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Reinforced Concrete Members with Subsequently Bonded Steel Sheets

Eléments en béton armé renforcés ultérieurement par collage d'armature

Stahlbetonbauteile mit nachträglich aufgeklebter Bewehrung

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Born in 1939, Marc Ladner received his diploma as a civil engineer in 1962 and his Dr. sc.techn. degree in 1968 from the Swiss Federal Institute of Technology (ETH) in Zürich. Since 1970, he is working at the Swiss Federal Laboratories for Materials Testing and Research where he is the head of the reinforced concrete structure department.

SUMMARY

The paper deals with the design of reinforced concrete members, strengthened by subsequently bonded steel sheets. The amount of additional reinforcement is limited for several reasons which are discussed. Moreover, two methods for determining the stress distribution between concrete and external reinforcing steel are summarized, from which practical design rules are drawn.

RESUME

L'exposé traite du dimensionnement d'éléments en béton armé dont le renforcement ultérieur fait appel à la méthode d'armature collée. Les considérants portent tout d'abord sur la détermination du taux d'armature extérieure pouvant être appliqué à titre de renforcement sur une section existante de béton armé. Il est ensuite fait état de deux procédés pour le calcul des contraintes entre l'acier et le béton. Des conclusions sont tirées pour la pratique.

ZUSAMMENFASSUNG

Der vorliegende Artikel befasst sich mit der Bemessung von Stahlbetonbauteilen mit nachträglich aufgeklebter Bewehrung. Dabei wird zunächst auf die Ermittlung der Grösse des Bewehrungsanteiles eingegangen, der als äussere Bewehrung auf einen bestehenden Stahlbetonquerschnitt aufgebracht werden kann. Sodann werden kurz zwei Verfahren zur Berechnung der Spannungen zwischen Stahl und Beton gestreift und daraus Folgerungen für die Bemessungspraxis gezogen.

1. ASSESSMENT OF THE QUANTITY OF EXTERNALLY BONDED STEEL

1.1 Brittleness of Cross Section

In order to avoid a brittle failure of a strengthened reinforced concrete member with externally bonded steel sheets, the total area of reinforcement (Fig. 1) should not exceed an upper limit of

$$\rho_{tot,max} = \frac{A_{s,tot}}{A_c} \qquad (1)$$

where $A_{s,tot} = A_{si} + A_{sa}$ = total area of reinforcement
$A_c = bd_i$ = area of concrete.

This limiting value has to be chosen in such a way, that both the inside reinforcement and the externally bonded steel come to yield before the concrete fails by crushing in the compression zone.

On the other hand, neither of the two reinforcements should reach its rupture strain, before the concrete fails in compression. Therefore, the total ratio of the reinforcing steel should be higher than

$$\rho_{tot,min} = \frac{A_{s,tot}}{A_c} \qquad (2)$$

as a lower limit.

In the following, the expressions for $\rho_{tot,max}$ and $\rho_{tot,min}$ will be derived under the assumption, that the cross section of a rectangular beam remains plane during bending.

With the notations of Fig. 1, the depth x of the compression zone can be expressed by

$$x = \frac{f_{si,y}A_{si} + f_{sa,y}A_{sa}}{f_c bk_1} \qquad (3)$$

The factor k_1 describes the stress distribution in the concrete compression zone.

