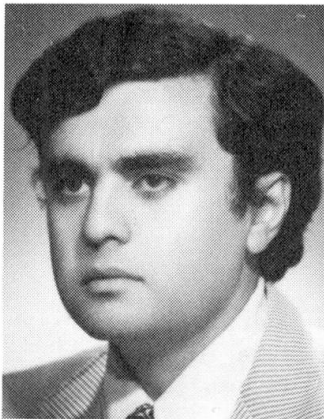# Safety of Earth Slopes: a Probabilistic Approach

Sécurité des talus de terre: une approche probabiliste

Sicherheit von Böschungen: ein probabilistischer Ansatz

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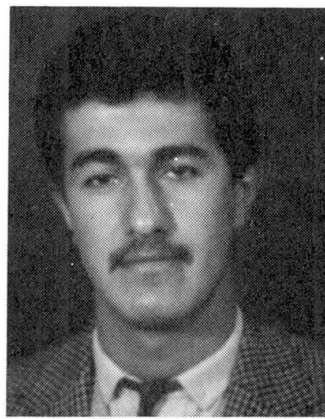
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SUMMARY

A probabilistic model to assess the three-dimensional stability of earth slopes under long-term conditions is described. A landslide is studied in detail to illustrate the implementation of the proposed model. The slope formed after the occurrence of this landslide is also analysed, and the optimal alternative to improve its safety is selected in view of the associated failure probabilities and costs.

RÉSUMÉ

Un modèle probabliste pour évaluer la stabilité tridimensionelle des talus de terre à long terme est décrit. Un éboulement est étudié en détail pour illustrer l'application du modèle proposé. Le talus formé après l'éboulement est aussi analysé, et la meilleure alternative pour l'amélioration de sa sécurité est choisie, en vue des probabilités de ruine et des coûts.

ZUSAMMENFASSUNG

Es wird ein auf der Zuverlässigkeits-Theorie beruhendes Modell für die Überprüfung der dreidimensionalen Langzeit-Stabilität von Böschungen beschrieben. Die Anwendung des Modells wird an einem Beispiel gezeigt. Auch die durch den Erdrutsch neu entstandene Geländeform wird untersucht. Schliesslich wird die optimale Alternative zur Anhebung der Sicherheit untersucht auf der Basis von Versagenswahrscheinlichkeit und Kosten.

1. INTRODUCTION

In the past twenty years several landslides have been observed and reported along many Jordanian highways. One of these highways is the Irbid-Amman highway. Along this route a number of landslides have occurred at different locations in the past. Various researchers (e.g. Saket /2/) have analysed these landslides in detail with the purpose of estimating the values of the shear strength parameters at the time of failure. The existing slopes at the sites where these landslides have occurred were also analysed, and a set of recommendations were proposed to improve their stability. However, these studies were carried out within a deterministic framework. The uncertainties in the mechanical model and in soil properties were not accounted for explicitly. Also, there was no systematic basis for comparing various alternatives proposed to improve the safety of the existing slopes and thus select the optimal measure.

In this study, such a landslide is analysed based on the probabilistic three-dimensional slope stability analysis (PTDSSA) model developed by the authors. The formulation of this probabilistic model and the related assumptions are outlined in the first part of the paper. The analysis of the landslide through this model is presented in the second part of the study. To improve the reliability of the existing slope different remedial measures are compared. A risk-based optimization method is used in the selection of the optimal stabilization measure.

2. PROBABILISTIC 3-D SLOPE STABILITY MODEL

In our study, the PTDSSA model is developed by following the general framework outlined by Vanmarcke /3/. The details of the PTDSSA model can be found in /1/. Because of space limitation, here we shall only present an outline of the model.

