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Stability of Column-Girt-Diaphragm Systems in Industrial Buildings

Stabilité des systèmes poteaux-entremises-diaphragme dans les bâtiments industriels

Stabilität des Systems Stütze-Riegel-Wandscheibe in Industriegebäuden

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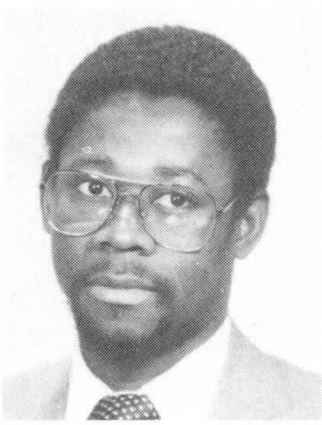

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SUMMARY

Two finite element models to study column-girt-diaphragm systems are presented. The first one is a general model for all components; the second is a simplified model using beam elements for the column and equivalent elastic elements for the other components at girt level. The simplified model is mostly oriented towards practical design. The numerical results show that column resistance not only depends on the physical properties of the assemblage elements but is significantly influenced by the arrangement of these elements.

RÉSUMÉ

L'article présente deux modèles par éléments finis pour l'analyse des systèmes poteaux-entremises-diaphragme. Le premier est un modèle général où toutes les composantes sont modélisées tandis que le second, qui a été développé principalement pour les calculs pratiques, ne modélise que le poteau alors que les autres pièces du système sont remplacées par des ressorts équivalents au niveau des entremises. Les résultats numériques montrent que la capacité portante du poteau dépend non seulement des propriétés physiques des éléments du système, mais aussi de leur arrangement.

ZUSAMMENFASSUNG

Zwei Systemmodelle werden mit Hilfe der Finiten-Element-Methode formuliert. Das erste ist ein allgemeines Modell, das alle Systemteile einzeln erfasst. Das zweite ist ein vereinfachtes Modell, das ein Balkenelement für die Stütze und äquivalente elastische Elemente für die übrigen Teile benutzt. Das vereinfachte Modell ist für die Anwendung in der Ingenieurpraxis bestimmt. Die numerischen Resultate zeigen, dass die Stützenfestigkeit nicht nur von den Materialeigenschaften der Systemteile, sondern auch wesentlich von der Art ihrer Anordnung beeinflusst wird.

1. INTRODUCTION

Corrugated steel sheets generally attached to girts are frequently used as cladding in industrial buildings. The girts are either connected to the column web or external flange based on physical or economical considerations. For light columns, girts are generally attached to the web at the center of gravity of the column section (Fig. 1-a). For deep columns, girts are either attached to the web near the external flange, or simply lie outside the external flange and are attached to it (Fig. 1-b,c). The internal flange of deep columns is generally braced by means of diagonal elements attached to girts in order to reduce, if not eliminate, the possibility of column torsional buckling. In such column-girt-diaphragm (C.G.D.) systems, the lateral support provided by the diaphragm at girt level may substantially increase the column buckling resistance. In practical design, the girt-diaphragm system is either assumed fully rigid or is completely neglected when evaluating the column resistance. Due to the complex interaction existing between the different components of C.G.D. systems, it is necessary to use numerical models such as finite element models to correctly assess the stabilizing action of the girts and the diaphragms.

Some early designers have taken account of the influence of girt-diaphragm systems through column restraints [6,7,8,12]. These were represented by discrete or continuous systems whose equivalent rigidity was defined by some calibration criteria. Apparao, Errera and Fisher [2,3] carried out research on the stabilizing action of girt-diaphragm systems and developed design formulas for simple loading and boundary conditions.

The purpose of this paper is to present a tri-dimensional finite element model that simulates the behaviour of the global physical system in ordre to evaluate the load carrying capacity of the columns. Based on the numerical results obtained from this model, we developed a simplified model for practical design. A detailed numerical experimentation was undertaken to assess the efficiency and reliability of the simplified model.