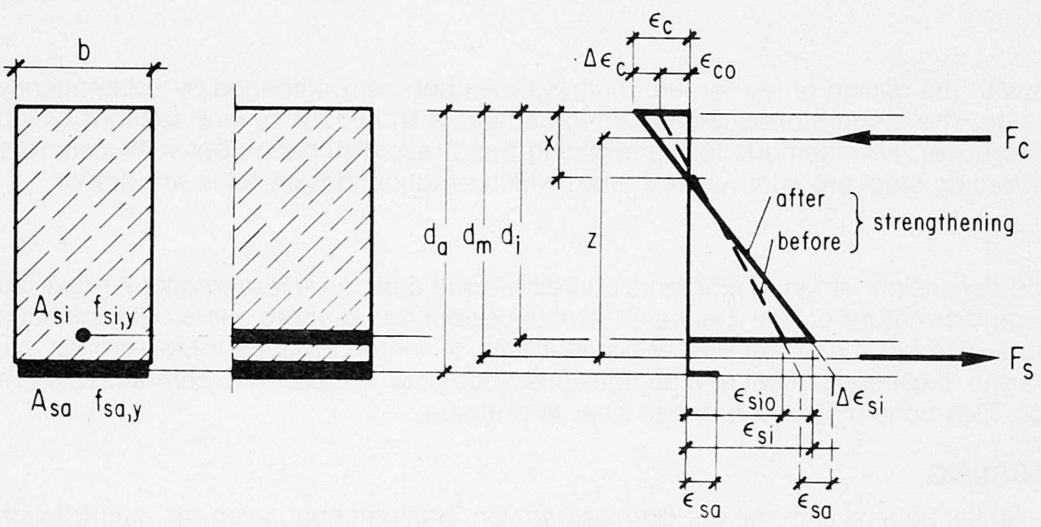

Fig. 1: Notations and dimensions

Using the mean value of the static depth of the cross section

$$d_m = \frac{f_{si,y} A_{si} d_i + f_{sa,y} A_{sa} d_a}{f_{si,y} A_{si} + f_{sa,y} A_{sa}} \qquad (4)$$

the ultimate bending moment of the cross section can then be written as

$$M_u = zF_s = (d_m - k_2 x)F_s \qquad (5)$$

where again k_2 depends on the stress distribution in the concrete compression zone.

With $x = \xi d_m$ equation (5) can be rewritten as

$$M_u = d_m(1 - k_2\xi)F_s \qquad (6)$$

or, after some calculations,

$$M_u \cong d_i\{f_{si,y} A_{si} + \frac{d_a}{d_i} f_{sa,y} A_{sa}\}\{1 - \frac{k_2}{k_1}(\bar{\rho}_i + \bar{\rho}_a)\} \qquad (7)$$

with the notations

$$\bar{\rho}_i = \frac{f_{si,y}}{f_c} \frac{A_{si}}{bd_i} \qquad (8)$$

and

$$\bar{\rho}_a = \frac{f_{sa,y}}{f_c} \frac{A_{sa}}{bd_i} . \qquad (9)$$

When a linear strain distribution is assumed over the cross section, an additional equation for x can be found

$$x = \frac{d_i(\Delta\varepsilon_c + \varepsilon_{co})}{\Delta\varepsilon_c + \varepsilon_{co} + \varepsilon_{sio} + \Delta\varepsilon_{si}} = \frac{d_i}{1 + \varepsilon_{si}/\varepsilon_c} \qquad (10)$$

The combination of the equations (3) and (10) leads to an expression for the sum of $\bar{\rho}_i$ and $\bar{\rho}_a$ as a function of $\varepsilon_{si}/\varepsilon_c$

$$\bar{\rho}_i + \bar{\rho}_a = \frac{k_1}{1 + \varepsilon_{si}/\varepsilon_c} . \qquad (11)$$

