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Numerical Models for the Non-Linear Analysis of Prestressed Concrete Frames

Modèles numériques pour l'analyse non-linéaire de cadres en béton précontraint

Numerische Modelle für die nichtlineare Berechnung von Spannbetonrahmen

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SUMMARY

After a brief overview of the models currently used for prestressed concrete structures, a model originally developed by the authors is discussed in detail. Its main goal seems to be the simplicity in which shear deformations and slippage of the cables are considered.

RÉSUMÉ

Après avoir passé en revue les modèles actuellement employés pour les structures en béton précontraint, les auteurs présentent leur propre modèle. Le résultat le plus important que l'on peut attendre de ce modèle est la simplicité dans l'expression de la déformation due à l'effort tranchant et de l'écoulement des câbles.

ZUSAMMENFASSUNG

Nach einer kurzen Übersicht über die Modelle, die heute für Spannbeton gebräuchlich sind, wird ein von den Autoren entwickeltes Modell im einzelnen besprochen. Das wichtigste Ergebnis ist, dass Schubverformung und Kabelschlupf sehr einfach behandelt werden können.

1. FOREWORD

The design of prestressed concrete structures must satisfy the requirements of safety and serviceability, as all other kinds of structures.
While this can be accomplished in most cases by following approximate or empirical procedures, it is also desirable to have refined models which can trace the structural response throughout their elastic, cracking, inelastic and ultimate ranges.
The use of these models can be either providing a firmer basis for the codes and analyzing unusual and complex structures.
The purpose of this paper is twofold: first a brief overview of the currently used models is given, then some more details are given about a model developed by the authors.

2. PROBLEMS IN MODELLING PRESTRESSED CONCRETE FRAMES

When dealing with prestressed members any model should be evaluated with respect to some important and peculiar problems.

Time dependent effects, due either to load history, temperature history, creep, shrinkage and aging of concrete and relaxation of the prestressing cables, have the outmost importance because of the character of imposed deformation of the prestressing action.

Bond between tendons, mortar and concrete is a qualifying point: refined model should give the possibility of simulating bonded and unbonded tendons, and in the former case should incorporate a law for the progressive deterioration and failure of bond connections.

Shear deformations and effects of the prestressing action on shear cannot always be neglected, particurarly when dealing with deep beams.

Planarity of the section is not guaranteed due to shear and torsions, and is not respected at all after some slippage between cables and concrete has occurred.

Connections between elements, mainly beam-column connections, may create problems to some models, effective for one-dimensional structures.

The constitutive relations are sometime based on well established laws for concrete and steel, sometimes on the contrary are obtained through euristic corrections of the usual relations for concrete beams.

Finally computer time and memory may be very penalizing for too refined or bad conditioned elements.

3. CURRENT MODELS

Models based on quite different ideas are presently implemented to simulate the behavior of PC beams. The most commonly used ones will be briefly presented in what follows.

3.1 Traditional beam elements whith corrections

This is conceptually the simplest and most euristic approach /2/. The usual moment-rotation relations are used, correcting at each time step the stiffness of the elements in order to take into account the actual non-linearity.
The prestressing action is introduced as an external force, but the reduction in stiffness keeps it equivalent to an imposed deformation. The sections are considered to remain plane, bond and shear are not explicity taken into account.
Most of the job consist in getting the right correction factors for cracking, time dependent effects, tension stiffening.

3.2. Integration of sectional M-ϕ relations

A second approach is based on the use of sectional results, obtained normally through programs which divide any section into layers with different stress-stain relations. Equilibrium and planarity of the section are then imposed to get moment-curvature relations /7/.
The prestressing action is taken into account prestressing some steel layers, bond is supposed to be perfect, time dependent effects are usually neglected.
A number of choices is then possible to get the overall stiffness of a beam element:

1. Each beam can be identified by one or more sections, using constant, linear or higher order variation of the stiffness properties along the member;
2. The moment-curvature relations can be piecewise linearized, or the actual M - ϕ can be used and the stiffness of each section computed at each step within the F.E. analysis /1/. In this case the procedure way be very time consuming.