2. FINITE ELEMENT MODEL OF C.G.D. SYSTEMS (FEM)

2.1 Linear buckling model: basic assumptions

The buckling model is obtained by equating the linearized expression of the second variation of the total potential energy to zero. It is also interpreted as the perturbed expression of the virtual work. The corresponding variational expression W may be written as follows:

$$\Delta W = \int_V (\delta e_{ij}\, \Delta\sigma_{ij} + \lambda_{cr}\, \sigma_{ij}\, \Delta\, \delta\eta_{ij})\, dV = 0 \qquad (1)$$

where $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$: linear part of Green-Lagrange strain

$\eta_{ij} = \frac{1}{2} u_{k,i} \cdot u_{k,j}$: nonlinear quadratic part

with $u_{i,j}$ representing the j^{th} spatial derivative of displacement component u_i

λ_{cr} = critical load factor for the stress state σ_{ij}

$\Delta\sigma_{ij}$ = perturbed stress due to the perturbation Δe_{ij}

$\Delta(\delta\eta_{ij})$ = perturbed expression of the first variation of nonlinear terms of Green-Lagrange strain (linear in $\Delta u_{i,j}$)

The finite element discretization of Eq. (1) leads to the following usual eigenvalue problem:

$$[K]\ \{\Delta u_n\} = \lambda_{cr}[K_g]\ \{\Delta u_n\} \qquad (2)$$

where [K] is the rigidity matrix at the state $[\sigma_{ij}]$ and $[K_g]$ is the geometrical matrix; λ_{cr} and Δu_n represent the set of eigenvalues and eigenvectors, respectively.

The solution of Eq. (2), using for example the subspace iteration scheme, leads to the set of lowest values of the critical load factors λ_{cr} [4]. Equation (1) may be adapted to structures with plastic deformations requiring a proper calculation of σ_{ij} and estimation of λ_{cr} close to unity. In this study, we assume that the structural behaviour is elastic up to buckling thus allowing the definition of buckling stress $\lambda_{cr}\sigma_{ij}$ in terms of a certain initial state of stresses σ_{ij}.

The model considers the contribution of all the components of the C.G.D. system (column, girts, diaphragm, connections and diagonal braces) when evaluating the rigidity matrix [K] of the system. The state of stresses σ_{ij} is assumed to be equal to zero for all elements except the column since we are mainly interessed in getting buckling load estimates for column stress states. It should be noted that the desired buckling model must be properly included in the perturbed configurations $\Delta(\delta\eta_{ij})$.

2.2 Column and girt modeling

Column and girt components are represented as tri-dimensional beam elements (Fig. 2) including axial, bending and torsional deformations. The displacement components u, v, w (u_1, u_2 and u_3) of a material point (x, y, z) are defined as follows, making use of the approximations of Vlassov [13] and Timoshenko [12] for thin-walled beams which are generalizations of Bernoulli-Navier hypotheses (thin beams) and St-Venant hypotheses for pure torsion:

$$\begin{aligned} u\ (x,y,z) &= u_o(z) - (y-y_o)\ \phi\ (z) \\ v\ (x,y,z) &= v_o(z) + (x-x_o)\ \phi\ (z) \\ w\ (x,y,z) &= w_c(z) - x\ u_o'(z) - y\ v_o'(z) + \omega(s)\ \phi'\ (z) \end{aligned} \qquad (3)$$

where u_o, v_o are transverse displacements at the shear axis, w_c the axial displacement along the neutral axis and ϕ the torsional rotation. The quantities u_o', v_o' and ϕ' are derivatives with respect to z. The warping function is defined as:

$$\omega = -\int_o^s r_o(s)\ ds, \text{ where s is the sectional coordinate.}$$

The linear strain components simply reduce to $\varepsilon_z = w_c' - xu_o'' - yv_o'' + \omega(s)\phi''$ and $\gamma_{zs} = 2n_h\,\phi'$ [10]. By using the displacement relations (3) and choosing the corresponding $\Delta(\delta n_{ij})$ for a known σ_{ij}, we obtain the column and girt contributions to the [K] and $[K_g]$ matrices. If σ_{ij} is zero in the girt, the corresponding geometrical matrix $[K_g]$ is equal to zero.

