Rate-dependent constitutive theory of concrete in tension

Rate-Dependent Constitutive Theory of Concrete in Tension

Théorie constitutive dépendant de ratios pour le béton en traction

Geschwindigkeitsabhängiges Werkstoffgesetz für Beton unter Zugbelastung

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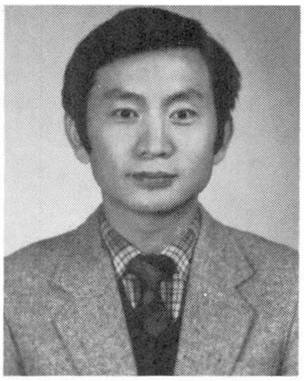

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SUMMARY
A realistic nonlinear constitutive model which can describe the dynamic tensile behavior of concrete is presented. The model is obtained by generalizing a rate-independent nonlinear tensile stress-strain relation for concrete. The static tensile behavior of concrete is modeled on the basis of the concept of microcrack planes. The affinity and shape transformations are employed to model the effect of strain-rate. The material parameters are characterized in terms of the strain-rate magnitude. The present theory is compared with the dynamic tensile test data available in the literature.

RÉSUMÉ
L'article présente un modèle constitutif réaliste et non-linéaire, du comportement dynamique du béton en traction. Le modèle est obtenu en généralisant une relation contrainte-déformation du béton à la traction pour un ratio indépendant non-linéaire. Le comportement statique du béton à la traction est exprimé dans un modèle sur la base du concept de plans de micro-fissures. L'affinité et la transformation de la forme sont utilisées pour modéliser l'effet du ratio de déformation. Les paramètres du matériaux sont caractérisés en termes d'amplitude du ratio de déformation. L'équation proposée permet de prédire l'augmentation de la résistance à la traction suite à l'augmentation du ratio de déformation.

ZUSAMMENFASSUNG
Der Beitrag behandelt ein wirklichkeitsnahes nichtlineares Werkstoffgesetz für das Stossverhalten von Beton unter Zugbelastung, wobei der Ansatz von Mikrorissebenen verwendet wird. Affinitäts- und Formtransformationen werden gebraucht um den Einfluss der Dehngeschwindigkeit zu modellieren. Nach einem Vergleich von Theorie und Versuchsergebnissen wird eine Beziehung für den Einfluss der Dehngeschwindigkeit auf die Zugfestigkeit vorgeschlagen.

1. INTRODUCTION

The mechanical behavior of concrete under dynamic loads induced from earthquakes, impacts, air blasts, wind gusts, and ocean waves is very complicated. In fact, the stiffness of material may significantly depend on the rate of loading under these conditions. The dynamic behavior of concrete in tension has been studied by Hatano[6], Suaris and Shah[8], Zielinski et al[10], and several other investigators. These outstanding studies have mainly focussed on the experimental aspects of concrete under high rates of tensile loadings.

Recently, Bazant and Oh[1] have developed a model to predict the strain-rate effect of concrete in compression. However, no model of this type exists to describe the dynamic behavior of concrete in tension. The purpose of this paper is, therefore, to propose a realistic model which can describe the dynamic tensile behavior of concrete. The model is obtained by generalizing a rate-independent nonlinear constitutive model for concrete in tension.

2. CONSTITUTIVE MODEL FOR STATIC TENSION

To develop a model to predict the dynamic tensile behavior of concrete, it is first necessary to clarify the static tensile behavior. Recently, Bazant and Oh[2, 3] have proposed a rate-indepent nonlinear constitutive model which can describe the static tensile behavior of concrete. This model considers that there exist certain weak planes within the material in which the stress relief due to microcracking takes place as a function of the stress and strain on each particular plane. It is then assumed that the orientations of the weak planes are distributed uniformly. The total strain tensor, ε_{ij}, is considered as a sum of an elastic strain tensor ε^{el}_{ij} and an inelastic strain tensor, e_{ij}, in which the latin lower-case subscripts refer to cartesian coordinates x_i ($i = 1, 2, 3$). The rheological model for this theory is depicted in fig. 1. The theory may be summarized as follows[2].