3.3. Layer and filament models

A beam element is decomposed in a number of straight layers or filament each of them with a monodimensional stress-strain law which can take into-account also time dependent effects.
Pretensioned, bonded and unbonded postensioned cables can be simulated, shear deformations are neglected.
The elements cross sections should have a simmetry in the case of layers models, can be of any shape in the case of filament models.
The tendons are straight, but not necessarily horizontal within the element /11/.

4. BIDIMENSIONAL LAYERS MODEL

This model, recently implemented by the authors, differs from the layer model mainly because of the layers are here modeled as plane stress elements instead of one-dimensional.
The advantages of such a model may be summarized as follows:

1. the shear deformations are considered, which is particulary important for deep beams.
2. It is possible to take into account the interactions between axial action, shear and bending moment.
3. The sections can assume up to a second order polynomial shape.
4. The number of layers is much smaller than in the case of a mono-dimensional layer model.

4.1. Analytical Formulation

The adopted approach is the Displacement Finite Element Formulation.
The displacement fields are defined over a bidimensional domain, subdivided into strips through the height of the beam.
The strain-displacement relations for the Timoshenko beam theory may be written as:

$$\begin{aligned} \varepsilon_{RR} &= - S \frac{\partial \theta}{\partial R} + \frac{\partial u}{\partial R} \\ \varepsilon_{RS} &= - \theta + \frac{\partial v}{\partial R} \end{aligned} \qquad (1)$$

Where R and S are coordinates related to a sectional reference system, which may vary along the beam because of the variability in the shape of the cross section; u , v and are the generalized dislacement components, according to a classical Lagrangian formulation, referred to an absolute reference system.

The commonly used Bernoulli-Navier formulation, which requires the planarity of the sections, assumes:

$$\theta = \frac{dv}{dR} \qquad (2)$$

and consequently equations (1) assumes the form:

$$\varepsilon_{RR} = - S \frac{d^2v}{dR^2} + \frac{du}{dR}$$
$$\varepsilon_{RS} = 0 \qquad (3)$$

This formulation requires the continuity in C^1 for the discrete variables in the shape functions: they have to be at least of the third order in R for v and of the first order for u (see table 1).
The use of the Timoshenko theory gives many advantages and one problem:
- the shape functions have to be continuous in C°, they can be the same for u, v and θ (see table 1).
- Three nodes elements can be used, so that curved beams can be represented, the layers can have curvilinear borders, sensible modifications in the cross section shape do not give problems.
- The shear deformations are directly taken into account.
- The problem is that the strain tensor components are not of the same order, so that the shear effects are overestimated when the height of the beam becomes smaller and smaller.

This difficulty has been successfully overjumped by fictitiously reducing the degree of ε_{RS} and setting the ε_{RS} and the ε_{RS} reduced at the same values at the Gauss integration points.
For the prestressing steel layers the strain displacement relations are modified as follows:

$$\varepsilon_{RR} = - \frac{\partial\theta}{\partial R} + \frac{\partial u}{\partial R} + \varepsilon_o + \frac{\partial \Delta u}{\partial R}$$
$$\varepsilon_{RS} = - \theta + \frac{\partial v}{\partial R} \qquad (4)$$

Where ε_o is the initial strain due to prestressing and Δu is the slippage between cable and concrete after the bond has been destroyed.

4.2. Constitutive Equations

The constitutive equations have to be biaxial for concrete and steel and have to treat the bond between steel and concrete.
Concrete. The relations presented by Kupfer and Gerstle /8/ have been used. They separate the hydrostatic and deviatoric behavior:

$$\sigma_o = 3K\, \varepsilon_o$$
$$\sigma_o = 2G\, \gamma_o \qquad (5)$$

finding a very good correlation between the experimental results and the following equations:

$$G_S/G_O = 1 - a\,(\tau_o/fcu)^m$$
$$G_T/G_O = \frac{(G_S/G_o)^2}{m - G_S/G_O\,(m - 1)}$$
$$K_S/K_O = G_S/G_O\, e^{(c\gamma_o)^P} \qquad (6)$$
$$K_T/K_O = \frac{G_T/G_o}{e^{-(c\gamma_o)^P}\,(1 - p(c\gamma_o)^P)}$$

where G_S and K_S are the secant moduli,
G_T and K_T are the tangent moduli,
G_o, K_o, a, m, c, and p are constants given as a function of ultimate monoaxial strength f_{cu}