2.3 Corrugated steel sheet modeling

We simulated the behaviour of corrugated steel sheets by a flat shell triangular element (DKT type) (Fig. 3) in order to consider equivalent orthotropic membrane and bending behaviour. More details are given in references [5,11]. Since the buckling load corresponds to a state of stresses in the column, the sheet steel cladding only contributes to the rigidity matrix [K] with corresponding $[K_g] = 0$.

2.4 Connection and brace modeling

In order to insure geometrical compatibility among the various eccentric components, special connection elements were used (Fig. 4) to introduce these constraints in a routine manner. The stiffness properties (K_x, K_y, K_z, $K_{\theta x}$,

$K_{\theta y}$, $K_{\theta z}$) of the connection elements (Fig. 4-a) may be adjusted to take account of flexible connections between girts and columns and between the diaphragm and the supporting elements. The diagonal braces are represented by the same type of elements and can be attached to any point on the column section (Fig. 4-b) even though the column is simulated by a linear finite element. More details are given in reference 1 on the development of connection and brace elements.

3. SIMPLIFIED MODEL (SM)

It is interesting to develop a simplified model for practical applications on micro-computers. The results of the general finite element model (FEM) may be used to define the accuracy of the simplified model (SM). In the first version of this model only the column is represented by a set of beam elements [9]. The other components of the system (diaphragm, girts, connections, braces) are modeled by equivalent elastic elements at girt level. The properties of these elements include the flexural rigidity of the girt about its strong axis, the girt-to-column connection rigidity and the shear rigidity of the diaphragm. The influence of braces was taken into account by increasing the flexural rigidity of girt elements. The diaphragm is assumed to act in simple shear by providing a rigidity evaluated from its effective shear modulus.

4. NUMERICAL EXAMPLES

4.1 Experimental test results and influence of girt-diaphragm eccentricity

Three tests were conducted at Cornell University to determine the buckling load (P_{cr}) of columns in C.G.D. systems. Details of these tests can be found in Ref. 2 . The test results are compared in Table 1 to the analytical results obtained from the general finite element model (FEM) and the simplified model (SM). The analytical results are less than 1% different from test results GT-2 and GT-3. However, there is a significant difference with the GT-1 test for which a perfectly pinned girt-to-column connection was considered. This difference may be attributed to the fact that it is difficult to realize a perfectly pinned connection in laboratory. The buckling load is highly sensitive to the rotational rigidity of the girt-to-column connection, as seen from test GT-3.

Test no.	$K_{\theta y}$ (1) (kNm/rad)	e (mm) (Fig.1)	Experimental results (kN)	FEM (error) (kN)	SM (error) (kN)	Buckling mode
GT-1	0	152	78.7	64.3 (18%)	64.3 (18%)	flexural-torsional (fig. 5)
GT-2	10^{10}	254	165.9	164.3 (1%)	164.3 (1%)	flexural
GT-3	1.469	152	113.4	112.0 (1%)	112.5 (1%)	flexural-torsional

(1) $K_{\theta y}$ is the rotational rigidity of the girt-to-column connection with respect to the column longitudinal axis.

Table 1 Comparison of experimental and theoretical results

The influence on column buckling load of girt and diaphragm eccentricities with respect to the column shear center (e_1 and e_2, respectively, as shown in Fig.1), was studied with the general model for the above three cases. The results are given in Table 2. The eccentricity of the girts with respect to the column shear center reduces the column buckling resistance in a significant manner for flexible girt-to-column connections. However, the buckling load for fully restrained connections is independent of the eccentricity of the girts for the cases studied.

