Limit load of masonry structures

Limit Load of Masonry Structures

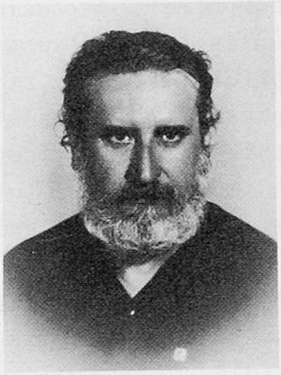
Charge ultime de structures en maçonnerie

Die Traglast von Mauerwerk

Matelda LO BIANCO
Research-worker
Univ. of Palermo
Palermo, Italy

Cesare MAZZARELLA
Professor
Univ. of Palermo
Palermo, Italy

M. Lo Bianco, born 1952, received her degree in Architecture at University of Palermo in 1975, and later began her activity as research-worker in the same Faculty. She published some papers about problems on optimum design and prestressed concrete.



C. Mazzarella, born 1932, received his civil engineering degree at the University of Palermo. He published some papers on problems in Structural Engineering, such as optimum design, work-hardening adaptation and prestressed concrete.

SUMMARY

The paper deals with the ultimate load of masonry structures conceived as discrete structures formed from rigid blocks and frictional joints. The problem is treated as one of limit analysis of a discrete rigid-plastic structure with non-associated flow rule. A numerical procedure is proposed which at first obtains a lower bound of the collapse factor and then, if necessary, calculates the true collapse factor by solving a non-linear program.

RESUME

Le problème de la charge ultime pour les structures en maçonnerie est résolu à l'aide d'un modèle formé de blocs rigides et de joints avec frottement. Il s'agit d'un problème d'analyse limite pour une structure rigide-plastique. Une procédure numérique fournit une valeur inférieure du facteur de charge, et calcule le facteur ultime exact à l'aide d'un programme non linéaire.

ZUSAMMENFASSUNG

Die Arbeit behandelt das Problem der Traglast von Mauerwerk. Das Mauerwerk wird als diskrete Struktur aus starren Blöcken und Fugen, welche als Reibungsflächen gedacht sind, aufgefasst. Der Grenzzustand wird mit Hilfe eines starr-plastischen Modells ohne zugeordnetes Fliessgesetz erfasst. Das Berechnungsverfahren bestimmt einen unteren Grenzwert der Traglast. Wenn notwendig wird dieser mit Hilfe eines nichtlinearen Programmes optimiert.

1. INTRODUCTION

The collapse load of masonry structures is the oldest problem in the structural mechanics field (see [1] for an exhaustive bibliography) and the first paper on the masonry arch or dome, allowing for friction and coesion is due to Coulomb [2]. More recently, Heymann [3-4] showed that the masonry's collapse load can be viewed as a limit analysis problem, and Livesley [5] provides a formal procedure for finding the collapse load of any structure formed by rigid blocks, with the block interfaces capable of carrying only compressive and shear stresses. Livesley's procedure maximizes the load factor ρ subjected to the linear equilibrium equations and the linearized constraints imposed by criteria of failure at block interfaces. Because the limit on the shear force is assumed to be that associated with Coulomb friction, the normality flow rule is not-satisfied and the obtained load factor is only an over-extimate of the true factor [6]. Consequentially it is necessary a post-optimality analysis to test the validity of the computed load factor.
The present paper studies the collapse load of masonry structures with the same Livesley basic assumption (*discrete rigid-plastic model with frictional interfaces*) in the general framework of limit analysis for material with non-associated flow rule [7]. For a such structure, the computation of the load collapse multiplier ρ implies the solution of a non linear, non convex mathematical program; instead it is relatively easy to construct [8] an upper bound ρ_u (or a lower bound ρ_ℓ) to ρ, solving, in both cases, a standard limit analysis problem of an auxiliary structure , which must have the same geometry of the original one but different constitutive equations, suitable choosen and with associated flow rule. You propose a numerical procedure formed by three steps: i) a lower bound ρ_ℓ of the real collapse multiplier ρ_c is obtained solving a linear programming (LP); ii) a post-optimality analysis of LP solution checks if ρ_ℓ is at the same time the real $\rho_c (\rho_\ell \equiv \rho_c)$; iii) only if the above test fails the real multiplier is obtained solving a non-linear program.
For semplicity sake the paper deals only with plane masonry structures (as simple or multiple arches and walls) but all the results of this paper can be easily extended to spatial structures (as domes).
Matrix and vector quantities are denoted by underlineed characters; $\underline{0}$ denotes the null matrix or vector; $\tilde{\underline{A}}$ or $\underline{A}^t$ denote the transpose of $\underline{A}$, and superimposed dot ($\dot{}$) denotes a time derivative.