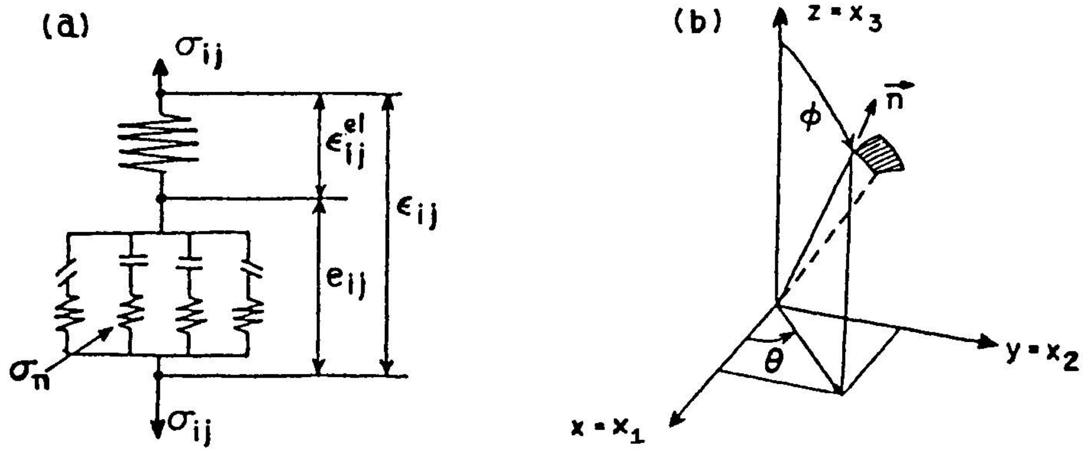

Fig. 1 (a) Rheological Model for concrete
(b) Spherical coordinate system.

$$d\sigma_{ij} = D_{ijkm}\, d\varepsilon_{km} \tag{1}$$

in which

$$D_{ijkm} = [C^{el}_{ijkm} + (B_{ijkm})^{-1}]^{-1} \tag{2}$$

$$C^{el}_{ijkm} = \frac{1}{9K}\, \delta_{ij}\delta_{km} + \frac{1}{2G}\left(\delta_{ik}\delta_{jm} - \frac{1}{3}\,\delta_{ij}\delta_{km}\right) \tag{3}$$

$$B_{ijkm} = \int_0^{2\pi} \int_0^{\pi/2} n_i n_j n_k n_m f'(e_n) \sin\phi \, d\phi \, d\theta \tag{4}$$

Here δ_{ij} = Kronecker delta, K = bulk modulus, G = shear modulus, and C^{el}_{ijkm} = elastic compliances corresponding to ε^{el}_{ij}.

The function $f(e_n)$ of Eq. 4 must be capable of describing the microcracking behavior of concrete. The simple and desirable expressions for this behavior are adopted as[2, 3]

$$\sigma_n = E_n \varepsilon_n e^{-(K\varepsilon_n^p)} \text{ for } \varepsilon_n > 0, \quad \sigma_n = E_n \varepsilon_n \text{ for } \varepsilon_n \leq 0 \tag{5}$$

Where E_n, k, and p are material parameters which are functions of concrete strength f'_c. If was found from this study that the average value of E_n = $5{,}322\sqrt{f'_c}$, k = $4.07 \times 10^8/\sqrt{f'_c}$, and p = 2, where f'_c is given in N/mm². Note here that the expression in Eq. 4 must be evaluated numerically. The numerical integration formulas for this purpose have been developed by the author and described in detail in ref. 4.

3. TENSILE STRESS-STRAIN RELATION WITH STRAIN RATE EFFECT

We now need to generalize the rate-independent constitutive model to include the effect of strain-rate. Recently, Bazant and the author[1] have developed a strain-rate dependent nonlinear constitutive theory for concrete in compression. This theory is extended here to model the dynamic tensile behavior of concrete. It was found from experiments[8] that the effect of strain rate is more sensitive in tension than in compression. This fact must be reflected in modeling the dynamic tensile behavior.

The shape of the uniaxial stress-strain curve of concrete largely depends on the strength. Generally, the peak portion of the curve is flatter for a lower strength concrete and becomes sharper for a higher strength concrete. This nature may be characterized by the parameter r[1].

$$r = \frac{E\varepsilon_p}{\sigma_p} \tag{6}$$

in which σ_p = peak stress, ε_p = strain at peak stress, and E = initial elastic modulus. The parameter r represents the ratio of strain at peak stress to the elastic strain corresponding to this stress. The general stress-strain curve may, therefore, be characterized by three basic parameters, i.e., σ_p, E, and r.

