogeneous, isotropic solid (Young modulus $E = 50,000 \text{ kgcm}^{-2}$; Poisson ratio 0.2; thermal dilatation coefficient $\alpha = .8 \cdot 10^{-5} (°C)^{-1}$; thermal diffusivity $a = 16 \text{ cm}^2\text{h}^{-1}$). We intended to assess on such a structure -in this preliminary phase- only the effects of deadweight and of yearly sinusoidal thermal variations. To this end a F.E. mesh was built up for one quarter of the dome, cut by two orthogonal vertical symmetry planes (fig. 1 a). In order to better define the constraint conditions at the dome basis a quarter of the underlying drum, as schematized by the F.E. mesh of fig. 1b, was later joined to the dome quarter of fig. 1 a. In this "dome plus drum" mesh there are 428 elements (2nd order, isoparametric ones with 20 or 15 nodes) and 2667 nodes. Some first-approach computations concerned only the dome, under different constraint hypotheses at the basis, assuming different degrees of participation for the outer shell, and also applying stiffness and deadweight either in one stage or in 7 successive stages broadly simulating the construction process. Passing over these first approaches, we intend here to illustrate the main results obtained by analysing the "dome plus drum" structure with complete constraints at the drum basis, under deadweight (including that of the skylight, acting on the upper rim of dome) and yearly periodic temperature variations. All these conditions were applied both to the intact and the cracked structure (see fig. 2, [1]). The position of cracks was defined by schematizing the actual cracking pattern, i.e. following vertical planes symmetrically intersecting four out of the eight dome panels (those at 45° from the plan cross axes). As concerns the extension in height of the cracks, an iterative computation was performed by raising in steps the upper tip of the cracks, until the tensile stresses that tended to

appear (in a direction perpendicular to the crack planes) above the tip became negligible. This condition obtained for cracks extending up to about 2/3 of the dome height, in good agreement with the observed present crack development. As concerns the stress analyses pertaining to thermal loads, it was necessary to obtain first the 3-D, time dependent temperature distribution inside the shells. Boundary thermal conditions were assumed with a common average of +15°C, on which sinusoidal waves were superposed,with amplitude ±15°C on the outher surface, ±5°C on the inner surface;for the latter also a time-lag of one month with respect to outside was introduced. Intermediate boundary conditions were assumed for the cavities between the two shells. Figs. 3 to 10 represent graphically some results of the above-cited analyses. By considering these results, together with observations on the crack pattern (on every other panel: actually, those panels are cracked that stem from the four massive pillars of the cross-vault, whereas the other four panels, springing from arch supports, are practically uncracked, see fig. 2), we deemed it useful to analyse in detail the structure underlying the drum. To this end we discretized, by the F.E. mesh of fig.11, the structure lying between the dome basis (third gallery level) and the first gallery level. 188 isoparametric, 2nd order elements with 1232 nodes were used for this mesh. This structure was constrained completely at the nodes of lower surface, whilst on the side vertical planes obvious symmetry conditions were applied for the intact structure. On the upper surface, two different load conditions were considered :

- a vertical, uniformly distributed pressure amounting to about 3 $kg cm^{-2}$ (the integral of which roughly balances the weight of overlying masonry masses);
- a unit horizontal load (1 $kg cm^{-2}$), uniformly distributed across the thickness, thrusting outward perpendicularly to each octagon side.

Some of the results are illustrated in figure 12. In particular, the stress pattern seen from fig.12a is confirmed by results of theoretical studies concerning curved wall-beams on interrupted supports [2] and continuous, curved intrados beams [3] . The upshot of it all is that the difference in stiffness between "solid" panels and "arch-supported" ones produces a bias in the horizontal tensile stresses at panel centers, especially under vertical loads. This bias is enhanced by the stress-concentration effect due to the "eyes" piercing the drum panels. If we analyze the possible superpositions of the previously considered stress states, it looks as if the structure possessed a natural propensity to cracking just in the "solid-supported" panels, starting from the "eyes" and following nearly vertical paths.

3. CONCLUSIVE REMARKS - The foregoing material aims at setting up a definite example of the possibilities offered by numerical analysis methods, as well as a first contribution - even if under sketchy hypotheses - to the diagnosis of the monument conditions. No more is attempted, in fact, than quantifying hints on the preferential bias of the structure to damage, as actually observed, and evaluating the broad lines of the present static conditions with open cracks. It is evident that even this diagnostic phase is far from concluded. It will, indeed, be necessary to effect further analyses - not only statical, but also dynamic ones - , improving the model with geometrical, thermal and mechanical characteristics more in tune with actual properties.

We shall, moreover, need accurate experimental data (thermal and mechanical measurements), in order to effect a more proper numerical simulation as well as to validate the mathematical model; this will allow, in turn, to orient more specific experimental checks. By thus proceeding, clearly, not only the set of informations needed to complete the diagnostic phase will be more accurately drawn up, but a possibility will be opened toward setting up means for checking the effects of any type of "therapy". Indeed, if we should pass to a phase of "design" of any intervention upon an already built structure, we should be able to forecast the actual response of the monument.

ACKNOWLEDGEMENTS
Heartfelt thanks are expressed to ISMES (Bergamo, Italy) who kindly allowed to use its stress-analysis "FIESTA" system for all numerical analyses. Special gratitude is reserved for Mr. Franco Pari, of ENEL/CRIS, who cared after all the numerical and graphical work with unrelenting competence and assiduousness.