2. THE MASONRY STRUCTURE AND THE IDEALIZED MODEL

Any (plane) masonry structure can be described as an assemblage of regular form stone blocks, with interposed mortar joints, which can carry only compressive and shear stresses. An useful approximation to the very complex behaviour of a such structure is obtained assuming the stone blocks as rigid and the joint's tickness as infinitesimal. The structural model is conceived as formed by rigid, discrete size, *nodes* (n in number) with interposed m *rigid-plastic sections* ($m>n$), which have yield limits on the (generalized) *stress* vector $\underline{\sigma}=[N\ T\ M]^t$. The limit on the *shear* T is linearly dependent on the *normal force* N ($N>0$) with non-associated flow rule (N A F R). Because no-interaction is assumed between the shear T and the *bending moment* M, the yield domain in the (N,M) space is the usual interaction curve, suitably linearized (see Fig.1 related to a rectangular

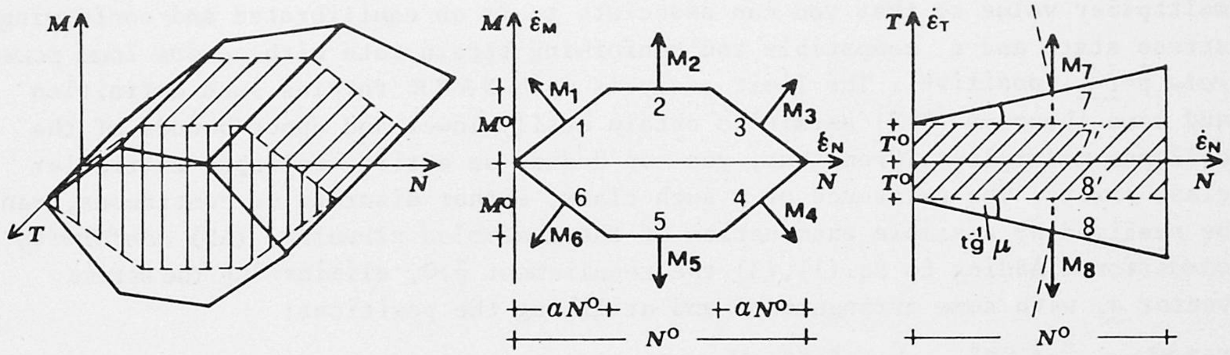


Fig.1 Elemental yield domains: axonometrie and sections.

cross-section). The analytical description of this constitutive law (*elemental conformity* conditions [9]), can be given the following matricial form:

$$\underline{\varphi}=\underline{\tilde{N}}^{\circ}\,\underline{\sigma}-\underline{k}^{\circ}\le\underline{0},\quad \underline{\Psi}=\underline{\tilde{M}}^{\circ}\,\underline{\sigma}-\underline{k}^{\circ},$$
$$\underline{\dot{\lambda}}\ge\underline{0},\quad \underline{\tilde{\varphi}}\,\underline{\dot{\lambda}}=0,\quad \underline{\dot{\varepsilon}}=\underline{M}^{\circ}\underline{\dot{\lambda}}, \tag{1}$$

which usually refers to a single section or element, but can refers to the full element set by a suitable redefinition of the vectors and matrices as super vectors and block diagonal supermatrices. In Eq.(1) $\underline{\varphi},\underline{\Psi},\underline{\dot{\lambda}},\underline{k}^{\circ}$, (with the same dimension) are, respectively, the (elemental) *yield function*, (*plastic*) *potential*, *multiplier rate* and (*plastic*)*resistance* vectors. Eq.(1c-d) define the *flow rule*: the *strain rate* vector $\underline{\dot{\varepsilon}}=[\dot{\varepsilon}_N\ \dot{\varepsilon}_T\ \dot{\varepsilon}_M]^t$ dual in the v.w. sense of vector $\underline{\sigma}$, is a linear combination of the *yield modes* M_i° (i.e. of the plastic potential gradient), with non-negative coefficients $\dot{\lambda}_i$, which can be non zero only if the corresponding y.f. φ_i vanishes. Some remarks will be useful: i) Adding to Eq.(1b) the sign requirement $\underline{\Psi}\le\underline{0}$, you can define the *reduced yield domain* with AFR, currently used to obtain a lower bound multiplier [8-9]; this definition requires a simple modify of the usual assumption: $\underline{M}_j^{\circ}$ and k_j° must be interpreted as proportional to unitary external normal and to distance from the origin of the $\Psi_j=0$ plane.
ii) The $\underline{\varphi}$ vector can be split in two subvectors $\underline{\varphi}=[\underline{\varphi}_a\ \underline{\varphi}_n]^t$, where $\underline{\varphi}_a$ collects the y.f. with AFR (1-6 in Fig.1) and $\underline{\varphi}_n$ the y.f. with NAFR (7-8 in Fig.1); breaking in the same way the $\underline{\Psi}$ vector, and observing that, as consequence of the assumed flow rule, $\underline{\varphi}_a\equiv\underline{\Psi}_a$ (whereas $\underline{\varphi}_n\neq\underline{\Psi}_n$), you obtain the following relations:

$$\underline{N}^{\circ}=\underline{M}^{\circ}+\underline{P}^{\circ},\ (\underline{N}^{\circ}=[\underline{N}_a^{\circ}\ \underline{N}_n^{\circ}],\ \underline{M}^{\circ}=[\underline{M}_a^{\circ}\ \underline{M}_n^{\circ}],\ \underline{P}^{\circ}=[\underline{0}\ \underline{P}_n^{\circ}]), \tag{2a-d}$$

where $N_a^{\circ}\equiv M_a^{\circ}$ depends on the cross-section type and the assumed linearization and:

$$\underline{P}_n^{\circ}=\begin{bmatrix}-\mu & 0 & 0\\ -\mu & 0 & 0\end{bmatrix}^t. \tag{2e}$$

With a *redundant stresses* formulation [10], the *equilibrium* conditions for a one-parameter loading (with parameter $\rho\ge0$) and the *compatibility* condition are:

$$\underline{\sigma}=\underline{\tilde{A}}(\underline{f}^{\circ}+\underline{f}\rho)+\underline{\tilde{B}}\,\underline{x},\quad \underline{\dot{u}}=\underline{A}\,\underline{\dot{\varepsilon}},\quad \underline{\dot{\theta}}=\underline{B}\,\underline{\dot{\varepsilon}}=\underline{0}, \tag{3}$$

where the vector $\underline{f}^{\circ}(\underline{f})$ collects the *nodal forces* equivalent to the fixed (variable) load sistem, and the vectors $\underline{x},\underline{\dot{u}},\underline{\dot{\theta}}$ are respectively the *redundant stress*, the (nodal) *displacement rate* and the *distorsion rate* vectors. The *compatibility* matrices $\underline{A}$ and $\underline{B}$ depend only on the structure's layout and on the assumed redundant stresses.

3. THE COLLAPSE MULTIPLIER CLASS AND ITS BOUNDS

In the limit analysis with AFR, the collapse multiplier is the only load multiplier value so that you can associate to it an equilibrated and conforming stress state and a compatible and conforming strain rate with *active load power rate* $\dot{p}=\underline{\tilde{f}}\,\underline{\dot{u}}$ positive . The limit analysis with NAFR retains such definition and same theorems [7-8] permit to obtain easily lower and upper bounds of the collapse multiplier. From that, you can deduce an entire collapse multiplier class exists. The existence of a such class, either discrete or continuous, can be realized by a simple examination of the *assembled structure* (AS) *conformity conditions*. Adding to Eq.(1),(3) the requirement $\dot{p}>0$, eliminating the stress vector $\underline{\sigma}$, with some arrangements and utilizing the positions:

$$\underline{N}=\underline{B}\,\underline{N}^\circ,\quad \underline{\tilde{n}}=\underline{\tilde{f}}\,\underline{A}\,\underline{N}^\circ,\quad \underline{\tilde{k}}_N=\underline{\tilde{k}}^\circ-\underline{\tilde{f}}^\circ\underline{A}\,\underline{N}^\circ$$
$$\underline{M}=\underline{B}\,\underline{M}^\circ,\quad \underline{\tilde{m}}=\underline{\tilde{f}}\,\underline{A}\,\underline{M}^\circ,\quad \underline{\tilde{k}}_M=\underline{\tilde{k}}^\circ-\underline{\tilde{f}}^\circ\underline{A}\,\underline{M}^\circ \tag{4}$$

you obtain the following AS conformity conditions:

$$\underline{\varphi}=\underline{\tilde{N}}\,\underline{x}+\underline{n}\,\rho-\underline{k}_N\leq\underline{0}\qquad \underline{\Psi}=\underline{\tilde{M}}\,\underline{x}+\underline{m}\,\rho-\underline{k}_M$$
$$\underline{\dot{\lambda}}\geq\underline{0}\,,\quad \underline{\tilde{\varphi}}\,\underline{\dot{\lambda}}=0,\quad \underline{\dot{\theta}}=\underline{M}\,\underline{\dot{\lambda}}=\underline{0},\quad \dot{p}=\underline{\tilde{m}}\,\underline{\dot{\lambda}}=\omega>0, \tag{5}$$

where $\underline{\varphi}(\underline{\Psi})$ can be called the AS yield function (plastic potential) vector, and ω is a positive number.

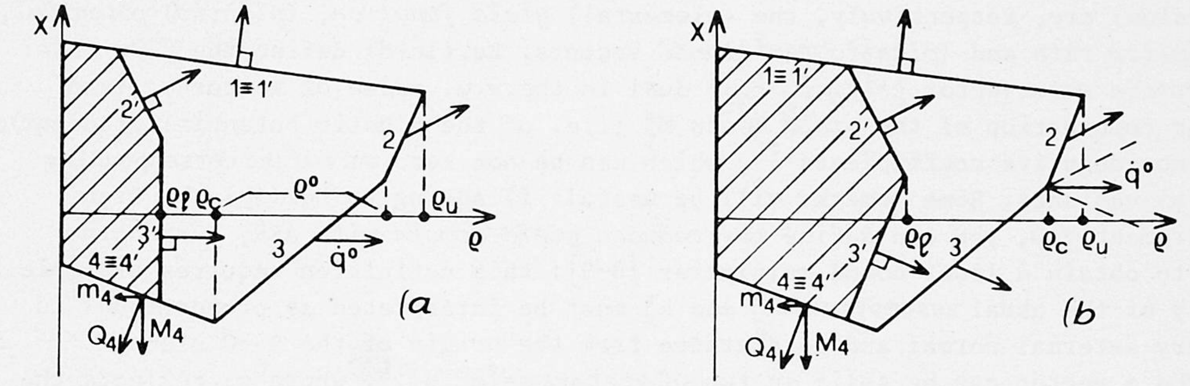


Fig.2 Assembled structure yield domains: two ideal cases.

In the $\underline{y}=[\underline{\tilde{x}}\,\rho]^t$ space (the AS *stress* space), Eq.(5a) defines the AS rigid region, whose boundaries are the AS yield planes $\underline{\varphi}=\underline{0}$. Denoting with $\underline{M}_j$ the j-th column of $\underline{M}$ and with m_j the j-th scalar component of $\underline{m}$, you can say that an AS *deformation mode* vector $\underline{Q}_j=[\underline{\tilde{M}}_j\ m_j]^t$ is associated to each plane $\varphi_j=0$, and that the AS *strain rate* vector $\underline{\dot{q}}=[\underline{\dot{\theta}}^t\ \dot{p}]^t$ is obtained as linear combination of these deformation mode vectors (i.e. of the AS plastic potential gradient) with non negative coefficient $\dot{\lambda}_j$, subjected to the complementary conditions (5d). The geometric interpretation of the AS conformity conditions (5) is formally similar to the usual for the elemental conformity conditions (1), but it is really most restrictive: it is easy to realize that the Eq.(5e-f) require only a value for $\dot{q}$, i.e. $\underline{\dot{q}}=\underline{\dot{q}}^\circ=[\underline{\tilde{0}}\ \omega]^t$, which is a vector orthogonal to the subspace $\underline{x}$ and directed as the positive ρ-axis.
Therefore a collapse condition can be represented by those points $\underline{y}^\circ$ of the AS rigid region boundaries to which you can associate, through the generalized flow law (5c-f), the AS strain rate vector $\underline{\dot{q}}^\circ=[\underline{\tilde{0}}\ \omega]^t$. For AFR structures, $\underline{Q}_j$ coincides with the external normal to the φ_j plane, and the $\underline{y}^\circ$ point set, either only a point or a continuous set, coincides with the boundaries point subset which have the greatest distance from the $\underline{x}=\underline{0}$ plane. For NAFR structures the $\underline{y}^\circ$ point set can be either discrete or continuous with a corresponding collapse multiplier ρ°

