Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 51 (1986)

Artikel: Optimisation of measures for quality assurance

Autor: Rackwitz, Rüdiger

DOI: https://doi.org/10.5169/seals-39565

Optimisation of Measures for Quality Assurance

Optimisation des mesures prises pour l'assurance de la qualité

Optimale Verteilung von Qualitätssicherungsmassnahmen

Rüdiger RACKWITZ
Dr.-Ing.
TU Munich
Munich, FR Germany

Rüdiger Rackwitz, born 1941, has been head of a research group in structural reliability at the Technical University of Munich for more than ten years.

SUMMARY

Planning for quality also involves quantitative optimisation provided that the overall reliability problem has been modelled appropriately. Importance and sensitivity measures are defined in the context of first-order reliability methods which can be used for optimisation of quality assurance measures.

RÉSUMÉ

La planification de la qualité est aussi une question d'optimisation quantitative lorsque le problème général de la fiabilité a pu être modélisé de façon appropriée. Des mesures d'importance et de sensibilité sont définies dans le cadre de la méthode de fiabilité du premier ordre. Celles-ci peuvent être utilisées avec avantage pour optimiser les mesures prises pour l'assurance de la qualité.

ZUSAMMENFASSUNG

Qualitätsplanung ist auch zahlenmässige Optimierung, wenn das allgemeine Zuverlässigkeitsproblem in geeigneter Weise modelliert werden konnte. Importanz- und Empfindlichkeitsmasse werden im Rahmen der Zuverlässigkeitsmethode 1. Ordnung definiert. Diese können mit Vorteil für die Optimierung von Qualitätssicherungsmassnahmen eingesetzt werden.

1. INTRODUCTION

"Quality" of a technical facility in the broad sense can be taken as is its property to fulfill its intended purpose reliably during the anticipated time of use. Quality must be produced. In particular, the facility must be designed, constructed and, possibly, maintained such that it withstands all foreseen internal and external actions. But both the capacities and the demands usually are highly uncertain and, thus, performance and safety requirements can only be met with a certain probability. Quality could exist without being assured. But, usually, efforts are being undertaken to specifically provide quality and this activity is called "quality assurance" herein. Clearly, the success of an activity needs to be verified. Thus, we shall include those passive actions into our notion of quality assurance although these are only of limited interest in our context. Here, any quality assurance action is understood to serve either for the control or, and preferably, for the reduction of prior uncertainties. On the other hand, the resources in money, man-power or time, individually or jointly, are always limited. Since, as a rule, increasing investments into the various means to achieve quality have a favourable effect on reliability a prominent engineering task is to specify the types of appropriate quality assurance measures and to allocate their intensity in the most efficient way.

Obviously, quality assurance activities should start in the very first phases of a project as they guide the amount of pre-investigations on the specific environmental parameters and potential building materials, the general lay-out of the system, and later, the scope of design calculations and, possibly, development tests, the constructions procedures and their control, and type and extent of the final qualification procedures; to name a few of those quality assurance measures. And it should also be clear that, under given performance and/or reliability constraints, each project has its own optimal distribution of quality assurance efforts.

Those uncertainties may be classified into several categories:

- classical (random) variations in the physical quantities such as material properties, structural geometry, internal and environmental actions on the structure.

- parameter and model uncertainties

- human errors

- professional ignorance

Almost nothing can be done about the last type of uncertainty. For the other types of uncertainties, however, reasonable quantitative models of varying realism and sophistication exist. The first type of uncertainty is the subject of modelling in classical structural reliability. Sufficient knowledge has been accumulated in the past years so that there is no need for further discussion, herein. The second type of uncertainties, which frequently dominates those of the first kind, can usually be removed by valid experiments, at least in principle. One of the primary aims of quality assurance, no doubt, is just to diminuish these uncertainties to a reasonable level. Human errors are the subject of a number of contributions at this conference, both with respect to the aspects of modelling and to the design of efficient prevention resp. detection systems. Formally, there is no specific difference in the treatment of human errors as uncertain events compared with the two other types of uncertainties. Supporting data are either available or are to be collected or even assessed subjectively during quality assurance for all of these types of uncertainties.