BIBLIOGRAPHIC LIST

[1] FONDELLI M., FERRI W., FRANCHI P., GRECO F. "Il rilevamento fotogrammetrico della Cupola di S. Maria del Fiore in Firenze", Istituto Geografico Militare di Firenze, 1971

[2] POZZATI P., "Tubo cilindrico ad asse verticale appoggiato in corrispondenza di alcuni tratti del bordo", Costruzioni in Cemento Armato, Studi e Rendiconti, Vol. 4, 1967

[3] CECCOLI C., MERLI M., "Trave-parete ad intradosso curvo" Giornale del Genio Civile, fasc. 7, 8, 9, 1976

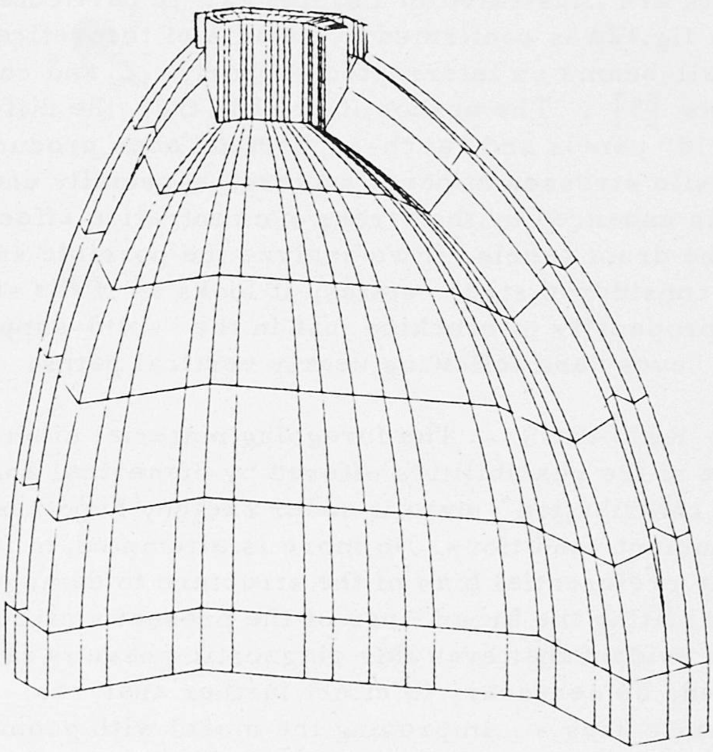


Fig. 1 a)

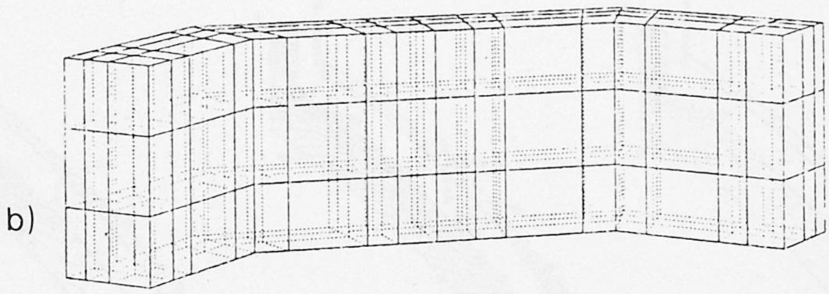


Fig. 1 - a) F.E. mesh of a quarter of the dome; b) F.E. mesh of the drum underlying the dome

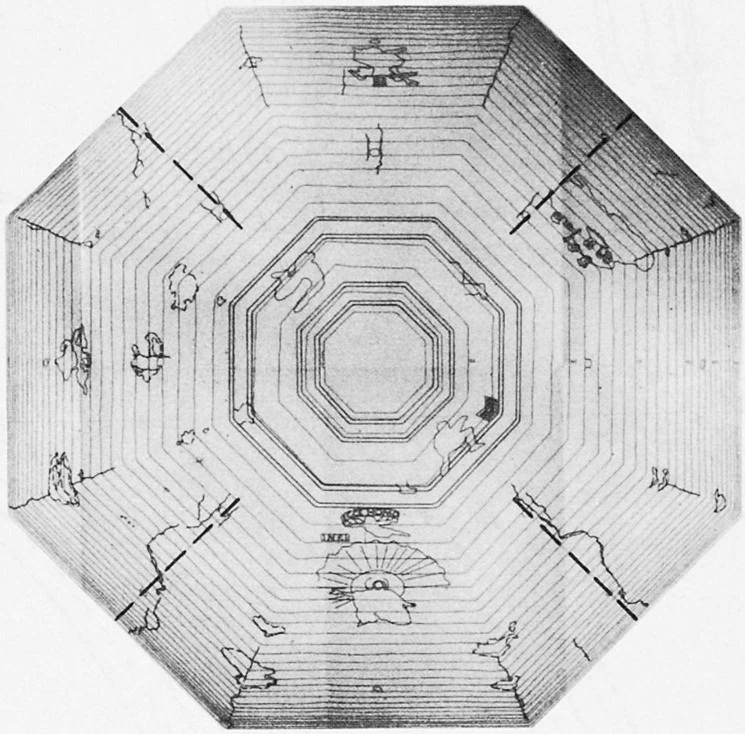


Fig. 2 - Nadiral view of crack pattern in Florence Cathedral dome (------) assumed cracks