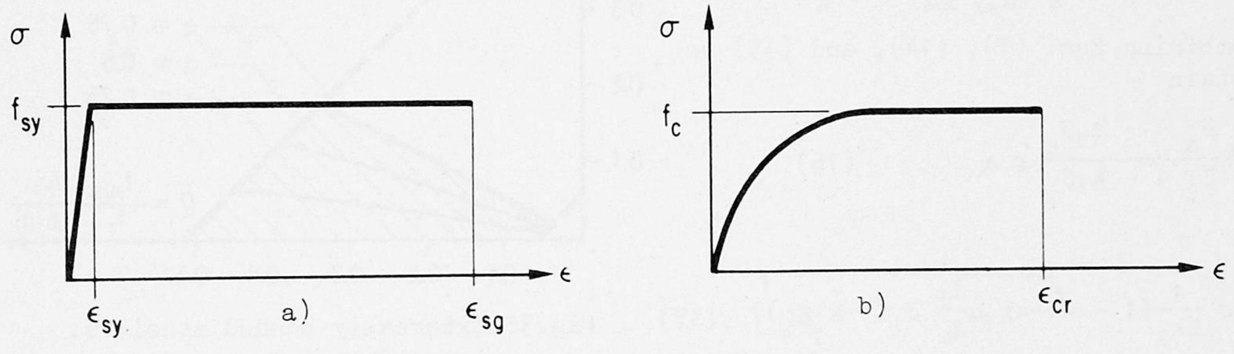

Fig. 2: Idealized σ-ε-diagram for steel (a) and concrete (b)

The σ-ε-diagram for steel can be simplified in a bi-linear form (Fig. 2a). Here, ε_{sy} denotes the strain, when the yield stress is reached for the first time, and ε_{sg} is the strain at rupture. On the other hand, the stress-strain curve for concrete is given in Fig. 2b with ε_{cr} being the strain at rupture.

From the condition

$$\varepsilon_{si,y} \leq \varepsilon_{si} \leq \varepsilon_{si,g}$$

a first set of lower and upper limits for the reinforcement ratio can be obtained

$$\frac{k_1}{1 + \varepsilon_{si,g}/\varepsilon_{cr}} \leq \bar{\rho}_i + \bar{\rho}_a \leq \frac{k_1}{1 + \varepsilon_{si,y}/\varepsilon_{cr}} \qquad (12)$$

In a similar way, the deformation of the externally bonded steel can also be checked. When ε_{sio} and ε_{co} denote the strains of the inner reinforcing steel and of the concrete before strengthening, an additional set of lower and upper limits for the total reinforcing steel area can be established (Eq. 13):

$$\frac{\frac{d_a}{d_i} k_1}{1 + \frac{\varepsilon_{sio}}{\varepsilon_{cr}} \{1+(1+\frac{\varepsilon_{co}}{\varepsilon_{sio}})(\frac{d_a}{d_i}-1)\} + \frac{\varepsilon_{sa,g}}{\varepsilon_{cr}}} \leq \bar{\rho}_i + \bar{\rho}_a \leq \frac{\frac{d_a}{d_i} k_1}{1+ \frac{\varepsilon_{sio}}{\varepsilon_{cr}} \{1+(1+\frac{\varepsilon_{co}}{\varepsilon_{sio}})(\frac{d_a}{d_i}-1)\}+\frac{\varepsilon_{sa,y}}{\varepsilon_{cr}}}$$

Equations (12) and (13) are graphically represented in Fig. 3. For any combination of $\bar{\rho}_i$ and $\bar{\rho}_a$ which lays between the limiting straight lines, it is assured that both inner and outer steel will reach the yield stress before the concrete will fail in the compression zone.

1.2 Limitation by ΔM_u

Some codes or regulations suggest that the increase of the ultimate bending moment ΔM_u, due to strengthening, should be limited

$$\frac{\Delta M_u}{M_{uo}} \leq \alpha \qquad (14)$$

where M_{uo} = ultimate bending moment before strengthening (Eq.7, A_{sa}=0)

M_u = ultimate bending moment after strengthening (Eq.7)

$$\Delta M_u = d_a f_{sa,y} A_{sa} (1-k_3\bar{\rho}_a) \qquad (15)$$

Combining Eqs. (7), (14), and (15) we obtain

$$\frac{d_a}{d_i}\frac{\bar{\rho}_a}{\bar{\rho}_i}\frac{1-k_3\bar{\rho}_a}{1-k_3\bar{\rho}_i} \leq \alpha \qquad (16)$$

or

$$\bar{\rho}_a \leq \frac{1}{2k_3}\{1 - \sqrt{1-4k_3\alpha\frac{d_i}{d_a}\bar{\rho}_i(1-k_3\bar{\rho}_i)}\} \qquad (17)$$