In the following some technical tools will be given both for the reliability analysis of complex systems and the optimal allocation of quality assurance actions. It will be demonstrated that a crucial ingredient of quantitative planning of quality assurance is the existence of importance and sensitivity measures for the parameters in quality assurance. At present, no commonly agreed definitions appear to exist. Therefore, suitable measures will be derived and discussed to some degree. Furthermore, these measures must be computable in the sense that a quantitative assessment of quality assurance can be carried out during practical work. Again, some proposals for suitable methods will be given. Finally, a few remarks on optimisation are made in order to outline the general methodology to be followed when planning quality assurance measures.

2. SYSTEM MODELLING AND RELIABILITY ANALYSIS

A convenient way to describe the logical structure of a system is by means of event trees for componential failures [2]. In general, a number of event trees have to be analysed each of those associated with an initiating event defining also the leading event of the hazard scenario under consideration. Then, the failure probability of a time-invariant problem can be given as [2]

$$P_f = P\left(\bigcup_i \left(\bigcup_j \bigcap_k F_{ijk}\right)\right) \tag{1}$$

where i runs over the index set of the hazard scenarios, j over the index set of all branch tips in the event tree to failure in the corresponding scenario i and k over the index set of all componential failure events along the j-th branch in scenario i. F_{ijk} denotes the failure event which always can be given in the form

$$F_{ijk} = \{\underline{X} \in V_{ijk}\} = \{h_{ijk}(\underline{X};\underline{p}) \le 0\} \tag{2}$$

with $\underline{X}$ the vector of uncertain variables and $\underline{p}$ a vector of deterministic parameters to be defined later. Let further a probability distribution transformation $\underline{X} = \underline{T}(\underline{U})$ exist where $\underline{U}$ is an independent standard normal vector [3]. Then, a first-order approximation (bound) for the failure probability is

$$P_f \le \sum_i \sum_j P\left(\bigcap_k h_{ijk}(\underline{X};\underline{p}) \le 0\right)$$

$$= \sum_i \sum_j P\left(\bigcap_k g_{ijk}(\underline{U};p) \le 0\right)$$

$$\approx \sum_i \sum_j P\left(\bigcap_k \{\underline{\alpha}_{ijk}(\underline{p})\underline{U} + \beta_{ijk}(\underline{p}) \leq 0\}\right)$$

$$= \sum_i \sum_j \Phi_k(\underline{\beta}_{ij};\underline{\underline{R}}_{ij}) \tag{3}$$

with $\underline{\beta}_{ij} = (\beta_{ij1},\ldots,\beta_{ijk})^T$ and $\underline{\underline{R}}_{ij} = \{\underline{\alpha}_{ijr}^T\underline{\alpha}_{ijs};\ r,s=1,\ldots k\}$. Herein, the vectors $\underline{\alpha}_{ijk}$ are the negative gradients (normalized by $\|\underline{\alpha}_{ijk}\|$) of $g_{ijk}(\underline{u};\underline{p}) = 0$ at the so-called individual β-points $\underline{u}_{ijk}^*$ defined as [2,4]

$$\beta_{ijk} = \|\underline{u}_{ijk}^*\| = \min\{\|\underline{u}\|\} \quad \text{for } \{\underline{u}:g_{ijk}(\underline{u})\leq 0\} \tag{4}$$

Theory and numerical procedures of this first-order reliability method (FORM) are now well developed and are not further discussed. The method can be simplified to a certain extent as well as refined towards an asymptotically exact second-order reliability method [4]. Usually, the first-order results are totally satisfying from a practical point of view and it can be shown that a first-order approach is even sufficient for the derivations of sensitivity and importance measures to come [5]. As mentioned, an easy calculation of these quantities is essential for the optimal distribution of quality assurance efforts in practice.

