Design of slender webs in steel structures

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Dimensionnement des âmes élancées dans les structures en acier

Bemessung schlanker Stege von Stahlkonstruktionen

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SUMMARY

The paper deals with the effects of initial geometrical imperfections, residual stresses and in-plane boundary conditions of slender webs. The limit state of the web is defined by the elastic and plastic portions of the pre- and postbuckling range. The coefficient of local sheet buckling of a web, subjected to compression for boundary flange sheet panel and for inner sheet panel is discussed.

RÉSUMÉ

Cet article étudie l'effet des imperfections géométriques initiales, des contraintes résiduelles et des conditions limites pour les âmes élancées de poutres d'acier. L'état limite de l'âme est défini par les portions élastique et plastique dans le domaine pré-et post-voilement. L'auteur étudie le coefficient de voilement local de l'âme soumise à la compression dans le cas d'un panneau de bord et d'un panneau intérieur.

ZUSAMMENFASSUNG

Der Artikel befasst sich mit der Wirkung anfänglicher geometrischer Imperfektionen, mit der Wirkung der Eigenspannungen und der Randbedingungen schlanker Stege. Der Grenzzustand eines Steges wird mit dem elastischen und plastischen Anteil des elastischen und überkritischen Beanspruchungsbereiches definiert. Weiter wird der Koeffizient lokaler Ausbeulung des Stegbleches besprochen, welcher entweder in einem Randfeld oder in einem inneren Stegfeld druckbeansprucht wird.

1. INTRODUCTION

In book [1] we have dealt with geometrically and physically non-linear analysis of slender webs with initial imperfections among which the effect of residual stresses was also accounted for. The limit state of the web has been defined by the load-carrying capacity if both the elastic and plastic portions of the post-buckled range were taken into account.

In the following analysis the effects of residual stresses, of boundary conditions and that of initial geometrical imperfections are treated separately. Consequently, the objective of the analysis is to find out, for certain boundary conditions, the stress state in a compressed web with a/ initial geometrical imperfections and b/ residual stresses.

2. A RECTANGULAR WEB SUBJECT TO COMPRESSION

In the edition of Czechoslovak Standard ČSN 73 6205 "Design of Steel Bridge Structures" 1984, pp. 23-24, the ultimate load of the web /or flange/ subject to compression is written as follows:

$$\sigma_{ult} = \varrho_n R_d \qquad /1/$$

where ϱ_n is the coefficient of local web sheet buckling /isotropic web/and R_d the s. c. design strength of the web material /which in the Czechoslovak Limit State Design approach replaces the yield stress R_y; $R_d = 0{,}87\ R_y$ if $R_y \leq 300$ MPa and $R_d = 0{,}80\ R_y$ if $R_y > 300$ MPa/.

The local reduction factor ϱ_n is worked out as follows:
Type I - for boundary flange sheet panels

$$\varrho_n = \frac{40}{b/t + 10}\sqrt{\frac{210}{R_d}} \qquad /1a/$$

Type II - for inner sheet panel

$$\varrho_n = \frac{40\sqrt{210/R_d}}{b/t + 10}\left(1.3 - \frac{12\sqrt{210/R_d}}{b/t + 10}\right) \qquad /1b/$$

/but no more than one/,
where t is the thickness of the web /flange/ sheet and b_{st} the spacing of the longitudinal ribs /or web panel width/.

The above strength, R_d, of the material being inserted in MPa. The above formula /1a, was derived by Z. Sadovský and the 1b by the Author /Table 1/.

The aforesaid isotropic reduction factor ϱ_n is a part of orthotropic plate approach, discussed in the final report of 12th Congress of IABSE in Vancouver.

Table 1

$\frac{b}{t}\sqrt{\frac{R_d}{210}}$	30	40	50	60	70	80	90	100	120	140	160	200
ϱ_n I.	1	0,80	0,67	0,57	0,50	0,44	0,40	0,36	0,31	0,27	0,23	0,19
ϱ_n II.	1	0,85	0,73	0,64	0,57	0,52	0,47	0,43	0,37	0,32	0,29	0,24

In the paper we deal with an isotropic web loaded in compression. The following results demand some deviation from the solution of the formulae /1a/ and /1b/.