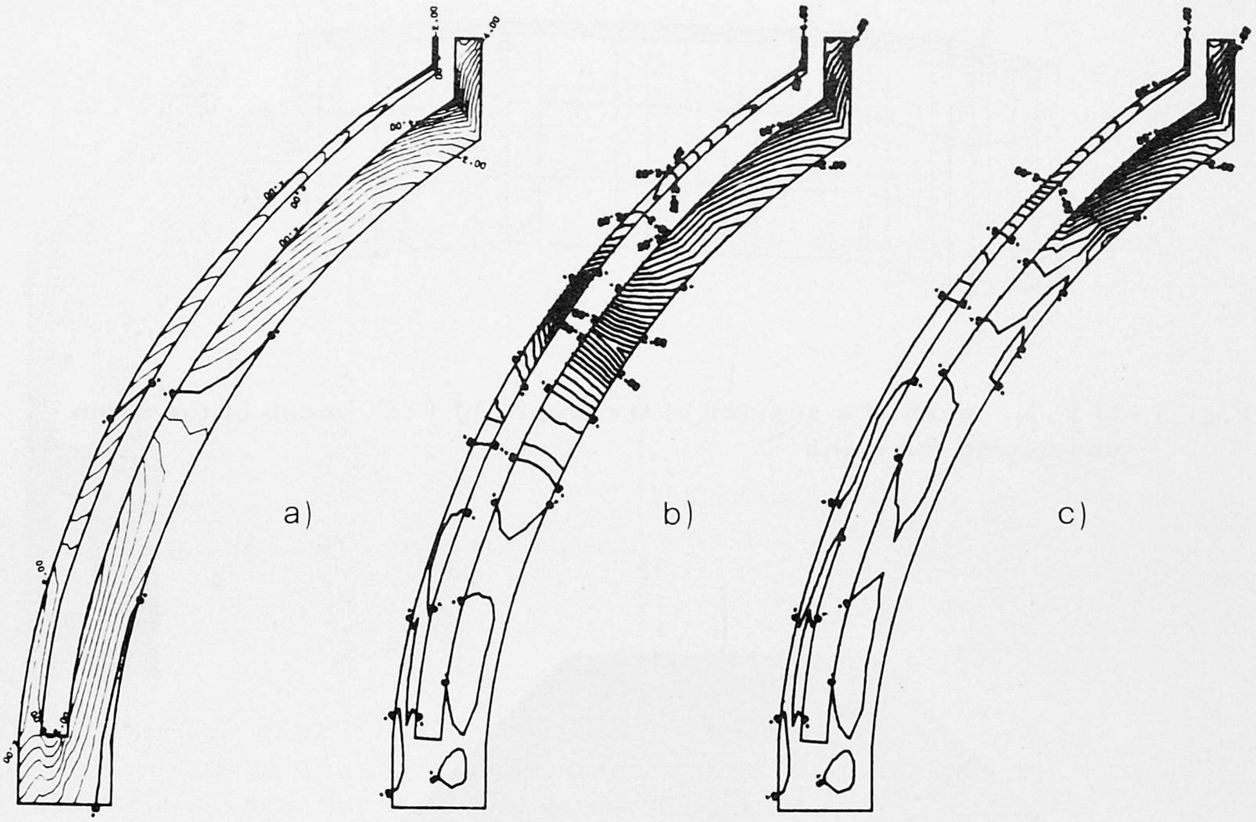


Fig. 3 - Contour lines of σ_x in the cross-section at mid-panel (load = deadweight) ; a) uncracked structure; b) crack up to one-half height; c) crack up to two-third height