3. SENSITIVITY AND IMPORTANCE MEASURES

We are now going to define several relevant additional quantities. It is useful to introduce first the so-called equivalent safety index [6,7].

$$\beta_E = -\Phi^{-1}\,[P\,(\underline{U} \in V)] \tag{5}$$

Consider the elementary case $V = \{\underline{\alpha}^T\underline{U} + \beta \leq 0\}$. Let the coordinate origin be translated by a small quantity $\underline{\epsilon}$ or $\underline{U}$ be changed into $\underline{U} + \underline{\epsilon}$. Then,

$$\beta_E = -\Phi^{-1}[P(\underline{U} + \underline{\epsilon}) \in V]$$

$$= -\Phi^{-1}[P(\underline{\alpha}^T(\underline{U} + \underline{\epsilon}) + \beta \leq 0]$$

$$= -\Phi^{-1}[\Phi(-\beta - \underline{\alpha}^T\epsilon)]$$

$$= \beta + \underline{\alpha}^T\underline{\epsilon} \tag{6}$$

and

$$\left.\frac{\partial\beta_E}{\partial\epsilon_i}\right|_{\underline{\epsilon}\to\underline{0}} = \alpha_i \tag{7}$$

Alternatively, one derives

$$\left.\frac{\partial P_f(\underline{\epsilon})}{\partial \epsilon_i}\right|_{\underline{\epsilon}\to\underline{0}} = \left.\frac{\partial\, \phi(-\beta_E(\underline{\epsilon})}{\partial \epsilon_i}\right|_{\underline{\epsilon}\to\underline{0}} = -\varphi(\beta_E)\,\alpha_i \tag{8}$$

Obviously, α_i is a measure of sensitivity of β_E against (small) changes in the variable U_i. Moreover, let $\underline{U}$ now be replaced by an independent normal vector with mean $\underline{m}$ and covariance matrix $\underline{\underline{\Sigma}} = \{\text{diag } \sigma_i^2\}$. It is easily shown that

$$V(\underline{m},\underline{\underline{\Sigma}}) = \{\sum_{i=1}^{n} \alpha_i(\sigma_i V_i + m_i) + \beta \le 0\}$$

$$= \{Z \le -\frac{\beta + \sum_{i=1}^{n} \alpha_i m_i}{(\sum_{i=1}^{n} (\alpha_i\sigma_i)^2)^{1/2}}\}$$

$$= \{Z \le -\beta_E(\underline{m},\underline{\underline{\Sigma}})\} \tag{9}$$

and, therefore,

$$\alpha_{m,i} = \frac{\partial \beta_E(\underline{m}\to\underline{0},\underline{\underline{\Sigma}}\to\underline{\underline{I}})}{\partial m_i} = \alpha_i \tag{10}$$

$$\alpha_{\sigma,i} = \frac{\partial \beta_E(\underline{m}\to\underline{0},\underline{\underline{\Sigma}}\to\underline{\underline{I}})}{\partial \sigma_i} = -\beta\alpha_i^2 \tag{11}$$

Hence, eq. (10) which formally coincides with eq. (7) is a sensitivity measure of β_E against (small) changes in the mean of a variable whereas eq. (11) can be interpreted as a measure of the stochastic importance of a variable.

These measures can, of course, also be defined for non-linear failure surfaces by using their β-point linearisation and they carry over to sets of unions of failure events. For the cut set it is

$$\beta_E(\underline{m},\ \underline{\underline{\Sigma}}) = -\phi^{-1}[P(\bigcap_{k=1}^{K} \{Z_k \le -\frac{\beta_k + \sum_{i=1}^{n} \alpha_{ki} m_i}{(\sum_{i=1}^{n} (\alpha_{ki}\sigma_i)^2)^{1/2}}\}] \tag{12}$$