The biaxial strength is then given by the expressions:

compression-compression $(\sigma_1/fcu + \sigma_2/fcu)^2 + \sigma_1/fcu + 3.65\ \sigma_2/fcu = 0$

compression-tension $\sigma_2/ftu = 1 + 0.8\ \sqrt[3]{\sigma_1/fcu}$ (7)

tension-tension $\sigma_2 = f_{tu} = 0.64\ \sqrt[3]{f_{cu}^2}$

The material stiffness matrix is:

$$4G\ \frac{3K+G}{3K+4G}\begin{bmatrix} 1 & \frac{3K-2G}{2(3K+G)} & 0 \\ \frac{3K-2G}{2(3K+G)} & 1 & 0 \\ 0 & 0 & \frac{3K+4G}{4(3K+G)} \end{bmatrix} \tag{8}$$

setting $\sigma_y = 0$ the final relations for concrete are obtained:

$$\begin{bmatrix} \sigma_{RR} \\ \sigma_{RS} \end{bmatrix} = \begin{bmatrix} \frac{9KG}{3K+D} & 0 \\ 0 & G \end{bmatrix} \times \begin{bmatrix} \varepsilon_{RR} \\ \varepsilon_{RS} \end{bmatrix} \tag{9}$$

<u>Steel</u>. The Von Mises plastic potential has been used, with an isotropic hardening taken from the CEB quintic for prestressing steel:

$$\varepsilon_S = 0.823\ \left(\frac{\sigma_S}{f_{y0.2}} - 0.77\right)^5 \qquad \sigma_S > 0.7\ \frac{f_{y0.2}}{E_S}$$

$$\varepsilon_S = \frac{\sigma_S}{E_S} \qquad \sigma_S < 0.7\ \frac{f_{y0.2}}{E_S} \tag{10}$$

By conformity and normality the following expression for the plastic velocity of deformation is obtained:

$$\dot{\varepsilon}_{ij}^{(p)} = \bar{H}\ \frac{\partial f}{\partial \sigma_{kl}}\ \dot{\sigma}_{kl}\ \frac{\partial f}{\partial \sigma_{ij}} \tag{11}$$

where

$$\bar{H} = \frac{1}{\frac{\partial f}{\partial \varepsilon_{mn}^{(P)}} + \frac{\partial f}{\partial \chi}\ \frac{\partial \chi}{\partial \varepsilon_{mn}^{(P)}}\ \frac{\partial f}{\partial \sigma_{mn}}}$$

and

$$f(\sigma_{ij}, \dot{\varepsilon}_{ij}^{(P)}, \chi(\dot{\varepsilon}_{ij}^{(P)})) = 0 \tag{12}$$

by consistency.

With some manipulation the material stiffness matrix is obtained:

$$\begin{bmatrix} \Delta\sigma_{RR} \\ \\ \Delta\sigma_{RS} \end{bmatrix} = \frac{1}{\frac{4\bar{H}}{9G}\sigma_{RR}^2 + \frac{4\bar{H}}{E}\sigma_{RR}^2 + \frac{1}{EG}} \begin{bmatrix} 4\bar{H}\sigma_{RS}^2 + \frac{1}{G} & -\frac{3}{4}\bar{H}\sigma_{RR}\sigma_{RS} \\ \\ -\frac{4}{3}\bar{H}\sigma_{RR}\sigma_{RS} & \frac{4}{9}\bar{H}\sigma_{RR}^2 + \frac{1}{E} \end{bmatrix} \begin{bmatrix} \Delta\varepsilon_{RR} \\ \\ \Delta\varepsilon_{RS} \end{bmatrix} \tag{13}$$

with

$$\bar{H} = 9.2587534 \frac{\sqrt[5]{(\varepsilon_p/0.823)^4}}{f_{y0.2}^2\,\sigma_{RR}\,(0.7 + \sqrt[5]{\varepsilon_p/0.823})}$$

For the ordinary steel hardening has been neglected because it is not usually reached: the ultimate strain of the prestressing steel is normally lower than the hardening strain of the ordinary steel.
Its constitutive equation is consequently simply elastic-perfectly plastic.