The potential sliding soil mass centered at $x = x_o$ and bounded by vertical end sections at $x_1 = x_o - b/2$ and $x_2 = x_o + b/2$ is assumed to be a portion of cylinder with a finite length b (Fig. 1). For this soil mass the three-dimensional (3-D) safety factor, $F_b(x_o)$, is defined as follows /3/:

$$F_b(x_o) = (\int_{x_1}^{x_2} M_R(x) \, dx + R_e)/ \int_{x_1}^{x_2} M_o(x) \, dx \qquad (1)$$

where, $M_R(x)$ and $M_o(x)$ are cross-sectional resisting and driving moments, respectively, and R_e is the contribution of the end sections of the failure surface to the resisting moment.

The probability of failure, $p_f(b)$, of a soil volume of width b and centered at a specified location along the slope axis is defined as follows:

$$p_f(b) = Pr(F_b \leq 1.0) \qquad (2)$$

Note that in this case the 3-D safety factor is treated as a random variable, since the location of the failure mass is fixed. On the other hand, the evaluation of the safety of an earth slope along its total length, B, is carried out by utilizing the level crossing concepts of random processes /1, 3, 4/. In this case, $F_b(x)$ forms a random process, since the location of the potential sliding soil mass is not fixed but random. A failure over a specified width of b will occur anywhere along the axis of the slope, whenever the random process $F_b(x)$ crosses into the unsafe domain defined by $\{F_b \leq 1.0\}$. Equations to compute the risk of failure at a specific location, $p_f(b)$, and the probability that a failure may take place at any location along the slope, $p_F(b)$, are
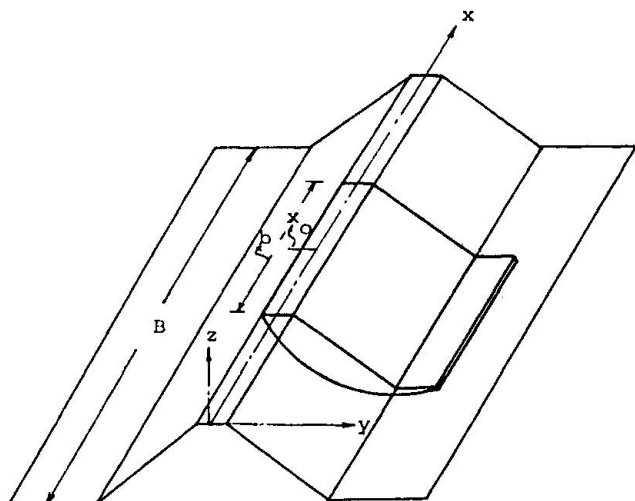
Fig. 1 Geometry of the sliding soil mass (from /3/)

derived and presented in /1/.

In Eq. 1 the most significant random function is $M_R(x)$. The ordinary method of slices is taken as the deterministic model of slope stability analysis and forms the basis for the computation of $M_R(x)$ and $M_o(x)$. Under the long-term conditions, the shear strength depends on the angle of friction, ϕ, cohesion, c, and the pore pressure, u. The spatial averages of these shear strength parameters are taken as the basic random variables, whereas the unit weight of soil and the geometric parameters are treated as deterministic basic variables, since the associated uncertainties are relatively small.

The proposed model takes into consideration all sources of uncertainties, quantifies them and systematically incorporates them into the assessment of the reliability of an earth slope. Besides the spatial variability of shear strength parameters in x, y, z directions, inaccuracies in the mechanics of the deterministic slope stability model as well as discrepancies between the in situ and laboratory-measured values of soil properties are taken into consideration. Random correction factors, denoted by N, are introduced to adjust for these inaccuracies and discrepancies.