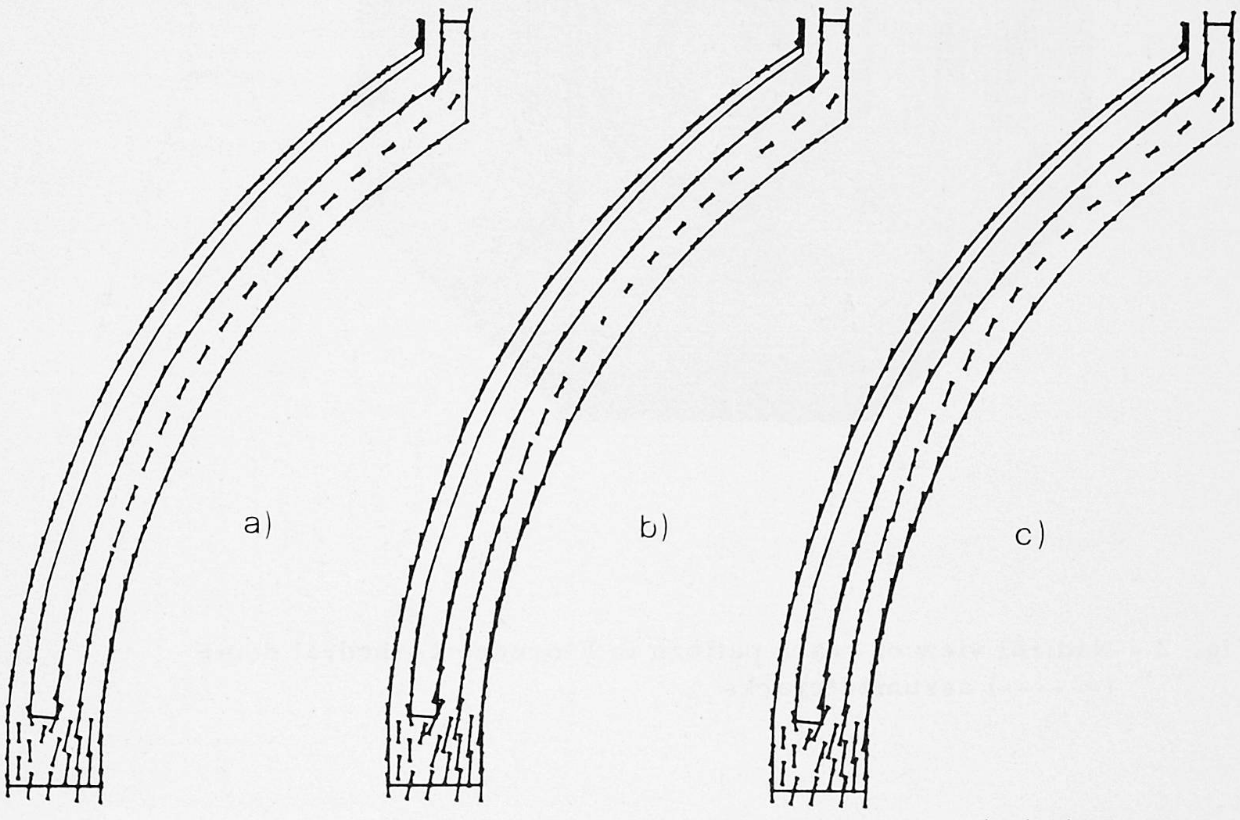


Fig. 4 - Principal stresses in cross-section at mid-panel: a) b) c) as in fig. 3 (load=deadweight)

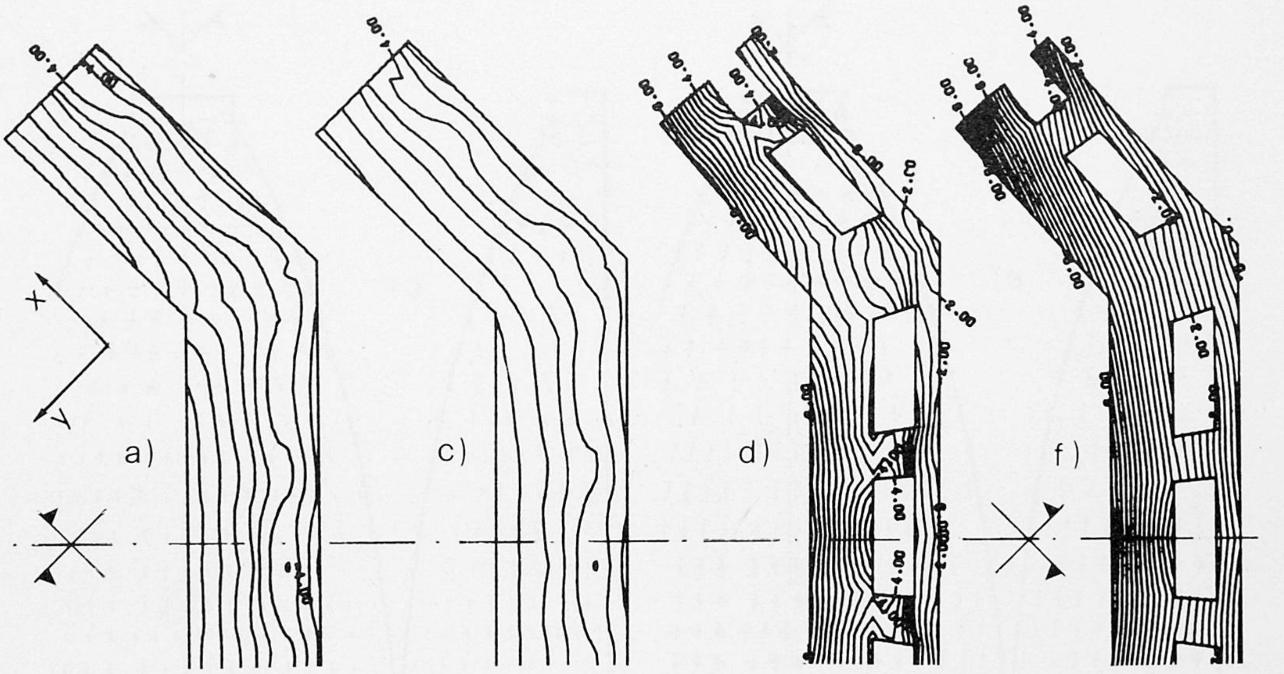


Fig. 5 - Contour lines of σ_z in horizontal sections: a) c) as in fig. 3 for hor. sect. at level of first gallery (load=deadweight); d) f) same as a) c) but for hor. sect. at level between second and third layer of elements in fig. 1 a)