$$\text{with } k_3 = k_2/k_1 \qquad (18)$$

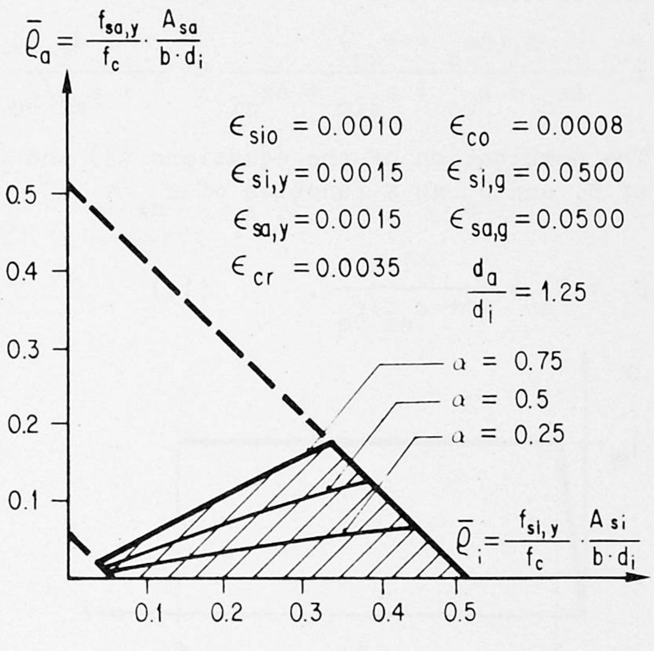

Fig.3: Externally bonded steel vs. internal reinforcement

2. LOAD TRANSFER BETWEEN EXTERNALLY BONDED STEEL AND CONCRETE

2.1 General Remarks

Although the mathematical models used for describing the load transfer from one material to another throughout an adhesive are of simple nature, the formulas given in the next paragraphs do not show a simple form and may therefore not be applicable in practice. In spite of this fact, the results obtained this way may help to understand some of this process and, together with test results, may lead to some advices for dimensioning and design of the anchorage zone of an additional reinforcement.

2.2 Elastic Solution for the Bond Stress Distribution

Let us consider first a concrete prism with bonded steel sheets on two opposite sides (Fig. 4). Assuming that

- the materials concrete, steel, and resin follow Hook's law;
- the resin takes shear stresses only;
- the thicknesses t and d as well as the width b are constant over the total bond length ℓ_a;
- the stresses and strains are distributed uniformly over the total cross section (bending effects are neglected)

then, according to [1], the bond stress distribution is given by

$$\tau(x) = \sigma_{so} t\omega \frac{Ch(\omega x)}{Sh(\omega \ell_a)} \qquad (19)$$

with

$$\omega^2 = \frac{G}{s}\left(\frac{1}{E_s t} + \frac{2}{E_c d}\right) \qquad (20)$$