The cross-sectional resisting moment, $M_R(x)$, with appropriate correction factors and consistent with the method of slices is expressed as follows /1/:
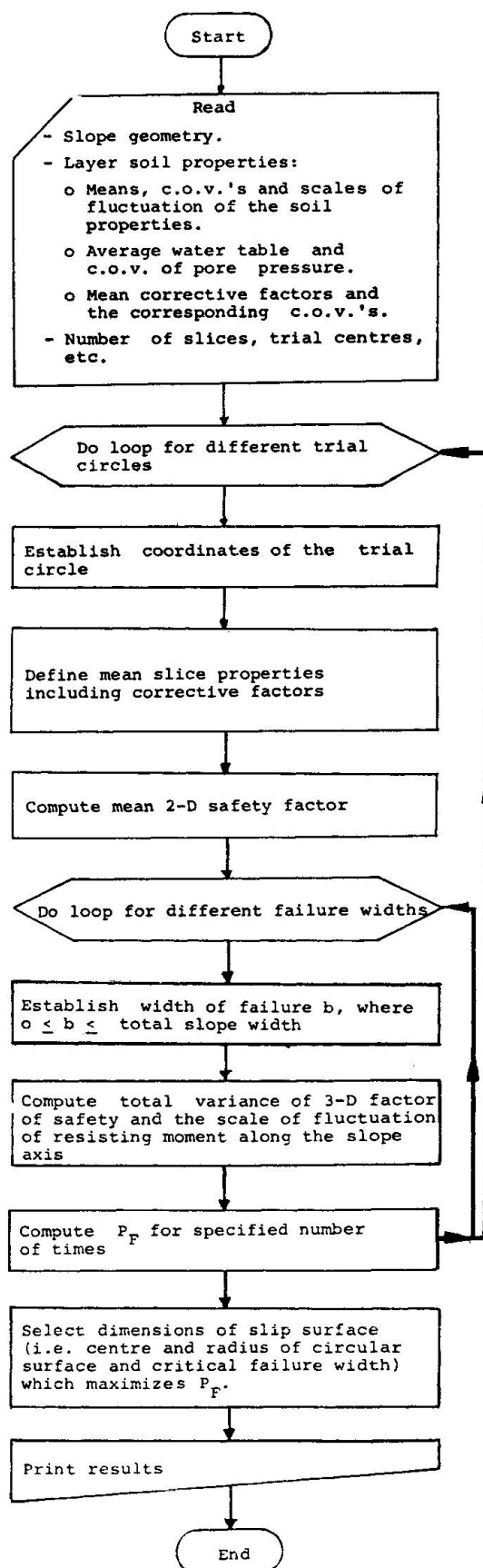

Fig. 2 Flowchart for the PTDSSA model

$$M_R(x) = N_m \, N_p \, r \, \sum_{i=1}^{m} \{ \, N_{c_i} \, c_i \, l_i + (W_i \, \cos \Theta_i - u_i \, l_i \, \cos^2 \Theta_i) \, \tan (N_{\phi_i} \, \phi_i) \} \quad (3)$$

where, N_m and N_p are correction factors for the modeling error and the effect of progressive failure, repectively; l_i = length of the ith segment of the failure surface, m = number of slices, u_i = pore pressure acting on the ith slice computed from the average pore pressure distribution along the failure surface, W_i = weight of the ith slice, Θ_i = the inclination of the base of the ith slice to the horizontal axis; r = radius of the failure surface.

In Eq. 3, N_c and N_ϕ are correction factors accounting for the systematic uncertainties resulting from the discrepancies between laboratory and in situ conditions in the estimation of the values of c and ϕ. Each of these factors can be written as the product of component factors, $N_j(.)$'s, to accomodate for errors resulting from disturbance during sampling, size of specimen, rate of shearing, sample orientation and anisotropy /1, 5/.

The mean and variance of $M_R(x)$ is obtained by using the first-order approximation. In this case, the statistical parameters required are the means and point variances of the basic variables and of the correction factors. Also any type of existing correlations have to be quantified. The variance of the cross-sectional resisting moment, denoted by, $\tilde{M}_R^2$, will be composed of contributions from all of the component variables. In the computation of this variance, the spatial and other sources of correlations among the component variables are taken into consideration.