One obtains

$$\tilde{\alpha}_{E,i} = \frac{\partial \beta_E(\underline{m}\to\underline{0},\ \underline{\underline{\Sigma}}\to\underline{\underline{I}})}{\partial m_i}$$

$$= - \frac{1}{\varphi(\beta_E)} \frac{\partial}{\partial m_i} P(\bigcap_{k=1}^{K} (Z_k \leq - \beta_k - \underline{\alpha}_k^T \underline{m}))$$

$$= - \frac{1}{\varphi(\beta_E)} \sum_{k=1}^{K} \alpha_{ki} \frac{\partial}{\partial \beta_k} P(\bigcap_{k=1}^{K} (Z_k \leq - \beta_k)) \qquad (13)$$

with

$$\tilde{\gamma}_K = \frac{\partial}{\partial \beta_k} P(\bigcap_{k=1}^{K} (Z_k \leq -\beta_k)) = \frac{\partial}{\partial \beta_k} \int_{-\infty}^{-\beta_k} P(\bigcap_{j=1, j\neq k}^{K} (Z_j \leq -\beta_j | Z_k = u))\varphi(u)dn$$

$$= \varphi(\beta_k) \; P(\bigcap_{j=1, j\neq k}^{K} (Z_j \leq -\beta_j | Z_k = -\beta_k))$$

$$= \phi_{K-1}(\underline{\beta}_k ; \underline{\underline{R}}_k) \qquad (14)$$

For example, let k=1 and the Rosenblatt-transformation of the vector $\underline{Z}$ be given by [2,3]:

$$Z_j = \sum_{r=1}^{j} \delta_{jr} V_r \qquad (\delta_{11} = 1) \qquad (15)$$

Then, $\underline{\beta}_k = - \underline{\beta} + \underline{\delta}_k \beta_k$ and $\underline{\underline{R}}_k = \{\underline{\delta}_r^T \underline{\delta}_s ;\ r,s \neq k\}$ in eq. (14). Normalization yields:

$$\underline{\alpha}_{m,i} = \frac{1}{\|\tilde{\underline{\alpha}}_m\|} \sum_{k=1}^{K} \alpha_{ki} \; \varphi(\beta_k) \; \phi_{K-1} \; (\underline{\beta}_k ; \underline{\underline{R}}_k) \qquad (16)$$

Similarly,

$$\underline{\alpha}_{\sigma,i} = \frac{1}{\|\tilde{\underline{\alpha}}_\sigma\|} = \sum_{k=1}^{K} (-\beta_k \alpha_{ki}^2) \; \varphi(\beta_k) \; \phi_{K-1}(\underline{\beta}_k ; \underline{\underline{R}}_k) \qquad (17)$$

Note that $\tilde{\gamma}_k$ or the normalized version $\gamma_k = \tilde{\gamma}_k / \|\tilde{\underline{\gamma}}\|$ can be interpreted as importance measures for the components. A large value of γ_k indicates that this component is significant for system reliability and quality assurance activities should be directed towards improvement of its individual reliability. Alternatively, one may add additional (redundant) components. Further, it is added that the corresponding measures for (minimal) cut set representations in the form $V = \bigcup \bigcap V_{ij}$ are easily derived if one makes use of $P(\bigcup A_i) \leq \sum_i P(A_i)$. Finally, one might wish to introduce sensitivity measures for the deterministic parameter vector $\underline{p}$. This is most easily done by treating it as a vector of uncertain variables with vanishing standard deviation. In any case and if the logical structure of the system is retained, a change of the location parameters of the uncertain variables resp. those constants in the direction of $-\underline{\alpha}_m$ will improve reliability at the fastest possible rate. If the system originally had the equivalent safety index $\beta_{E,O}$ but $\beta_{E,1} = \beta_{E,O} + \Delta$ is required we need to modify $\underline{p}_O$ to $p_1 = p_O - \Delta\underline{\alpha}_E$. Clearly, this might not result in an optimal solution in general.