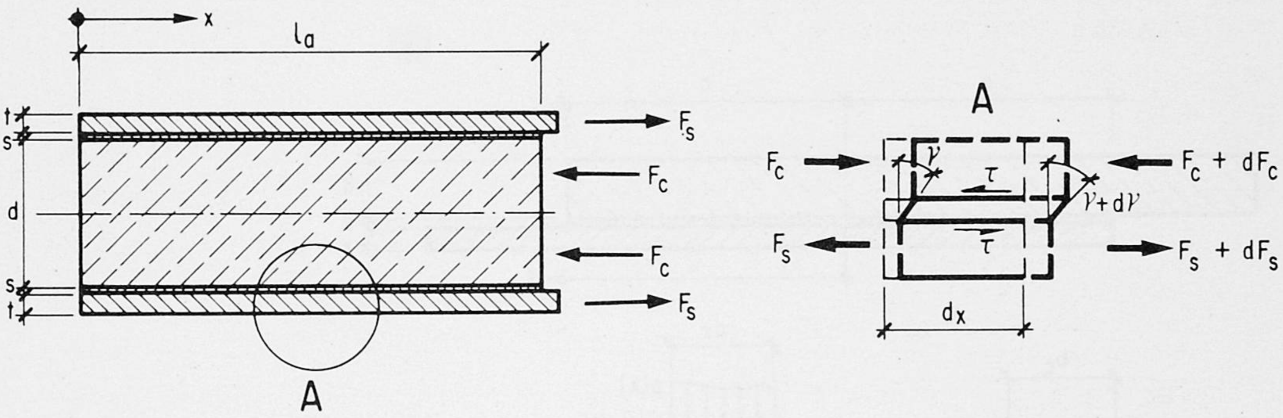

Fig. 4: Concrete prism with bonded steel sheets

The general shape of the bond stress distribution $\tau(x)$ is shown in Fig. 5. From this, it can be seen clearly, that only a very small part of the total anchorage length ℓ_a is needed for load transfer. To find the length ℓ' which is needed for tranferring k percent of the total tensile force F_s from the steel to the concrete ($0 \leq k \leq 1$) the bond stress $\tau(x)$ has to be integrated from $x = x_o$ to $x = \ell_a$:

$$\int_{x_o}^{\ell_a} \tau b dx = kF_s = k\sigma_{so} bt \qquad (21)$$

$$\sigma_{so} tb(1 - \frac{Sh(\omega x_o)}{Sh(\omega \ell_a)}) = k\sigma_{so} bt \quad (22)$$

For $\omega x_o > 1$ this expression can be simplified to

$$\frac{e^{\omega x_o}}{e^{\omega \ell_a}} = 1 - k \quad (23)$$

or

$$x_o = \ell_a + \frac{1}{\omega} \ln(1-k). \quad (24)$$

Finally, the effective anchorage length ℓ' can be written as

$$\ell' = \ell_a - x_o = \frac{-1}{\omega} \ln(1-k). \quad (25)$$

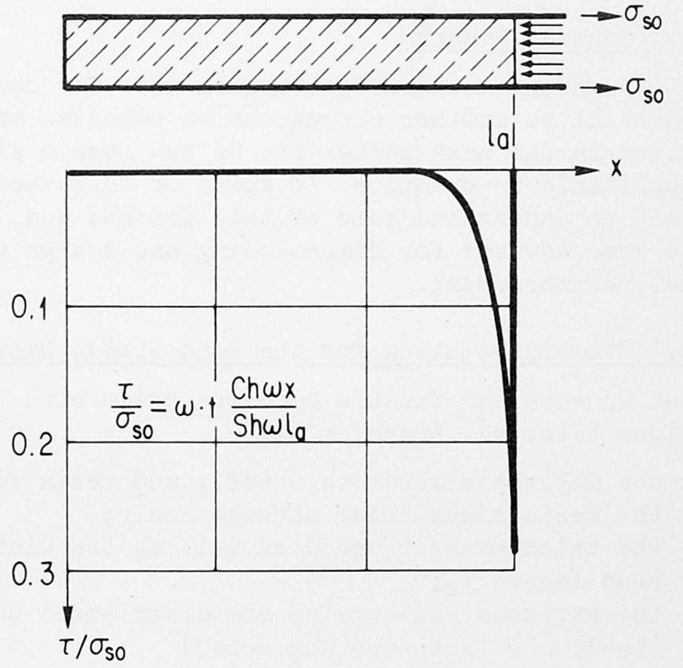

Fig. 5: Bond stress distribution between steel and concrete

2.3 Bond and Normal Stress Distribution in a Glued Joint

A very early study about the normal and bond stress distribution σ and τ in a glued joint was performed by Goland and Reissner in 1944 [2]. Based on their investigation a more general form of the corresponding differential equations for σ and τ will be given in the following (Fig. 6).