The degree of spatial correlation associated with the shear strength parameters is quantified by the scales of fluctuation in the x, y and z directions, denoted by λ_x, λ_y and λ_z, respectively. The scale of fluctuation, λ, which is introduced by Vanmarcke /3, 4/ is a convenient measure of the degree of correlation in a soil medium. Physically interpreted it represents the distance over which a soil property (in a certain direction) shows a relatively strong correlation.

The 3-D slope stability analysis requires the spatial averaging to be carried out over the soil volume, and this necessitates the consideration of 3-D correlation functions. However, in the proposed model the (volumetric) spatial averaging is carried out first over the arc length, then over the slope axis. Such a procedure simplifies the computations, since the knowledge of the 1-D correlation functions (or scales of fluctuation) in the x, y and z directions becomes sufficient.

It is to be noted that spatial averaging or integration have a smoothing effect, and thus cause a reduction in the variance. The degree of reduction depends on the degree of correlation, and in our study the dimensionless standard deviation reduction factor $\Gamma(.)$ is used to quantify this effect. For example, the standard deviation of the first term in Eq. 1, involving the integration becomes equal to "b $\tilde{M}_R$ $\Gamma(b)$", where, $\tilde{M}_R$ is the standard deviation of M_R, and $\Gamma(b)$ is the standard deviation reduction factor associated with the integration of M_R over a length of b. The exact functional form of $\Gamma(b)$ depends on the correlation function. However, by using the approximate relations proposed by Vanmarcke /3, 4/, it is possible to express $\Gamma(b)$ in a simple way based on the associated scale of fluctuation /1, 3, 4/.

The uncertainty in the driving moment M_O is neglected here, since the variables involved in the computation of M_O have comparatively small uncertainties. Thus, it is treated as a deterministic variable in Eq. 1.

The numerical computations associated with the PTDSSA model are to be carried out by using the computer program prepared for this purpose. The corresponding flow-chart is shown in Fig. 2.

3. ANALYSIS OF THE LANDSLIDE

3.1 Description of the Landslide

This landslide has occurred in early March 1967 at 37th km of the Sweileh-Jerash road. The failure was of a rotational nature. The total width of the slope was about 800 m whereas the width of the landslide was approximately 80 m. Prior to the movement, the slope had an inclination of 32° to the horizontal /2/. The slope cross-section before the landslide took place and the actual failure surface are shown in Fig. 3.

Undisturbed samples were taken throughout the soil deposit. The peak and residual strength parameters were measured from the triaxial and direct shear box tests, respectively. Based on the reported results of these tests the following mean values and inherent variabilities (δ) are computed:

$$\bar{c}_p = 23.3 \text{ kPa}, \qquad \bar{\phi}_p = 16.9^\circ, \qquad \bar{c}_r = 17.7 \text{ kPa}$$

$$\phi_r = 11.5^\circ, \qquad \delta_{c_p} = \delta_{c_r} = 0.21, \qquad \delta_{\phi_p} = \delta_{\phi_r} = 0.06$$

In Table 1, the mean correction factors and the corresponding coefficients of variation (c.o.v.) denoted by Δ, accounting for different sources of discrepancies between laboratory-measured and in situ values of c and ϕ are listed. These values are selected according to the guidelines given in /5/ and considering the soil properties reported for this site in /2/.

Correction Factors	$\bar{N}_j(c)$	$\Delta_j(c)$	$\bar{N}_j(\phi)$	$\Delta_j(\phi)$
Mechanical disturbance (N_1)	1.35	0.15	1.20	0.10
Specimen size (N_2)	0.73	0.10	0.93	0.05
Rate of shearing (N_3)	0.80	0.14	0.80	0.14
Anisotropy (N_4)	0.98	0.04	0.98	0.04

Table 1. Summary of the statistics of correction factors

There is no data available to calculate the scales of fluctuation for cohesion and angle of friction in the x, y and z directions. Therefore, estimates of these scales of fluctuation are obtained based on the ranges proposed in /1/. The selected values are: $\lambda_{c_x} = \lambda_{c_y} = \lambda_{\phi_x} = \lambda_{\phi_y} = 30$ m and $\lambda_{c_z} = \lambda_{\phi_z} = 1.5$ m. A sensitivity study carried out with respect to the scales of fluctuation indicated that within the range of reasonable values for the scales of fluctuation in the horizontal direction, the failure width is not sensitive to λ_x and λ_y /1/.