set either discrete or continuous. Fig.2 represents the AS rigid region of hypothetical structures with N A F R: in Fig.2a you have a continuous $\underline{y}^\circ$ set and a continuous collapse multiplier ρ° set, and in Fig.2b you have only a $\underline{y}^\circ$ point and only a ρ° collapse multiplier, but the point $\underline{y}^\circ$ don't coincide with the boundaries point subset with the greatest distance from $\underline{x}=\underline{0}$ plane. For the structural safety, the only interesting value of the ρ° collapse multiplier set is the minimum value ρ_c which can be defined from the following *non linear program* (NLP):

$$\rho_c=\left\{ \min \rho \;\middle|\; \begin{array}{l} \underline{\varphi}=\underline{\tilde{N}}\,\underline{x}+\underline{n}\,\rho-\underline{k}_N \le \underline{0} \quad \underline{\dot{\lambda}} \ge \underline{0}, \quad \underline{\tilde{\varphi}}\,\underline{\dot{\lambda}}=0 \\ \underline{M}\,\underline{\dot{\lambda}}=\underline{0}, \quad \underline{\tilde{m}}\,\underline{\dot{\lambda}}=\omega>0, \quad \rho\ge 0 \end{array} \right\}. \tag{6}$$

The NLP (6), often encountered in the structural engineering field [11-13] is a marginally non linear program (in the sense that it is a linear program at all but with the complementarity relations $\underline{\tilde{\varphi}}\,\underline{\dot{\lambda}}=0$, which makes NLP (6), strictly speaking a non convex non linear program) whose solution, theoretically and computationally speaking, is not easy.
On the contrary, it is relatively easy to construct [8] an upper bound ρ_u and a lower bound ρ_ℓ to ρ_c, i.e. $\rho_\ell\le\rho_c\le\rho_u$. The upper bound ρ_u can be obtained as the collapse multiplier of an auxiliary structure which has the same geometry, load sistem and rigid region of the assigned one, but with AFR. The value of ρ_u can be obtained [5] solving the following LP:

$$\rho_u=\{\max \rho \mid \underline{\varphi}_u=\underline{\tilde{N}}\,\underline{x}+\underline{n}\,\rho-\underline{k}_N \le \underline{0}, \quad \rho\ge 0\}, \tag{7}$$

where $\underline{\varphi}_u$ is defined in Eq.(5a), i.e. $\underline{\varphi}_u=\underline{\varphi}$.
The lower bound ρ_ℓ can be obtained as the collapse multiplier of a second auxiliary structure which has the same geometry and load sistem of the assigned one, but a reduced yield domain with AFR. You can define [9] the elemental reduced yield domain as the envelope of the planes with external normal $\underline{M}_j^\circ$ and passing through that points of the original polyedron face from which ortogonal proiection on $\underline{M}_j^\circ$ has the minimum value; in Fig.1 e 2 this reduced domain is depicted with dashed lines. The value of ρ_ℓ can be obtained solving the following LP:

$$\rho_\ell=\{\max \rho \mid \underline{\varphi}_\ell=\underline{\tilde{M}}\,\underline{x}+\underline{m}\rho-\underline{k}_M \le \underline{0}, \quad \rho\ge 0\}, \tag{8}$$

where $\underline{\varphi}_\ell$ is defined from Eq.(5b), i.e. $\underline{\varphi}_\ell=\underline{\Psi}$. In both cases considered in Fig.2, we have depicted the points ρ_ℓ and ρ_u.

4. THE LOWER BOUND APPROACH AND THE NUMERICAL PROCEDURE

We propose a numerical procedure based on a lower bound approach, i.e. on the relation $\rho_\ell\le\rho_c$, where ρ_ℓ and ρ_c are obtained solving the LP (8) and the NLP (6), respectively. We call *lower bound solution* (l.b.s) the optimal solution of the LP (8), i.e. the value set $\rho_\ell,\underline{x}_\ell,\underline{\dot{\lambda}}_\ell$, which verifies the *optimality condition* [10] of the LP (8):

$$\begin{array}{l} \underline{\varphi}_\ell=\underline{\tilde{M}}\,\underline{x}_\ell+\underline{m}\rho_\ell-\underline{k}_M \le \underline{0}, \quad \underline{\dot{\lambda}}_\ell \ge \underline{0}, \quad \underline{\tilde{\varphi}}_\ell\,\underline{\dot{\lambda}}_\ell=0 \\ \underline{M}\,\underline{\dot{\lambda}}_\ell=\underline{0}, \quad \underline{m}\,\underline{\dot{\lambda}}_\ell=\omega>0, \quad \rho_\ell \ge 0. \end{array} \tag{9}$$