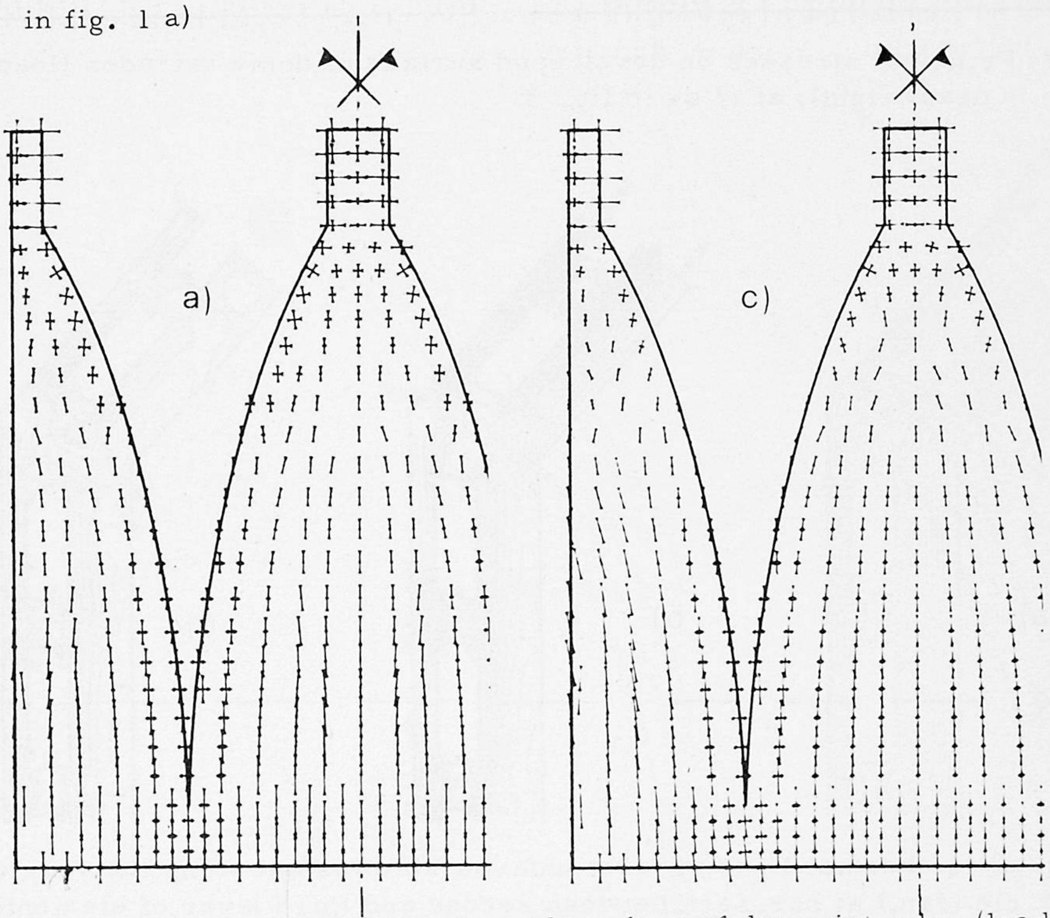


Fig. 6 - Principal stresses on developped surface of dome intrados (load = = deadweight); a) c) as in fig. 3

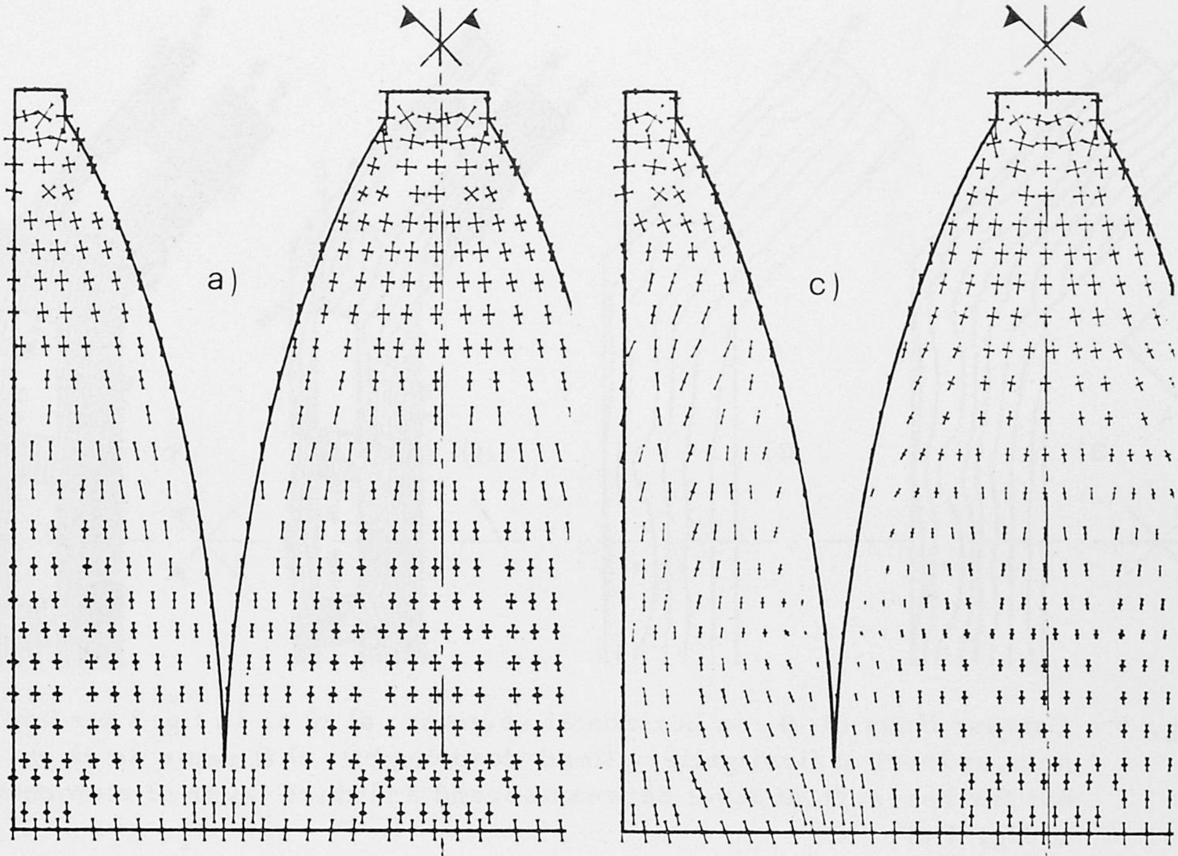


Fig. 7 - Principal stresses on developped surface of dome extrados (load = = deadweight); a) c) as in fig. 3