3. BASIC SYSTEM OF DIFFERENTIAL EQUATIONS

The problem under consideration is described by the system of von Kármán-Marguerre´s differential equations, generalized for elasto-plastic webs. The problem is solved by Papkovich´s method, that means the compatibility equation is solved exactly and the equilibrium one approximately. The equilibrium equation is solved by Bubnov-Galerkin method. The obtained equations are algebraic cubic equations [1].

The support of the web on its boundaries is assumed to be hinged which in the case of larger depth to thickness ratios is fully compatible with the behaviour of the web. The functions of the initial and additional deflections are considered in the form of series

$$w(x,y) = \sum_{m,n} w_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad /2/$$

all term of which satisfy boundary conditions. As far as the deflection function $w(x,y)$ is concerned, one coefficient, w_{11}, was sufficient for small width-to-thickness ratios of the web, while for larger width-to thickness ratios three coefficients /i.e. w_{11}, w_{13}, w_{33}/ were considered.

3.1 Unrestrained and restrained non-loaded edges

It is necessary to distinguish between the low and high width-to-thickness ratio of the web.

In the case of lower width-to-thickness ratio the deflections of the web are distributed relatively uniformly /the influence of membrane stresses is negligible/. For lower width-to-thickness ratios there is no difference between flexible and inflexible non loaded edges in the web plate plane /e.g. type I and type II of the boundary conditions, table 3/.

If the web slenderness grows the disagreement of the local reduction factor ρ_n for two boundary conditions rises. In Fig. 2 that difference for real initial imperfections $w_0 = b/100$ and $w_0 = b/200$ is remarkable.
Comment: The results of equal reduction factor ρ_n for the treated boundary conditions /unrestrained and restrained non-loaded edges/, becomes obvious in the theory of slender webs. In the stability theory of ideal web plates /e.g. linear problems in the solution of critical stresses, there exist no boundary conditions in the web plate plane. There are no flexible edges in the web plane either.

3.2 The influence of geometrical imperfections

It has been known that unavoidable initial imperfections can exhibit a significant effect on the behaviour of steel thin-walled plated structures. For this reason an international Task Group "Tolerances in Steel Plated Structures" chaired by Professor Ch. Massonnet, some time ago was formed within the framework of IABSE.

To obtain reliable information regarding the magnitude of imperfections occuring on ordinary steel bridges or other similar steel constructions we can start with the initial imperfections on four Czechoslovak steel box-girder bridges. The initial curvature /the stiffeners and sheet panels of both webs and flanges were measured/ was conducted on erected bridges, i.e. inclusive the effect of dead weight of the structure. The contribution is confined to the initial imperfections of the lower flange, since it is in the compressed portion of these flanges where the effect of the unavoidable "dishing" is expected to be most significant. The measured maximum geometrical imperfections are as follows: a/ the motor-way bridge over the river Oslava at Velké Meziříčí - $w_0 = b/109$, b/ the railway bridge at Ivančice - $w_0 = b/320$, c/ the fly-over crossing the railway station Prague-Centre - $w_0 = b/227$ and d/ the new structure of the Barricade Fighters Bridge over the river Vltava in Prague.

By the large deflection theory analysis was determined the coefficient of local web buckling φ_n /for initial imperfections $w_0 = b/200$ and $w_0 = b/100$/ on the relative slenderness $b_{st}/t \cdot \sqrt{R_d/210}$.

The buckling coefficient φ_n is depending not only from the width-to-thickness ratio of the web, but depends also from the side ratio of the web panel $\alpha = a/b$. The smallest value of the buckling coefficient φ_n of the web subject to compression /for the same value of the width-to-thickness ratio/ is approximately for a/b = 0.8, that have been used in calculation.

The influences of geometrical imperfections for high width-to-thickness ratio is negligible. On the other hand for the low width-to-thickness ratio this influence is greater /for the relative slenderness $b/t \cdot \sqrt{R_d/210} = 40$ the difference φ_n is about 20-25%/.