It was found from previous study[1] that the parameter r depends on the strength f'_c. This relation may be written as $f'_c = f_1(r)$. Since the elastic modulus E of concrete is known as a function of strength, one may reasonably write that $E = f_2(r)$.

Since the constitutive equation expressed in Eq. 1 is a function of current strain, stress, and concrete strength, this relation may be rewritten as

$$d\sigma_{ij} = D_{ijkm}[\boldsymbol{\varepsilon}, \underset{\sim}{\sigma}\,;\ f_1(r)]\,d\varepsilon_{km} \tag{7}$$

For the given value of r = r*, these constitutive relations will yield the peak stress $f_1(r^*)$, and initial tangent modulus $f_2(r^*)$.

It is now needed to make a transformation that preserves the value r* but changes the peak stress and the initial elastic modulus. The affinity transformations may be applied for this purpose. Namely, the strain values are replaced by $a\,\underset{\sim}{\varepsilon}$ and the stress values by $b\underset{\sim}{\sigma}$. Theorefore, Eq. 7 may be written as

$$b\, d\sigma_{ij} = D_{ijkm}[a\underset{\sim}{\varepsilon},\ b\underset{\sim}{\sigma};\ f_1(r^*)]\, a\, d\varepsilon_{km} \tag{8}$$

It is noted here that the parameter r is not affected by such transformations. However, the peak stress, $f_1(r^*)$, and the initial elastic modulus, $f_2(r^*)$, will be transformed to the following values.

$$\sigma_p^* = \frac{1}{b} f_1(r^*)\ ,\ E^* = \frac{a}{b} f_2(r^*) \tag{9}$$

The transformation coefficients a and b, may now be determined from Eq. 9.

$$b = \frac{f_1(r^*)}{\sigma_p^*}\ ,\qquad a = \frac{E^*}{f_2(r^*)}\, b \tag{10}$$

The following formulas have been obtained by fitting the availble dynamic tensile test data[6, 8, 10].

$$g(\dot{\varepsilon}) = \frac{1 - \dot{\varepsilon}^{1/8}}{2.2 + 3.2\ \dot{\varepsilon}^{1/8}} \tag{11}$$

$$r^* = 2.09 - 0.0215\sigma_{po} + g(\dot{\varepsilon}) \tag{12}$$

$$\sigma_p^* = [1.95 - 3.32g(\dot{\varepsilon})]\ \sigma_{po} \tag{13}$$

in which $\dot{\varepsilon}$ = strain rate, given in strain per second, and σ_{po} = uniaxial static strength in N/mm². It is noted here that the formulas in Eqs. 11-13 differ from those for dynamic compression because the effect of strain rate is more sensitive in dynamic tension[8]. This is probably due to the fact that the concrete cracking influences greatly the strain rate sensitivity.

As mentioned previously, the relation $f_c' = f_1(r)$ is formalized here from the test data as follows.

$$f_1(r^*) = (110 - 46r^*) \tag{14}$$

$$f_2(r^*) = 4{,}740\sqrt{f_1(r^*)} \tag{15}$$

in which $f_1(r^*)$ and $f_2(r^*)$ are expressed in N/mm², and $f_2(r^*)$ is obtained from the relation between the elastic modulus and the compressive strength.

The behavior of concrete under initial stage may be characterized by the compliance function, J(t, t'), which is defined as the strain at time t produced by a constant unit stress acting since time t'[5]. The appropriate form for the compliance function of concrete may be written as[5]

$$J(t,\ t') = \frac{1}{E_o}\,[1 + \phi_o(t - t')^n] \tag{16}$$

in which $\phi_o = \phi_1(t'^{-m} + \alpha)$, E_o = asymtotic modulus. The typical values of the material parameters are $m = 1/3$, $n = 0.1$, $\phi_1 = 5$, $\alpha = 0.03$, $E_o = 1.5E_{28}$ where E_{28} = standard 28-day elastic moduls. It was shown the Eq. 16 gives reasonable values even for the rapid loading[5]. Since the aging of concrete during rapid loading is negligible, the effective modulus, defined as $E_{eff} = 1/J(t,\ t')$, may be efficiently used to model the concrete behavior.
One may, therefore, take the initial elastic modulus E* as the effective modulus for load duration equal to the time to reach strain $0.05\varepsilon_p$. Since the peak strain ε_p is about 0.0002 for concrete in tension, the load duration $(t - t') = 0.05\varepsilon_p/\dot{\varepsilon} = (1 \times 10^{-5}/\dot{\varepsilon})$ days $\simeq (1/\dot{\varepsilon})$ seconds. Therefore one may write

$$E^* = \frac{1}{J(t,\ t')} \simeq \frac{E_o}{1 + \phi_o(\dot{\varepsilon})^{-n}} \tag{17}$$