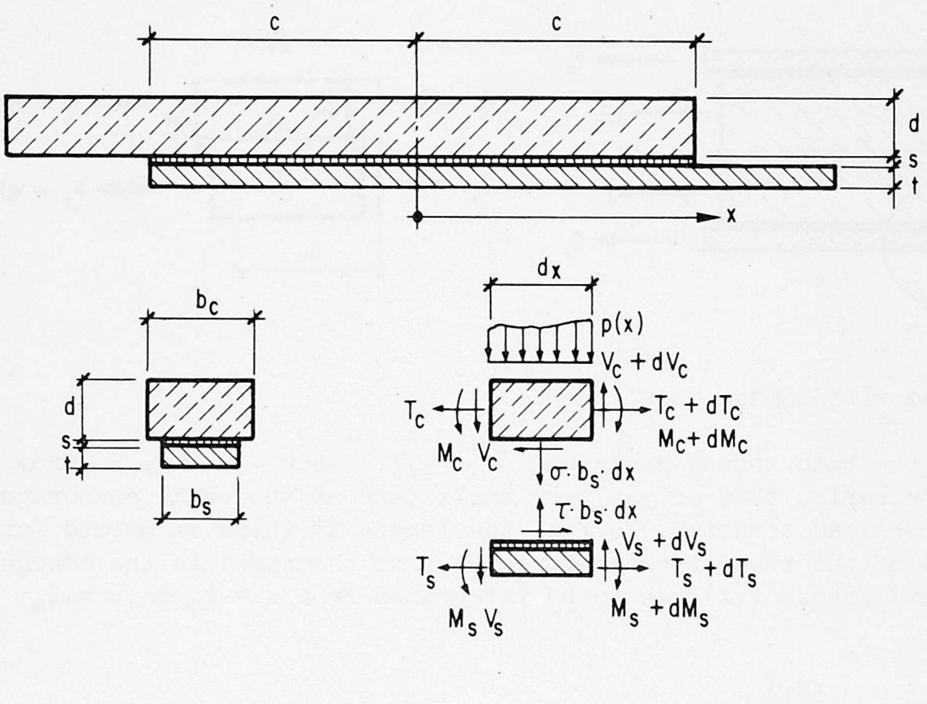

Fig. 6: Notations and dimensions

A first differential equation for τ can be established from the conditions of equilibrium and compatibility

$$\frac{d^3\tau}{dx^3} - \frac{4G_a}{E_s s}\frac{nb_s t+b_c d}{b_c dt}\frac{d\tau}{dx} + \frac{6G_a}{E_s s}\left\{\frac{nb_s t^2-b_c d^2}{b_c d^2t^2}\sigma + \frac{np(x)}{b_c d^2}\right\} = 0 \qquad (26)$$

with E_s = Modulus of elasticity, steel
E_c = Modulus of elasticity, concrete
G_a = Shear modulus, adhesive
$n = E_s/E_c$

In a similar way, a differential equation for σ can be found

$$\frac{d^4\sigma}{dx^4} + \frac{12E_a}{E_s s}\frac{b_c d^3+nb_s t^3}{b_c d^3t^3}\sigma + \frac{6E_a}{E_s s}\frac{b_c d^2-nb_s t^2}{b_c d^2t^2}\frac{d\tau}{dx} + \frac{12E_a}{E_c s b_c d^3}p(x) = 0 \qquad (27)$$

with E_a = Modulus of elasticity, adhesive.

Combining the two differential equations (26) and (27) an equation for σ is obtained

$$\frac{d^6\sigma}{dx^6} - B'\frac{d^4\sigma}{dx^4} + C'\frac{d^2\sigma}{dx^2} - D'\sigma + F' = 0 \qquad (28)$$

The constants B', C', D', and F' contain all geometric and mechanical properties of the specific problem.

At the beginning of a joint, a compression stress σ is present which should be superimposed to the bond stress τ.

For the case, where $E_s = E_c = E$ $(n = 1)$, $b_s = b_c = b$, $d = t$, and $p(x) = 0$, equation (26) becomes independent of σ

$$\frac{d^3\tau}{dx^3} - \frac{8G_a}{Est}\frac{d\tau}{dx} = 0 \qquad (29)$$

and equation (27) is then also independent of τ

$$\frac{d^4\sigma}{dx^4} + \frac{24E_a}{Est^3}\sigma = 0 \qquad (30)$$

The differential equations (29) and (30) were found and solved by Goland and Reissner in [2].