3.2 Assessment of Failure Probability

The stability analysis in terms of effective stresses is carried out for an approximate cross-section before the landslide took place, as shown in Fig. 3. Here the average peak strength parameters are to be used in the analysis, since this landslide is considered to be a first-time landslide. For the progressive failure effect the correction factor N_p, with $\bar{N}_p = 0.73$ and $\Omega_{N_p} = 0.10$ is applied, where Ω denotes the total c.o.v.

The exact position of the water table at the time of landslide was not known. However, the main cause of this landslide was indicated to be the heavy rain and consequent saturation of the soil /2/. In our analysis, the ground water table is assumed to be parallel to the natural ground surface with its top level

Fig. 3 Approximate cross-section of the original slope and the critical slip surface
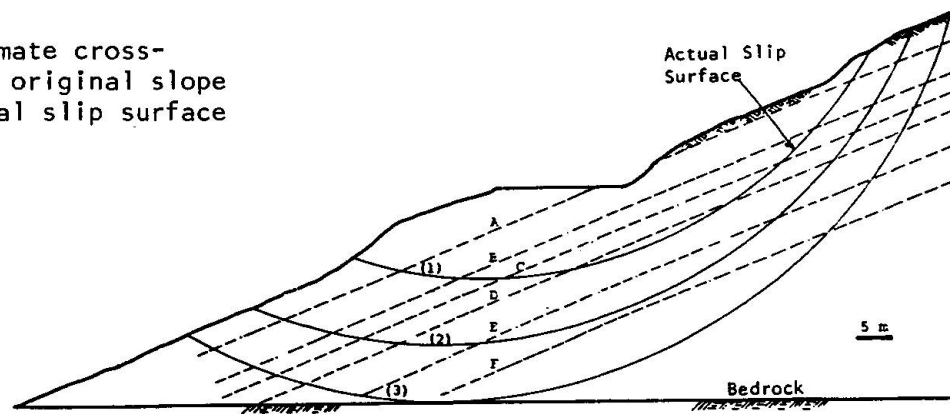
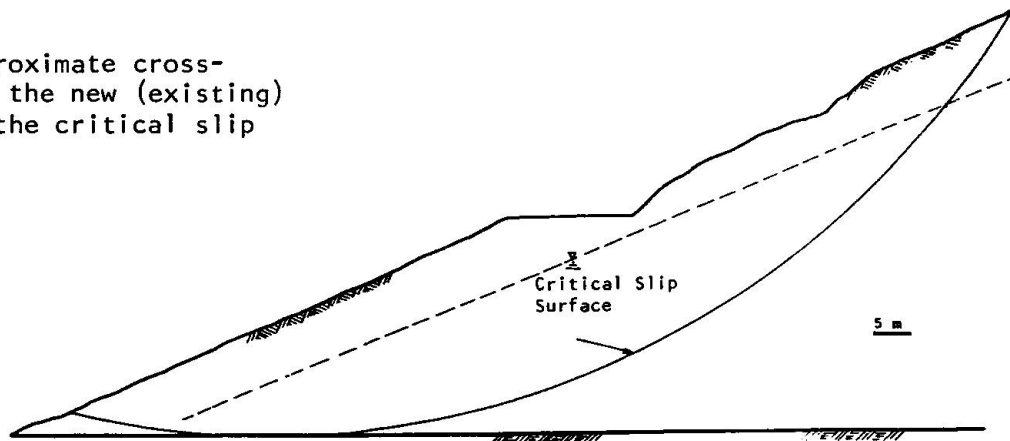**Fig. 4** Approximate cross-section of the new (existing) slope and the critical slip surface