We call *admissible collapse solution* (a.c.s.) a value set $\rho^\circ,\underline{x}^\circ,\underline{\dot{\lambda}}^\circ$ which verifies the constraint set of NLP (6) and the relation:

$$\rho_c \leq \rho^\circ . \qquad (10)$$

The following theorems can now be given:

1-st Theorem. *If the lower bound solution is an admissible collapse solution, then the collapse multiplier and the lower bound multiplier values coincide* (i.e. $\rho_c=\rho_\ell$)

Proof. If the l.b.s. $\rho_\ell, \underline{x}_\ell, \underline{\dot{\lambda}}_\ell$, is an a.c.s. then, by Eq.(10), you have $\rho_c \leq \rho_\ell$, whereas $\rho_\ell \leq \rho_c$ by definition. The continuous inequality $\rho_\ell \leq \rho_c \leq \rho_\ell$ implies $\rho_c=\rho_\ell$.

2-nd Theorem. *If the lower bound solution verifies the relation:*

$$\underline{\tilde{\sigma}}_\ell \, \underline{P}^\circ \, \underline{\dot{\lambda}}_\ell = 0, \qquad (\underline{\sigma}_\ell = \underline{\tilde{A}}(\underline{f}^\circ + \underline{f}\,\rho_\ell) + \underline{\tilde{B}}\,\underline{x}_\ell), \qquad (11)$$

then the l.b.s. *is an admissible collapse solution, and, by* th.1, $\rho_c=\rho_\ell$.

Proof. We must demonstrate that the l.b.s. $\rho_\ell, \underline{x}_\ell, \underline{\dot{\lambda}}_\ell$, with the added condition (11) verifies the constraint set of NLP (6). This set differs from Eq.(9) only for the yield function (i.e. $\underline{\varphi}_\ell \neq \underline{\varphi}$) and the complementary condition (i.e. $\underline{\varphi}_\ell \, \underline{\dot{\lambda}} \neq \underline{\varphi} \, \underline{\dot{\lambda}}$), so it is necessary to demonstrate only that $\underline{\varphi}(\underline{x}_\ell, \rho_\ell) \leq \underline{0}$ and $\underline{\tilde{\varphi}}(\underline{x}_\ell, \rho_\ell) \underline{\dot{\lambda}}_\ell = 0$. In order to this, we observe:

i) $\underline{\varphi}_\ell(\underline{x}_\ell, \rho_\ell) \leq \underline{0}$ implies $\underline{\varphi}(\underline{x}_\ell, \rho_\ell) \leq \underline{0}$, because the reduced yield domain $\underline{\varphi}_\ell \leq \underline{0}$ is not external to yield domain $\underline{\varphi} \leq \underline{0}$ by definition.

ii) $\underline{\tilde{\varphi}}_\ell(\underline{x}_\ell, \rho_\ell) \underline{\dot{\lambda}} = 0$, with the cond.(11), implies $\underline{\tilde{\varphi}}(\underline{x}_\ell, \rho_\ell) \underline{\dot{\lambda}}_\ell = 0$; in the elemental stress space $\underline{\sigma}$, the c.c.(9c) can be written as $(\underline{\tilde{M}}^\circ \underline{\sigma}_\ell - \underline{k}^\circ)^t \underline{\dot{\lambda}}_\ell = 0$, and adding to it the cond.(11), you obtain $((\underline{\tilde{M}}^\circ + \underline{\tilde{P}}^\circ)\underline{\sigma}_\ell - \underline{k}^\circ)^t \underline{\dot{\lambda}}_\ell = 0$, i.e. $\underline{\tilde{\varphi}}(\underline{x}_\ell, \rho_\ell) \underline{\dot{\lambda}}_\ell = 0$ by position (2a).

Corollary. *If the lower bound solution implies a collapse mechanism without relative sliding rate at critical sections, i.e. with* $\dot{\varepsilon}_{Tj}=0$ $(j=1,2,\ldots m)$, *then the* l.b.s. *is an admissible collapse solution, and, by* th.1, $\rho_c=\rho_\ell$.