Since the static uniaxial tension test is normally conducted at $\dot{\varepsilon} \simeq 2 \times 10^{-6}$/sec as indicated by Hatano et al[6], the value E for static tension may be expressed as $E = E_o/[1 + \phi_o(2 \times 10^{-6})^{-n}]$.

The dynamic tensile behavior of concrete may now be described through Eqs. 1, 8-9. One may finally write the constitutive relations as

$$d\sigma_{ij} = D_{ijkm}(\underset{\sim}{\varepsilon}, \underset{\sim}{\sigma}, \dot{\varepsilon})d\varepsilon_{km} \tag{18}$$

4. COMPARISONS WITH TEST DATA

The rate-dependent tensile stress-strain relation derived in the previous section has been compared with the existing dynamic thensile test data for concrete.

Fig. 2 shows the comparion of the uniaxial tensile stress-strain curves obtained for different strain rates by Hatano[6] in which the solied lines indicate the results from the present theory and the dashed lines indicate the test data. It can be seen that the tensile strength of concrete is greatly increased with the increase of strain rate.

It was possible to obtain from the present study a prediction formula for the dynamic uniaxial tensile strength of concrete. The formula obtained is

$$f_{td} = [1.95 - 3.32(\frac{1 - \dot{\varepsilon}^{1/8}}{2.2 + 3.2\dot{\varepsilon}^{1/8}})]\ f_{to} \tag{19}$$

in which f_{td} = dynamic tensile strength, and f_{to} = static tensile strength.

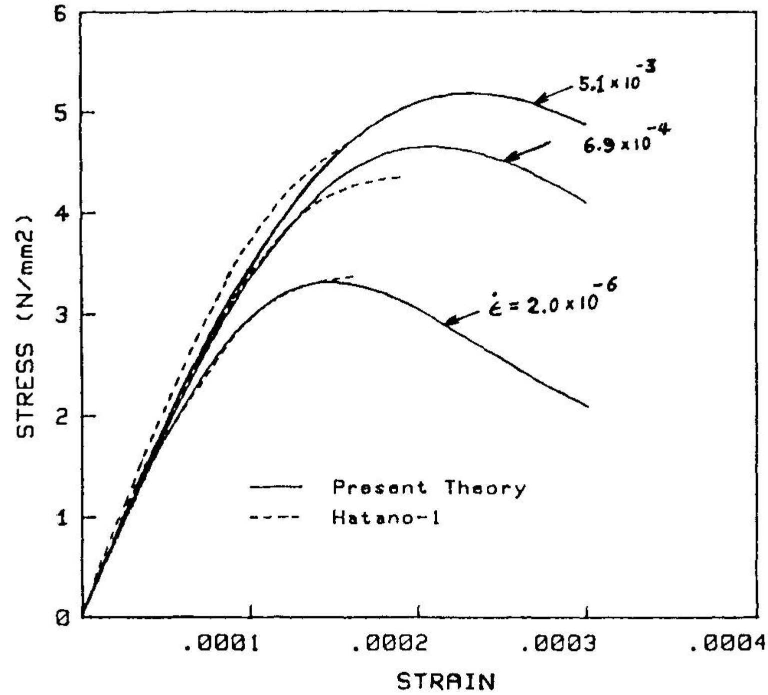

Fig. 2 Comparison of present theory with the dynamic tensile tests

5. CONCLUSION

The rate-dependent nonlinear constitutive relation for concrete in tension is proposed. The model is obtained by generalizing a recently developed, rate-independent nonlinear tensile constitutive relation for concrete. The static tensile behavior is modeled on the basis of the concept of micro-crack planes which may be considered to be uniformly distributed within the concrete.

The affinity and shape transformations are used to include the effect of strain rate. The present theory, which can model the dynamic tensile behavior of concrete, is compared with the test data available in the literature. The model adequately predicts the dynamic tensile properites of concrete and allows more realistic dynamic analysis of concrete structures.

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