4. OPTIMISATION

It is not possible and not necessary to elaborate in detail on various suitable optimisation techniques. These are standard especially if derivatives of the objective function and/or the constraints are available. To show that those can be computed rather easily with FORM was the primary purpose of the foregoing section. Here, we additionally assume that quality assurance cost can be given as $C(\underline{p})$ and that the gradient

$$\tilde{\underline{c}} = \text{grad}(C(p_O)) \tag{18}$$

exists. For convenience, the normalized gradient $\underline{c} = \tilde{\underline{c}}/\|\tilde{\underline{c}}\|$ is also introduced. Then, two basic tasks have to be solved. The initial set-up for quality assurance, i.e. the parameter $\underline{p}_O$, does not fulfill the safety requirements but a new parameter $\underline{p}_1 = \underline{p}_O + \Delta\underline{p} = \underline{p}_O + \Delta\underline{d}$ is required. It can be shown that the optimal direction $\underline{d}$ (the direction in which the increase in reliability is maximal without substantially increasing cost) is:

$$\underline{d}^R = \underline{\alpha}_E - (\underline{\alpha}_E^T\underline{c})\underline{c} \tag{19}$$

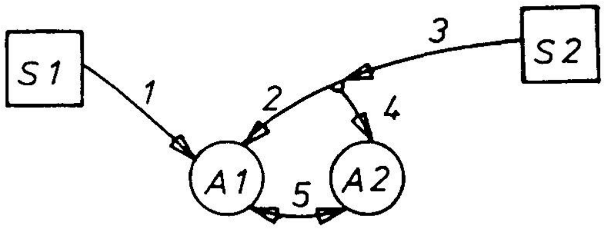
On the other hand, if the reliability requirements are already met but the possibility of cost savings has to be investigated, the optimal direction (maximal cost savings at essentially constant reliability) is:

$$\underline{d}^C = -[\underline{c} - (\underline{c}^T\underline{\alpha}_E)\underline{\alpha}_E] \tag{20}$$

A globally optimal quality assurance setup is obtained if $\underline{d}^R$ and $\underline{d}^C$ approach zero which implies that $\underline{\alpha}_E = \underline{c}$. This situation will, however, rarely occur in practice.

5. NUMERICAL ILLUSTRATION

As an example the reliability of a simple life-line system is studied (see figure below). Two sources S1 and S2 supply two areas A1 and A2 by a network of life-lines. The arrows indicate the possible direction of flow. The system is exposed to some extreme



event, e.g. an earthquake. The system is said to fail if either of the areas are no longer supplied. The system failure event can be written down directly or by construction of a fault tree.

$$V = \{[V_1 \cap (V_2 \cup V_3) \cap (V_3 \cup V_4 \cup V_5)] \cup [V_1 \cap V_5) \cup (V_3 \cap V_4) \cup (V_2 \cap V_3 \cap V_5)]\}$$

The minimal cut set representation is:

$$V = \{V_1 \cap V_3) \cup (V_3 \cap V_5) \cup (V_4 \cap V_5) \cup (V_1 \cap V_2 \cap V_4) \cup (V_1 \cap V_2 \cap V_5)\}$$

This simplistic system is now investigated on the lines presented in sections 2 and 3. We assume a very simple componential state function:

$$g(\underline{X}) = R_k - S \leq 0$$

The "resistances" R_k are assumed to be independently log-normally distributed with mean 6 and standard deviation 2 (in appropriate units). The "loads" S on all components are Rayleigh-distributed with mean 2/3 and independent of the R_k's. Applying the corresponding probability distribution yields

$$g_k(\underline{U}) = \exp[U_{k+1}\,\eta + \xi] - \{2\sigma[-\ln \phi(-U_i)]\}^{1/2} \leq 0$$

with $\eta = 0.3246$, $\xi = 1.7391$ and $\sigma = 0.5319$. The componential safety index is $\beta_k = 3.722$ ($P_{f,k} \approx 10^{-4}$) and the componential sensitivity factors are $\alpha_1 = -0.655$ and $\alpha_k = 0.755$. On a component level the location parameter of the resistances, therefore, is slightly more relevant than the location parameter of the load. The system reliability analysis yields an equivalent safety index of $\beta = 4.48$ ($P_f = 3.8 \cdot 10^{-6}$) indicating rather significant redundancy in the system.The analysis of the componential importance factors, which, by definition, range between 0 and 1 for coherent systems, are:

$$\underline{\gamma} = (0.329, 0.016, 0.625, 0.329, 0.625)$$

This illustrates that component 2 is not important for system reliability although it is not true that it could easily be removed from the system without substantially changing overall reliability. Components 3 and 5 are most important. Already here, common sense appears to be inefficient in explaining the result.