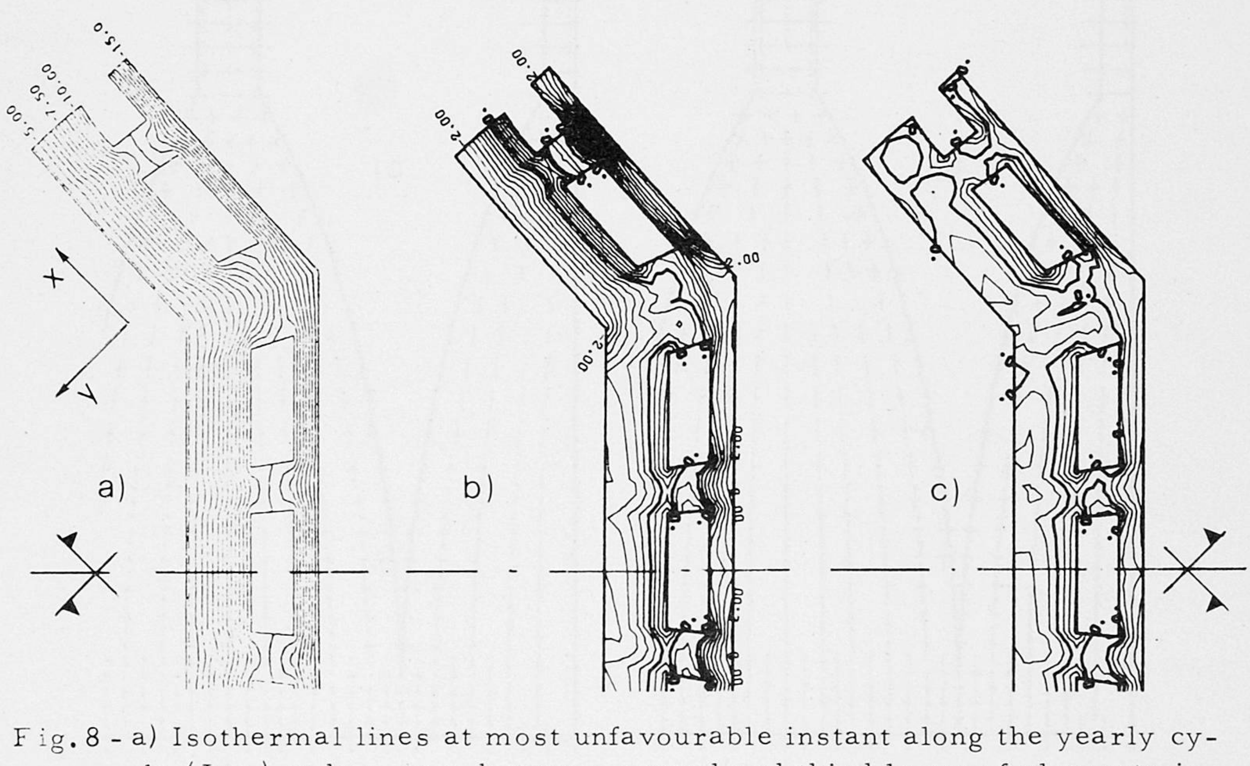


Fig. 8 - a) Isothermal lines at most unfavourable instant along the yearly cycle (Jan.) at hor. sect. between second and third layer of elements in fig. 1 a); b) Horiz. sect. between second and third layer of elements in fig. 1 a): contour lines for σ_x, uncracked structure, thermal load at Jan.; c) as b), but for cracked structure, maximum extension of cracks