e_2 (mm) (Fig.1)	e_1 (mm) (Fig.1)	GT-1 (P_{cr}/P_E)(2) FEM (kN)	GT-2 (P_{cr}/P_E) FEM (kN)	GT-3 (P_{cr}/P_E) FEM (kN)
e(1)	0	164.3 (1.0) (flexural)	164.3 (1.0) (flexural)	164.3 (1.0) (flexural)
62	($e-e_2$)	106.9 (0.65) (flexural-torsional)	164.3 (1.0) (flexural)	164.3 (1.0) (flexural)
32.4	($e-e_2$)	82.2 (0.50) (flexural-torsional)	164.3 (1.0) (flexural)	146.3 (1.0) (flexural-torsional)
15	($e-e_2$)	71.6 (0.44) (flexural-torsional	164.3 (1.0) (flexural)	126.2 (0.77) (flexural-torsional)
0	e	64.3 (0.39) (flexural-torsional)	164.3 (1.0) (flexural)	112.0 (0.68) (flexural-torsional)

(1) e = 152 mm for the GT-1 and GT-3 tests and 254 mm for the GT-2 test.

(2) P_E = 164.3 kN is the critical load assuming flexural buckling between girts.

Table 2 Influence of eccentricities

4.2 Influence of diagonal braces

The sub-system shown in Fig. 6 was used to study the influence of diagonal braces. Two different diaphragm rigidities were considered together with perfectly pinned and highly rigid girt-to-column connections. The critical column flexural load, considering the unsupported length between girts, was 5514 kN. The results given in Table 3 show a good agreement between the two models. It is interesting to note that for relatively high values of the diaphragm effective shear modulus (G_{eff}), the diagonal braces provide an efficient torsional restraint even if the girts are pin-connected to the column. In the case of more flexible diaphragms, the critical flexural load was never reached.

5. CONCLUSION

Two finite element models have been presented to analyse column-girt-diaphragm systems. The first model (FEM) is applicable to a wide variety of C.G.D. systems, including those with openings, and is used in the present case for assessing the accuracy of the simplified model. The second is oriented towards design and can easily be supported by micro-computers. The results indicate that the two models are practically equivalent in the elastic domain for the problems considered. This implies, considering the assumptions retained in each model, that the axial rigidity of the girts is negligible and that the axial and flexural stiffnesses of the cladding have no significant effect on the column buckling resistance.

The results also show that the critical buckling load of columns braced by girt-diaphragm systems is not only a function of the diaphragm shear resistance but also of parameters such as the girt-to-column connection rigidity, diaphragm and girt eccentricities, diagonal braces, and the flexural rigidity of the girts. Further developments are under way to include plastification, among others, in the finite element model.

Example	G_{eff} (MPa)	$K_{\theta y}$ (kN.m/rad)	Diagonal area $A(mm^2)$	FEM (P_{cr}/P_E) (kN)	SM (P_{cr}/P_E) (kN)	Buckling mode
CGD1	1000	0	0 (no brace)	2061 (0.37)	2071 (0.37)	flexural-torsional
CGD2	2300	0	0 (no brace)	2252 (0.41)	2264 (0.41)	flexural-torsional
CGD3	1000	0	216	4903 (0.89)	-	flexural-torsional
CGD4	2300	0	216	5514 (1.0)	-	flexural
CGD5	1000	10^{10}	216	4905 (0.89)	4876(1) (0.88)	flexural-torsional
CGD6	2300	10^{10}	216	5514 (1.0)	5514(1) (1.0)	flexural

(1) Diagonals are replaced by overlapped girts (Fig. 6-c).