The analysis of the location sensitivity factors according to eq.(16) gives:

$$\underline{\alpha} = (-0.855, 0.195, 0.007, 0.317, 0.170, 0.317)$$

The computed values are considered as quite a surprise by the author. First of all, the location parameter of the load now becomes most important in contrast to the componential consideration. Secondly, the location parameters of component 3 and 5 are most important followed by component 1 and then 4. Again, the median resistance of component 2 has little relevance. These results have been obtained by using so-called crude FORM but almost exact results can be produced with higher order methods which are not given here. Nevertheless, the general picture does not change with the application of more sophisticated reliability methods. That any possible effort should be directed towards a better quantification of the load in that system as opposed to the componential consideration certainly would not have been detected by a less systematic analysis. At most, intuition had suggested that component 5 ought to be made strong.

Given a reliability requirement, for example, $\beta = 4.76$ ($P_f = 10^{-6}$) and constant unit cost when increasing the resistance in the system components the location parameters of the R_k's should be increased proportional to the computed α-values until the required reliability is achieved. Obviously, because reliability is affected rather non-linearly by changes in those parameters, an exact solution requires iteration.

With eq. (19) and the assumption that the cost of the system components are proportional to the location parameters of the R_k's and to the length of the pipe it is easy to determine the appropriate direction $\underline{d}^R$ to modify the R_k's in a cost optimal manner. It is to be mentioned that in this case the direction $\underline{d}^R$ is almost identical to $\underline{\alpha}$ unless the lengths of the pipes differ dramatically.

6. CONCLUSIONS

Planning of efficient quality assurance measures is optimisation involving an appropriate modelling of the system and the computation of importance and sensitivity measures. Approximate methods suitable in practical applications are available. Such a formal reliability analysis quite frequently results in actions to improve the quality which are not expected from classical deterministic analysis. The author even presumes that a probabilistic sensitivity analysis makes more engineering sense than the substitution of classical safety provisions by their probabilistic counter part.

REFERENCES

[1] RACKWITZ R., Planning for Quality-Concept and Numerical Tools, Introductory Report of IABSE-Workshop on Quality Assurance within the Building Process, 08.06. - 10.06.1983, Rigi, Schweiz, pp. 49-64

[2] HOHENBICHLER M., RACKWITZ R., First-Order Concepts in System Reliability, Struct. Safety. 1, 3, 1983, pp. 177-188

[3] HOHENBICHLER M., RACKWITZ R., Non-Normal Dependent Vectors in Structural Reliability, Journ. of the Eng. Mech. Div., ASCE, Vol 107, No. 6, 1981, pp. 1227-1249

[4] HOHENBICHLER M., GOLLWITZER S., KRUSE W., RACKWITZ R., New Light on First- and Second-Order Reliability Methods, to be published

[5] HOHENBICHLER M., Mathematische Grundlagen der Zuverlaessigkeitsmethode erster Ordnung und einige Erweiterungen, Berichte zur Zuverlaessigkeitstheorie der Bauwerke, Technische Universitaet Muenchen, SFB 96, Heft 72, 1984

[6] DITLEVSEN O., Generalized Second Moment Reliability Index, Journ. of Struct. Mech., Vol. 7, 4, 1979, pp. 435-451

[7] GOLLWITZER S., RACKWITZ R., Equivalent Components in First-Order System Reliability, Rel. Eng., Vol. 5, 1983, pp. 99-115