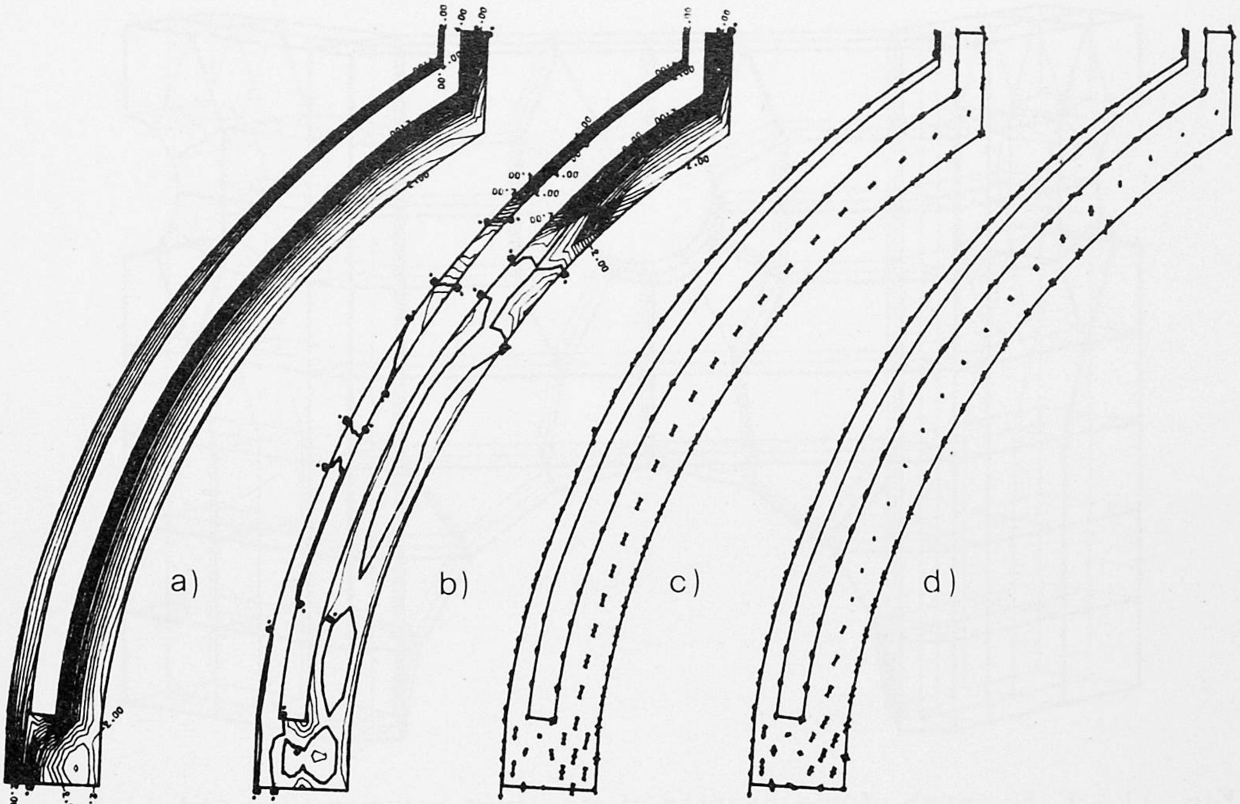


Fig. 9 - a) Contour lines for σ_x on vertical cross-section at mid-panel, uncracked structure, thermal load at Jan.; b) as a), but for cracked structure, max. extension of cracks; c) principal stresses on vertical cross-section at mid-panel, uncracked structure, thermal load at Jan.; d) as c), but for cracked structure, max. extension of cracks

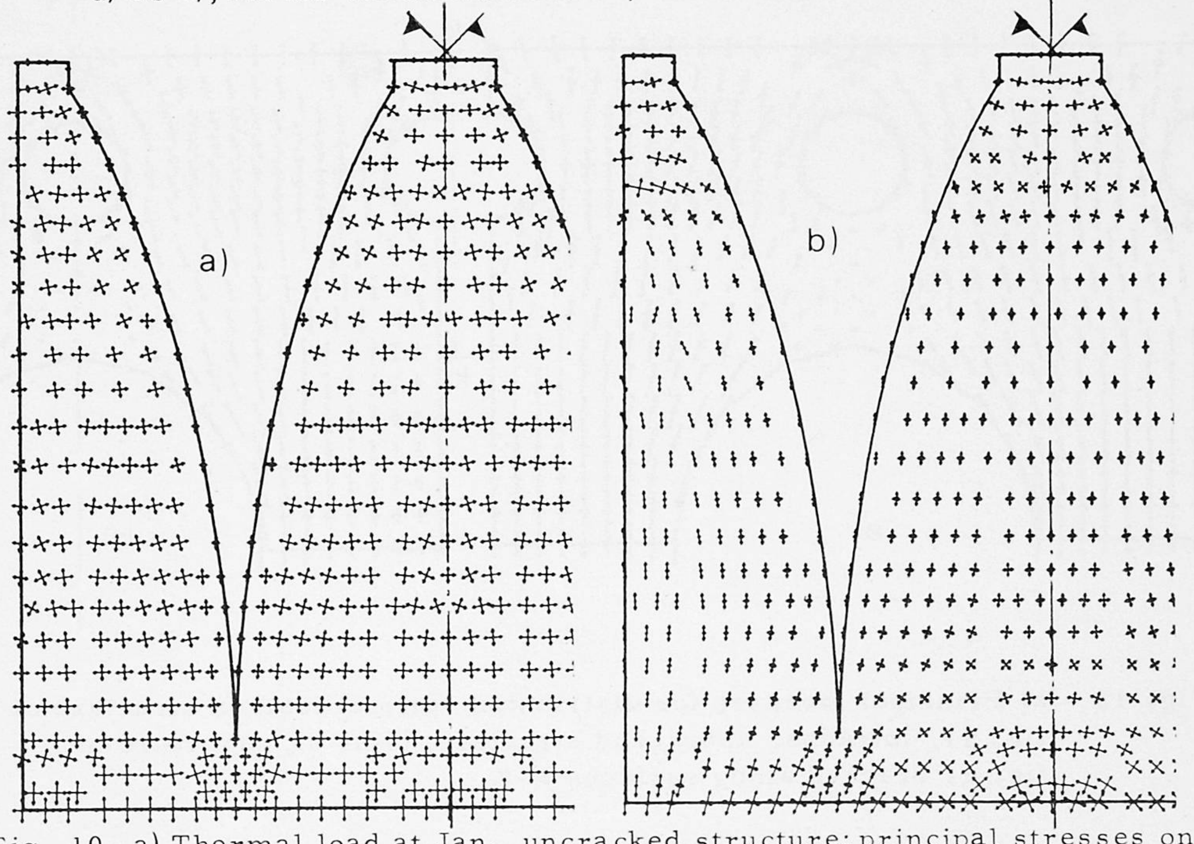


Fig. 10 - a) Thermal load at Jan., uncracked structure; principal stresses on extrados; b) as a), but for cracked structure, max. extension of cracks