Table 3 Influence of diagonal braces

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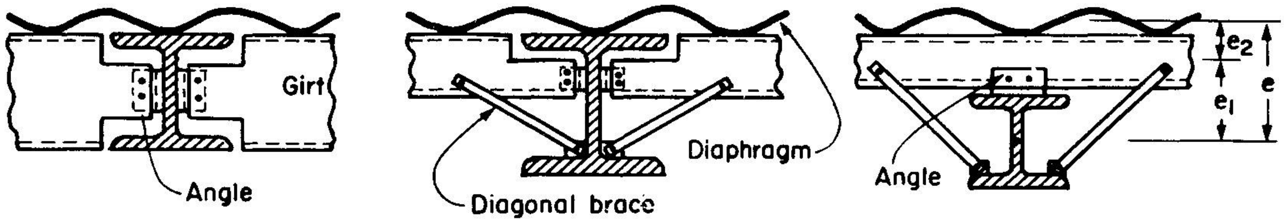
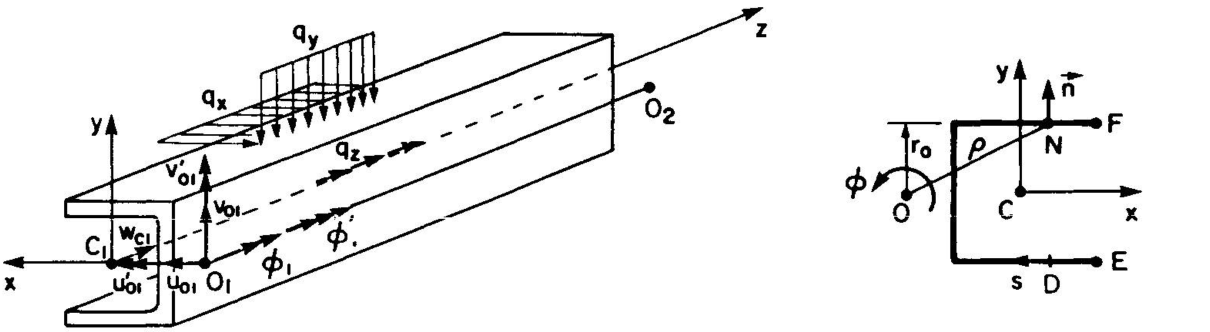


a) Light column *b) Deep column with girts between the flanges* *c) Girts outside the external flange*

Fig. 1 — Girt-to-column connections.



a) Degrees of freedom of the beam element *b) Thin-walled open cross section*

Fig. 2 — Tri-dimensional beam element (7 DOF per node).

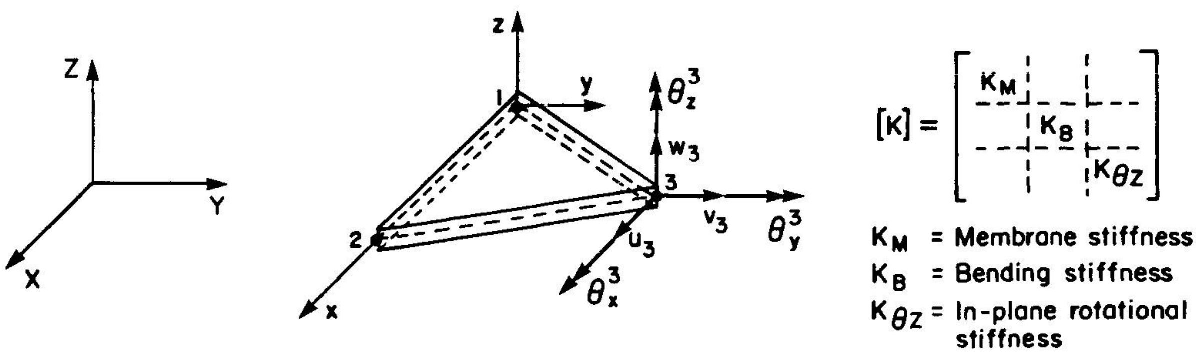
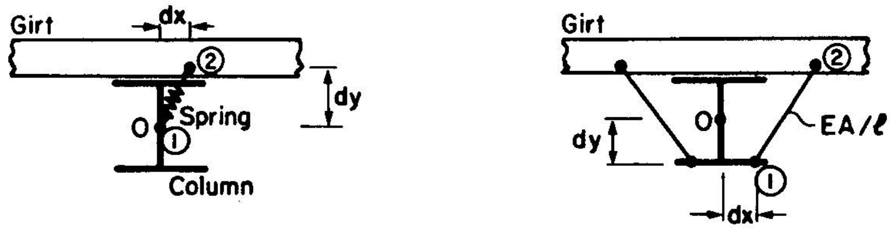
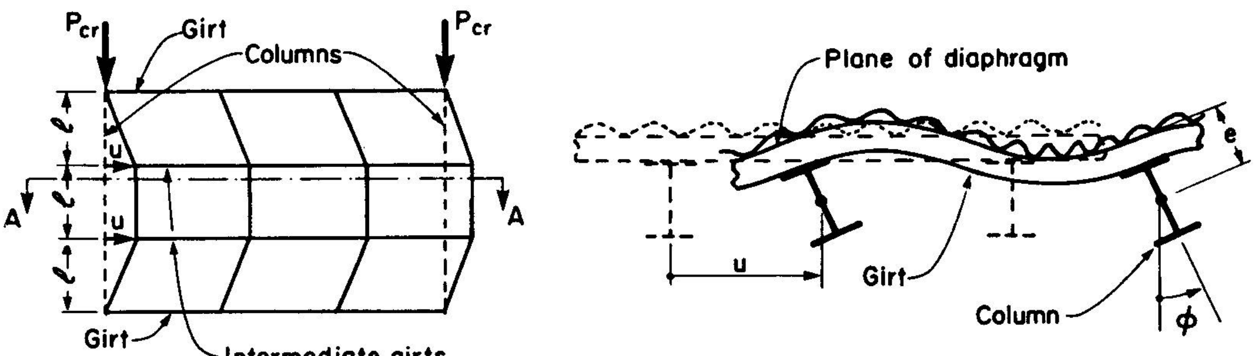