Proof. It is easy to see, by position (2d), that $\dot{\varepsilon}_{Tj}=0$ $(j=1,2,\ldots m)$ implies $\underline{\tilde{\sigma}}_\ell \, \underline{P}^\circ \, \underline{\dot{\lambda}} = 0$, i.e. the l.b.s. is an a.c.s.

Adopting the lower bound approach, we propose a *numerical procedure* in three steps; the first, which requires a not-heavy computational effort, must be always executed, whereas the second and the third, which one requires a heavy computational effort, can be sometime omitted. The steps are: 1) Solve the LP(8), obtaining the l.b.s. $\rho_\ell, \underline{x}_\ell, \underline{\dot{\lambda}}_\ell$; if the obtained ρ_ℓ is sufficient to ensure the structural safety, the procedure can be abandoned; 2) Test, using the previous corollary or Th.2, if the l.b.s. is an a.c.s.; in this case the procedure can be abandoned because $\rho_c=\rho_\ell$; 3) Solve the NLP (6) to obtain the collapse solution and the collapse multiplier ρ_c.

The solution of the NLP (6) can be obtained adopting espressly conceived algorithms [11,12,14], of the branch and bound type, or trasforming the NLP (6) in a mixed integer program (MIP) and solving it by commercial codes [15]. This trasformation can be easily obtained replacing the complementarity cond. (6c) with an equivalent set of constraints on the yield function $\underline{\varphi}$, the multiplier rate $\underline{\dot{\lambda}}$ and an auxiliary boolean vector $\underline{z}$. The MIP problem is:

$$\rho_c = \left\{ \min \rho \left| \begin{array}{lll} \underline{\varphi} = \underline{\tilde{N}}\,\underline{x} + \underline{n}\,\rho - \underline{k}_N \le \underline{0}, & \underline{M}\,\dot{\underline{\lambda}} = \underline{0}, & \underline{\tilde{m}}\,\dot{\underline{\lambda}} = \omega > 0, \dot{\underline{\lambda}} \ge \underline{0}, \\ \beta \underline{z} - \underline{\varphi} \le \underline{0}, & \underline{\lambda} + \beta \underline{z} \le \beta \underline{i}, & z_i = (0,1), \quad \rho \ge 0 \end{array} \right. \right\} \tag{12}$$

where $\underline{i}=(1\ldots1\ldots1)^t$ and $\beta \gg 1$ must be assigned to ensure always that $-\beta<\varphi_i$ and $\lambda_i<\beta$. Beginning the numerical experiences, we chose to solve the NLP in the derived form MIP (12) for two reasons: i) the commercial code, with some arrangements, permit to execute automatically the necessary steps; ii) the codes available to solve the NLP (6) are not sufficiently tested even for moderately large problems. The solved problem are not so many to give a judgement about the validity of formulation (12), but we notice that the assignement of value to β seems to be a critical problem.

5. NUMERICAL EXAMPLE

As a simple application we consider (Fig.3) a two trapeizodal blocks (*A*,*B*), three critical section (*a*,*b*,*c*) plane model, with a one parameter loading system, $f^\circ_{YA}=f^\circ_{YB}=-0.2\,N^\circ_b$, $f_{YA}=0.1\,N^\circ_b$, where N°_b is the yield limit to normal force N_b at sect.*b*, where we have assumed $M_b=T_b=0$. The yield domain at sect.*a* and *c* is as in Fig.1, with yield limits $N^\circ=N^\circ_b/\cos\gamma$ (where $\gamma=15°$ is the section's inclination), $M^\circ=N^\circ \ell/8$, $T^\circ=0.1\,N^\circ$, $\alpha=0.35$ and friction angles $\mu_a=17°$ and $\mu_c=13°$.