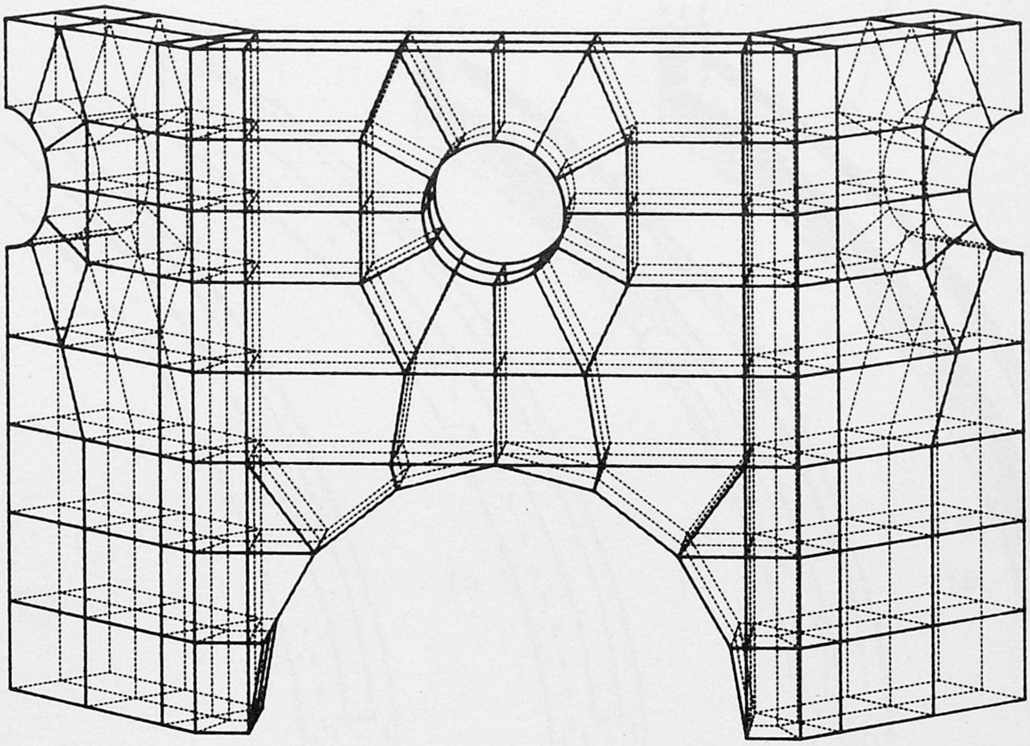


Fig. 11 - F.E. mesh of one-quarter of structure between first and third gallery

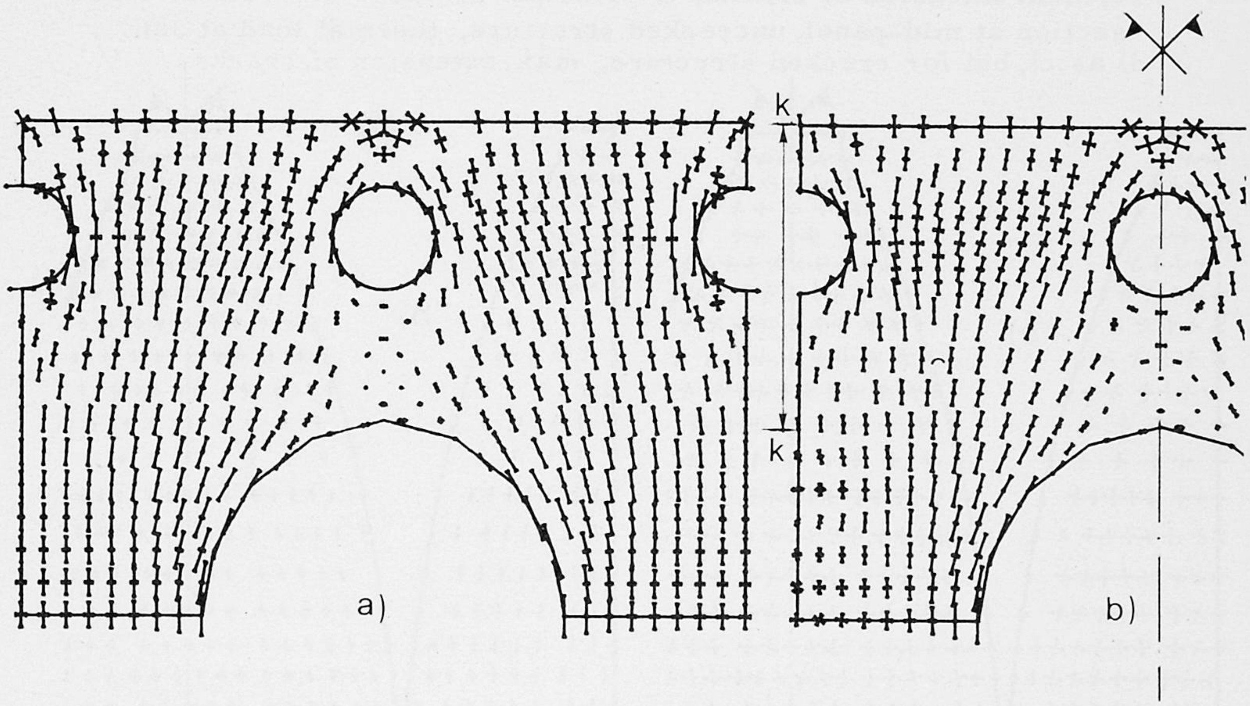


Fig. 12 - a) Principal stresses for distributed vertical load on structure of fig. 11, intrados, uncracked structure; b) as a), structure of fig. 11 cracked along surfaces K-K