3.3 Residual stresses

Residual stresses develop as results of the welding process. Welds as well as their immediate vicinity are subjected to tension equal to the yield stress, and the remainder of the cross-section is subject to residual compression. The residual tension and compression stresses satisfy equilibrium condition.

The depth of tension field, practically independent of the total depth of the web, can be found out with due regard to the welding speed, heat sources, preheating, web thickness and the like. In bridge structures or in civil engineering structures the tension zone is assumed to extend to the distance c = 2t to c = 4t, where t is the web thickness /Fig. 1/.

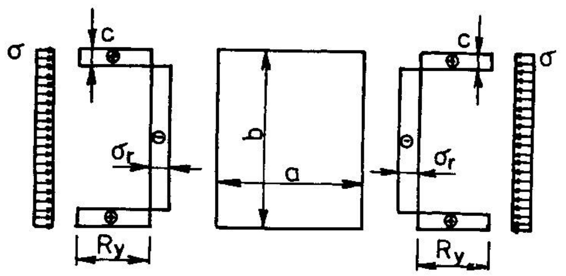

Fig. 1

When the web has been cooled, the longitudinal stresses near the weld remain close to the yield stress and are balanced by compressive stresses. It is advantageous /for the sake of calculations/ to work with idealized residual stresses, which very satisfactorily describe the actual stress distribution. A simple rectangular tension distribution of the residual stresses would be suitable for using in the analysis /Fig. 1/.

The residual stress on loaded boundaries of the web is of the form

$$n_r = R_y \frac{2c}{b - 2c} ,$$

where n_r is the compressive residual stress.

The compressive residual stress n_r reduces as the width-to-thickness ratio b/t increases. The value of n_r/R_y are taken as follows

b/t	30	45	60	90	120
n_r/R_y	0.25	0.15	0,11	0,07	0,05

In all cases the value c = 3t is introduced the yield stress of web material R_y = 230 MPa being considered.

The increment of the deflection w_{11}/t was taken by 0.0025 for the elasto-plastic region which did not affect the results.

The residual stresses were calculated as percentage of relative ultimate stress $\rho_n = \sigma/R_y$ for geometrical imperfections w_0 = b/100 and w_0 = b/200. The flexible unloaded edges in the web plane were considered. The resulting maximum residual stresses σ_r for width-to-thickness ratios of the web b/t are as follows

b/t	30	45	60	90
σ_r %	20.60	8.13	6.86	3.02

The stress σ_r for higher geometrical imperfections w_0 = b/100 is presented. For lower w_0 the residual stress would be smaller. So, for example, if w_0 = b/200 and b = 30, the residual compressed stress σ_r = 9.83 %, for b/t = 60, σ_r = 3.44 %.

The stress distribution in the slender web /for the points x,y,z = ± t/2 / can be followed in Table 2. The results refer for 1/4 of the compressed web. By c_1 = 1 are denoted the points, where the relative effective stress σ_e/R_y /where σ_e = $(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + \tau^2)^{1/2}$) is lower or equal to one.

Table 2

The relative effective stress σ_e/R_y in the web loaded in compression

1	1	1	1	1	1
1	1	1	1	1	1
1.15	1.25	1.31	1.33	1.33	1.32
1	1.04	1.14	1.24	1.31	1.33
1	1	1.03	1.16	1.27	1.32
1	1	1	1.13	1.25	1.31

Let us mention that the welding residual stress depends not only of the web slenderness but also from the geometrical imperfection.

4. COMPARISON OF RESULTS

According to the obtained results /the effect of residual stresses and that of the geometrical imperfections and boundary conditions is treated/ the conclusions can be written. The magnitude of initial imperfections was taken as w_0 = b/200, that it is also the conclusion of the work of International Task Group "Tolerances in Steel Plated Structures" formed a few years ago within the framework of IABSE. Welding residual stresses /that depend also on geometrical imperfection/ are taken also by the value w_0 = b/200.

There follows from the obtained results, that it is advantageous to design slender webs, with minimal effect of residual stresses, which refer to higher width-to thickness ratio /Fig. 2/.