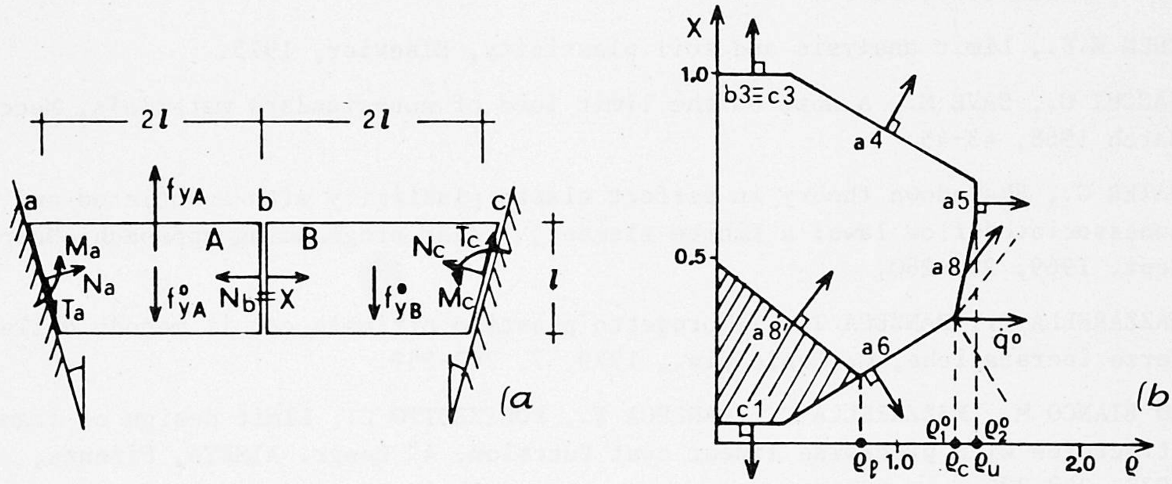


Fig.3 Structural model (a) and assembled structure yield domains (b).

We have depicted the yield domain (the reduced one with dashed lines) in the adimensional AS stress space $\underline{y}=[x\ \rho]^t$, where $x=N_b/N^\circ_b$ is the redundant stress and ρ is the multiplier of the load f_{YA}. The yield planes indication meaning is: the letter denotes the section, and the following digit denotes the yield plane number in the elemental stress space (Fig.1). We remark: i) the yield plane (a5) points and the corner (a8)-(a6) point are admissible collapse solution, i.e. they are two subsets of the $y°$ set defined in Ch.3; ii) the collapse point, i.e. the NLP (6) optimal solution is the corner (a6)-(a8), with $\rho_c=1.356$, $x_c=0.385$; iii) the upper bound multiplier $\rho_u=1.415$ is, at the same time, an admissible collapse multiplier; iv) the lower bound solution is at the corner (a6)-(a8') with $\rho_\ell=0.773$.

ACKNOWLEDGEMENTS

This work has been sponsored by C.N.R. (Italian Research Council) and M.P.I. (Italian Educational Department). This support is gratefully acknowledged.

REFERENCES

1. BENVENUTO E., La scienza delle costruzioni ed il suo sviluppo storico, Sansoni, 1981.
2. COULOMB M., Essai sur une application des règles de Maximis e Minimis à quelques Problèmes de Statique, relatifs à l'Architecture, Mem. Acad. Roy. Sc. Paris, 1776, (7), 343-382.
3. HEYMANN J., The stone skeleton, Int. J. Solids Struct., 1966, 2, 249-279.
4. HEYMANN J., On shell solution for masonry domes, Int. J. Solids Struct., 1967, 3, 227-241.
5. LIVESLEY R.K., Limit analysis of structures formed by rigid blocks, Int. J. Num. Meth. in Eng., 1978, 12, 1853-1871.
6. DRUCKER D.C., Coulomb friction, plasticity, and limit loads, J. Appl. Mech. ASME, March 1954, 71-74.
7. CHEN W.F., Limit analysis and soil plasticity, Elsevier, 1975.
8. SACCHI G., SAVE M., A note on the limit load of non-standard materials, Mecc., March 1968, 43-45.
9. MAIER G., Shakedown theory in perfect elasto plasticity with associated and nonassociated flow laws: a finite element, linear programming approach, Mecc., Sept. 1969, 250-260.
10. MAZZARELLA C., PANZECA T., Il progetto plastico ottimale con il metodo delle forze iperstatiche, G. Genio Civ., 1979, 7, 247-259.
11. LO BIANCO M., MAZZARELLA C., PANZECA T., POLIZZOTTO C., Limit design of frame structures with piecewise linear cost function, 4° Congr. AIMETA, Firenze, 1978, 313-323.
12. GIANNESSI F., JURINA L., MAIER G., Optimal excavation profile for a pipeline freely resting on the sea floor, Eng. Struct., Jan. 1979, 81-91.
13. LO BIANCO M., MAZZARELLA C., Sul progetto ottimale agli stati limite delle strutture in c.a.p., Giornate AICAP, Ravenna, 1981, 27-63.
14. HALLMAN W.P., Complementary problem, Doct. Thesis, Wisconsin Un. Madison, 1979.
15. MPSX/370 with MIP/370 extension, IBM, 1975.