<u>Bond</u>. The bond between steel and concrete is presently considered as rigid-plastic, but it could be easily transformed in a multi-linear relation.
At every step the stresses at the border of the concrete layers are checked at the joint position: if a failure occurs the bond ties are supposed to be broken.
As a consequence the height variable S for the steel layer is not any more taken with reference to the N.A.of the section, but with reference to the N.A. of the layer. In such a way the moment of inertia of the steel layer is not any more contributing to the whole section moment of inertia.
Moreover a new unknown variable Δu (eq.2) is considered, which is the displacement of the steel layer at the joint where the failure has occumed.
Of course this means that a new row and a new column are inserted in the stiffness matrix.
Two different strategies are possible, depending on the meaning of Δu: it may be taken as the total displacement of the cable joint or as the displacement of the cable joint with respect to the deformed position of the section. To clarify the ideas let's consider a two elements, three joints, two layers example in the simpler formulation.
The stiffness matrix can be written as follows:

$$\begin{bmatrix} F_{u1} \\ F_{v1} \\ F_{\theta 1} \\ F_{u2} \\ F_{v2} \\ F_{\theta 2} \\ F_{u3} \\ F_{v3} \\ F_{\theta 3} \\ F_{\Delta u} \end{bmatrix} = \begin{bmatrix} K_{1,1} & 0 & 0 & K_{1,4} & 0 & 0 & 0 & 0 & 0 & K_{1,10} \\ & K_{2,2} & K_{2,3} & 0 & K_{2,5} & K_{2,6} & 0 & 0 & 0 & 0 \\ & & K_{3,3} & 0 & K_{3,5} & K_{3,6} & 0 & 0 & 0 & 0 \\ & & & K_{4,4} & 0 & 0 & K_{4,7} & 0 & 0 & 0 \\ & & & & K_{5,5} & K_{5,6} & 0 & K_{5,8} & K_{5,9} & 0 \\ & & & & & K_{6,6} & 0 & K_{6,8} & K_{6,9} & 0 \\ & \text{sym.} & & & & & K_{7,7} & 0 & 0 & 0 \\ & & & & & & & K_{8,8} & K_{8,9} & 0 \\ & & & & & & & & K_{9,9} & 0 \\ & & & & & & & & & K_{10,10} \end{bmatrix} \begin{bmatrix} u_1 \\ v_1 \\ \theta_1 \\ u_2 \\ v_2 \\ \theta_2 \\ u_3 \\ v_3 \\ \theta_3 \\ \Delta u \end{bmatrix} \tag{14}$$

Some significant terms of the stiffness matrix are explicitly given in table 2, before and after the bond failure at joint 2.

5. CONCLUSIONS

The main points to be considered when dealing with modeling P.C. frames are discussed, showing that a choice should be operated case by case to take into account the most important effects and to disregard the others.
An original model is presented: it seems to be a good compromise between

simplicity of calculation and refinement in the material modeling.
The model deals satisfactorily with the problems of bond, shear, connections and constitutive relationships, but presently disregards the time dependent effects.
Many simulations of different structure are needed in order to check the effectiveness of the model; then further refinements might be implemented.
The most important of them seem to be to consider a multilinear law for bond and to take into account time dependent effects.
A particular problem deals with the constitutive equations for concrete, estensively discussed in /3/; in the present model it seems to be necessary to take into account the effect of confinement due to steel.