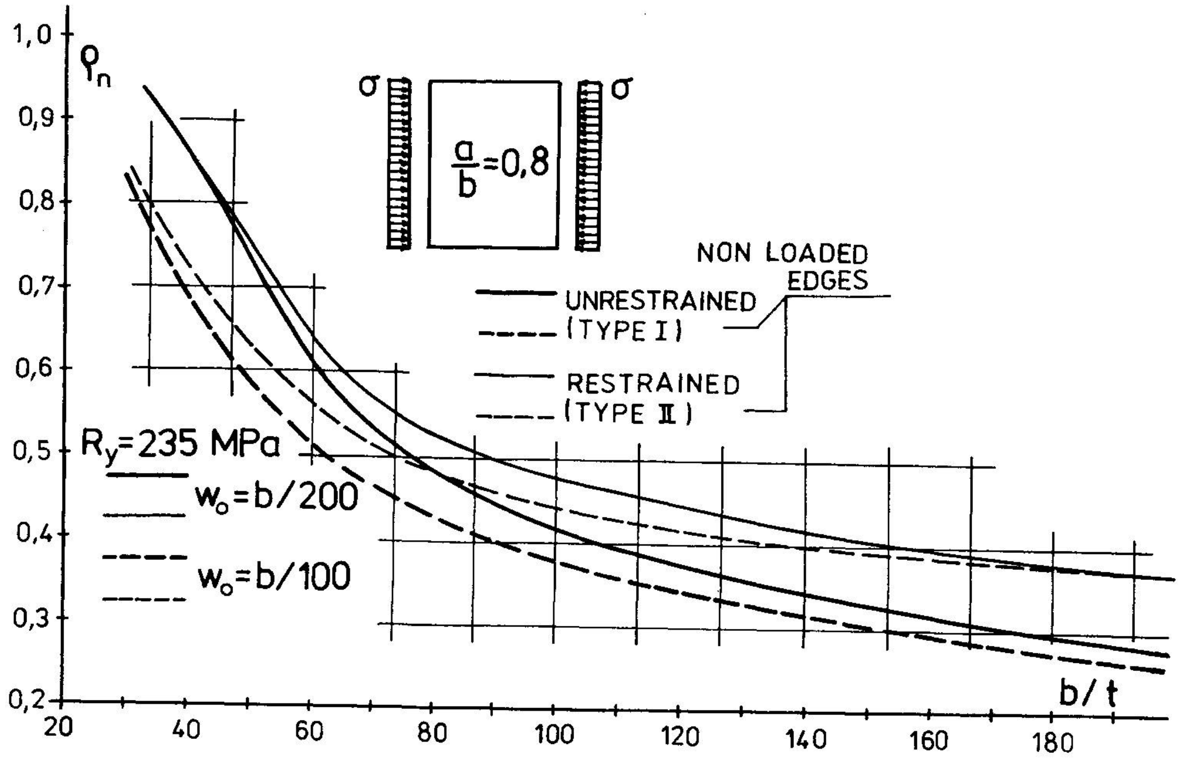

Fig. 2

If the initial imperfections /less or equal to w_0 = b/200/ are considered , the coefficients of local web buckling ρ_n /type I for boundary sheet panels, type II for inner sheet panels/ according to Table 3 should be used.

Table 3
Coefficient ρ_n of sheet buckling of rectangular simply supported web subject to compression

$\frac{b}{t}\sqrt{\frac{R_d}{210}}$	30	40	50	60	70	80	90	100	120	140	160	200
I.	1	0.80	0.67	0.58	0.53	0.48	0.45	0.42	0.38	0.35	0.32	0.28
II.	1	0.82	0.73	0.64	0.57	0.52	0.50	0.48	0.44	0.42	0.39	0.36

5. OTHER TYPES OF WEB LOADING

We have dealt in detail with the slender web subjected to compression. The obtained results /the effect of initial imperfections and residual stresses/ are of the same character for other types of web loadings such as bending, shear and combined bending and shear.

References

1. DJUBEK, J. - KODNÁR, R. - ŠKALOUD, M.: Limit State of the Plate Elements of Steel Structures. Birkhäuser Verlag, Basel-Boston-Stuttgart, 1983.