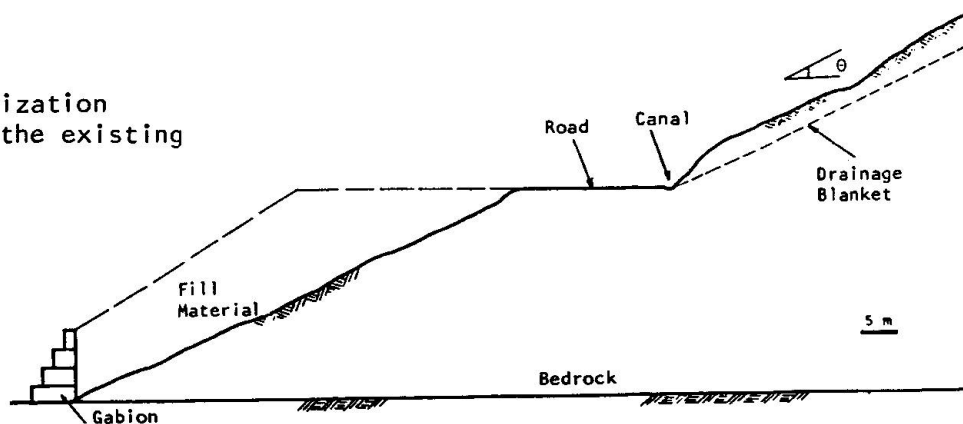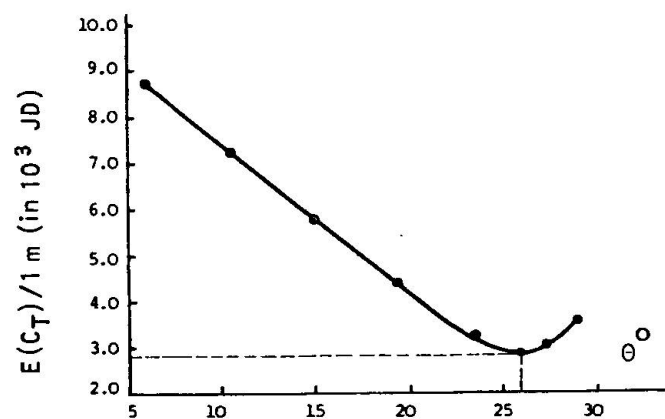**Fig. 5** Stabilization measures for the existing slope

Fig. 6 Variation of the expected total cost with the slope inclination

at a certain distance from the ground surface. To account for the rather erratic fluctuations in pore pressure resulting from the rapid fluctuations in the ground water level (G.W.L.) a value of 0.3 is taken as the total uncertainty in pore pressure.

Considering the best estimate values of all parameters, an analysis is carried out on the actual slip surface for different mean water table levels. Computations are performed by using the PTDSSA computer program. High failure probabilities are computed for all water tables. On the other hand, ground water table level designated by B in Fig. 3 yields to a failure width of 77 m which is closest to the actual failure width of 80 m. So G.W.L. described by level B is taken to be our best estimate. The corresponding failure probability is 0.80, which is quite high and consistent with the observation of the landslide.

3.3 Assessment of the Safety of the Existing Slope

In this section we shall analyse the safety of that portion of the slope which has experienced a downward movement and moved out from the original slope mass. The width of the slope that experienced this movement was observed to be 80 m. However, the total slope width to be analysed in this section is taken to be 100 m considering the extensions at either sides of the failed soil volume. It should also be noted that for this existing slope, the residual shear strength parameters will be used, since the slope has already gone through a slide.