3. CONCLUDING REMARKS

As it can be seen from equation (25), a relatively small amount of the total anchorage length ℓ_a is effective for load tranfer when all materials involved behave elastically. The total anchorage length ℓ_a will only be needed when, due to local failure of the bond, the peak value of the bond stress moves towards the unloaded end of the steel sheet. This phenomenon could be seen clearly in test results as reported in [3]. Thus, a redistribution of the bond stresses takes place.

Consider now in more detail the local force flow in the concrete. The concentrated load transfer at the beginning of the anchorage zone can be compared with a nodal point of a truss, where the tension chord is formed by the steel sheet and the compression diagonal member by the concrete. The vertical strut members are usually also represented by the concrete; however, since in ordinary reinforced concrete structures all tensile forces are usually taken by the reinforcing steel, additional "stirrups" should also be provided to equilibrate these vertical strut forces and to bring them up to the compression chord of the truss model (Fig. 7). In this way, a consistent and adequate model is used throughout the whole structure. Test results have shown, that these unfavourable tensile forces may lead to a failure in the concrete (Fig. 8).

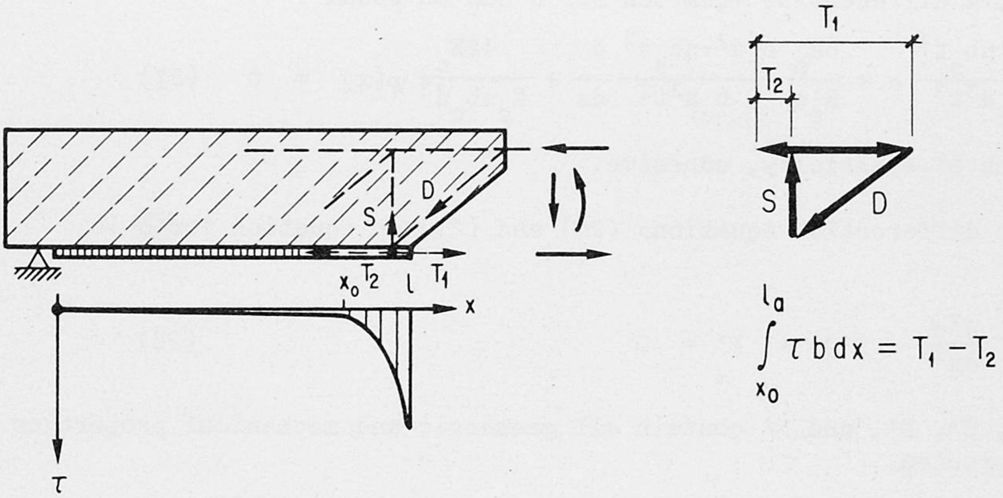

Fig. 7: Truss model

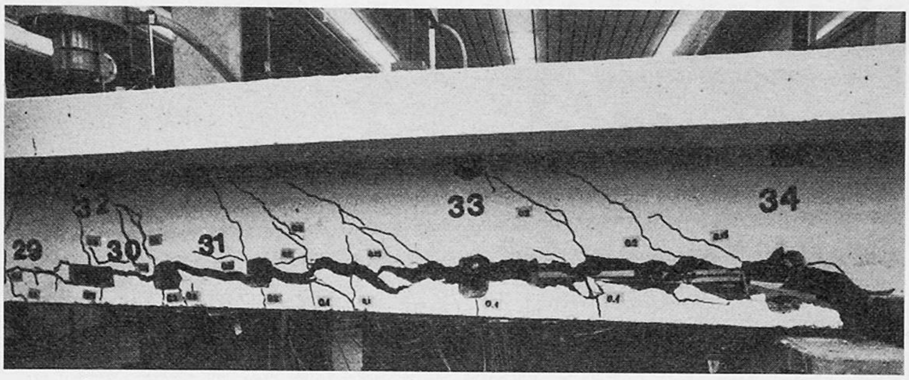

Fig. 8: Failure in a test beam

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