Fig. 3 — Orthotropic shell element (6 DOF per node).



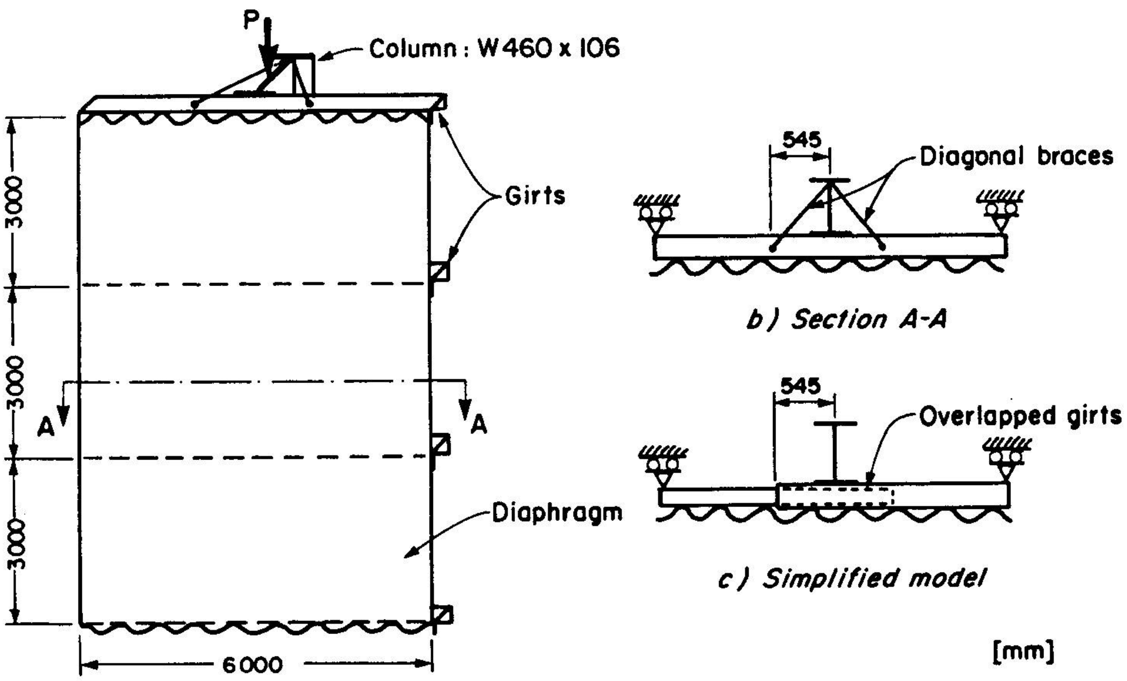
a) Girt-to-column spring *b) Diagonal braces*

Fig. 4 — Connection elements.



Section A-A

Fig. 5 — Flexural-torsional buckling mode.



a) Elevation

Fig. 6 — C.G.D. sub-system with diagonal braces (pin-ended column)