Figure 4 shows an approximate geometry of the existing slope. The existing slope is analysed using average residual strength parameters. The mean value and the c.o.v. of the correction factor that accounts for the progressive failure effect is taken to be 0.95 and 0.03, respectively. No change was made in the best estimates of the other parameters used in the analysis of the original slope. For the most critical slip surface and the mean ground water level shown in Fig. 4, the failure probability is found to be 0.96. This high failure probability implies that the slope is not in a safe state and remedial measures are needed to improve its safety.

3.4 Stabilization Measures for the Existing Slope

A set of remedial measures were suggested in /2/ to improve the safety of the existing slope. The first of these measures is the supply of adequate drainage to remove the surface water from the area and lower the G.W.L. If adequate drainage is achieved, then the G.W.L. will be shallow and we may reasonably assume that the pore pressure is zero in subsequent analysis. As a second remedial measure the resisting forces (or moments) against sliding will be increased by constructing a gabion wall at the toe of the original landslide and placing fill material behind the gabion as shown in Fig. 5.

The final remedial measure is the adjustment of the slope inclination, Θ, above the road level. For the purpose of selecting the optimal Θ value, different Θ values are assumed and the corresponding probability of total slope failure are calculated. For each alternative Θ value, the expected total cost is computed from the following equation:

$$E(C_T) = C_1 + C_2 + C_F \, p_F \tag{4}$$

where, C_1 = (volume of fill) x (unit cost of fill), C_2 = (volume of the soil cut) x (unit cost of cut), $E(.)$ = expected value operator, C_F = cost of failure, p_F = probability of slope failure.

For the purpose of cost analysis, the cost of failure is assumed to be 30,000 JD per 1 m of slope width (this cost is due to road damage, delay, etc.). Also let the cost of fill material be 1 JD/m^3 and the cost of cutting to be 10 JD/m^3. By substituting these assumed costs into Eq. 4 we obtain the following

expression for the expected total cost per 1 m of slope width:

$$E(C_T) = 627.5 + 10\ V_c(\theta) + 30,000\ p_F(\theta) \tag{5}$$

where, $V_c(\theta)$ = volume of the soil cut in order to achieve an inclination of θ for the slope above the road level. A plot of the total expected cost versus slope inclination θ is shown in Fig. 6. As observed from this figure, the optimum value of θ is found to be $25.8°$, which means a reduction of $3°$ in the original ground inclination above the road level (which is $28.8°$). The probability of slope failure for $\theta = 25.8°$ is 0.035. This failure probability appears to be high and further improvements may be required. In this case, it is necessary to specify the level of acceptable risk. Accordingly, additional improvements could be implemented by increasing the volume of the fill and/or decreasing the slope inclination.

It is to be emphasized that the costs used here were assumed just to show the procedure of selecting the optimum solution and may not reflect the actual costs.

4. CONCLUDING COMMENTS

The reliability of an earth slope depends closely on the various uncertainties involved in the stability analysis. The probabilistic model briefly discussed in this study evaluates the 3-D stability of slopes under long-term conditions taking into consideration the spatial variability (and correlations) of soil properties, as well as the uncertainties stemming from the discrepancies between laboratory-measured and in situ values of shear strength parameters. It also accounts for the effects of modeling errors and progressive failure. Consideration of the third dimension and the end effects are crucial for a realistic assessment of the safety of earth slopes. In the 3-D analysis, the critical and total slope widths become two new and important parameters.

Through the PTDSSA model a certain landslide that occurred along the Irbid-Amman highway is analysed in a systematic way including all sources of uncertainties. The failure width is predicted to be 77 m (versus 80 m of observed failure width) and the probability of slope failure is computed to be 0.80. These results agree well with those actually observed, supporting the predictive ability of the proposed probabilistic model.

To improve the safety of an existing slope, different remedial measures are compared and the optimal slope inclination is selected by using the failure probabilities associated with each alternative and based on the minimization of the expected cost criterion. Such an analysis shows one of the benefits gained through the probabilistic approach.

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