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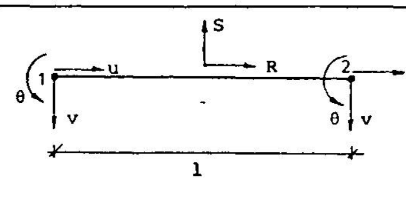
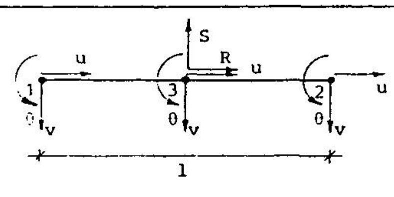
$N_{v,1} = \frac{1}{2} - \frac{3}{2}\frac{R}{l} + 2\frac{R^3}{l^3}$ $N_{v,2} = \frac{1}{2} + \frac{3}{2}\frac{R}{l} - 2\frac{R^3}{l^3}$ $N_{\theta,1} = \frac{l}{8} - \frac{R}{4} - \frac{R^2}{2l} + \frac{R^3}{l^3}$ $N_{\theta,2} = -\frac{l}{8} - \frac{R}{4} + \frac{R^2}{2l} + \frac{R^3}{l^3}$ $N_{u,1} = \frac{1}{2} - \frac{R}{l}$ $N_{u,2} = \frac{1}{2} + \frac{R}{l}$	$N_{u,v,\theta,1} = \frac{2R^2}{l^2} - \frac{R}{l}$ $N_{u,v,\theta,2} = \frac{2R^2}{l^2} + \frac{R}{l}$ $N_{u,v,\theta,3} = 1 - \frac{4R^2}{l^2}$
	

Table 1 - shape functions according to the Bernoulli-Navier and the Timoshenko formulations

	Before bond failure at joint 2	After bond failure at joint 2
$K_{1,1}$	$\int_I E_I N_1'^2\, dv + \int_{II} E_{II} N_1'^2\, dv^{(1)}$	same
$K_{1,4}$	$\int_I E_I N_1' N_2'\, dv + \int_{II} E_{II} N_1' N_2'\, dv^{(1)}$	$\int_I E_I N_1' N_2'\, dv^{(1)}$
$K_{1,10}$	0	$\int_{II} E_{II} N_1' N_2'\, dv^{(1)}$
$K_{2,2}$	$\int_I G_I N_1'^2\, dv + \int_{II} G_{II} N_1'^2\, dv^{(1)}$	same
$K_{2,3}$	$\int_I G_I N_1' N_1\, dv + \int_{II} G_{II} N_1' N_1\, dv^{(1)}$	same
$K_{2,5}$	$\int_I G_I N_1' N_2'\, dv + \int_{II} G_{II} N_1' N_2'\, dv^{(1)}$	same
$K_{2,6}$	$\int_I G_I N_1' N_2\, dv + \int_{II} G_{II} N_1' N_2\, dv^{(1)}$	same
$K_{3,3}$	$\int_I (E_I S_I^2 N_1'^2 + G_I N_1^2)\, dv + \int_{II} (E_{II} S_{II}^2 N_1'^2 + G_{II} N_1^2)\, dv^{(1)}$	same
$K_{3,5}$	$\int_I G_I N_1 N_2'\, dv + \int_{II} G_{II} N_1 N_2'\, dv^{(1)}$	same
$K_{3,6}$	$\int_I (G_I N_1 N_2 + E_I S_I^2 N_1 N_2)\, dv + \int_{II} (G_{II} N_1 N_2 + E_{II} S_{II}^2 N_1 N_2)\, dv^{(1)}$	$\int_I (G_I N_1 N_2 + E_I S_I^2 N_1 N_2)\, dv^{(1)}$
$K_{4,4}$	$\int_I E_I N_2'^2\, dv + \int_{II} E_{II} N_2'^2\, dv^{(1)} + \int_I E_I N_1'^2\, dv + \int_{II} E_{II} N_1'^2\, dv^{(2)}$	$\int_I E_I N_2'^2\, dv^{(1)} + \int_I E_I N_1'^2\, dv^{(2)}$
$K_{5,5}$	$\int_I G_I N_2'^2\, dv + \int_{II} G_{II} N_2'^2\, dv^{(1)} + \int_I G_I N_1'^2\, dv + \int_{II} G_{II} N_1'^2\, dv^{(2)}$	same
$K_{5,6}$	$\int_I G_I N_2' N_2\, dv + \int_{II} G_{II} N_2' N_2\, dv^{(1)} + \int_I G_I N_1' N_1\, dv + \int_{II} G_{II} N_1' N_1\, dv^{(2)}$	same
$K_{6,6}$	$\int_I (E_I S_I^2 N_2'^2 + G_I N_2^2)\, dv + \int_{II} (E_{II} S_{II}^2 N_2'^2 + G_{II} N_2^2)\, dv^{(1)} +$ $+ \int_I (E_I S_I^2 N_1'^2 + G_I N_1^2)\, dv + \int_{II} (E_{II} S_{II}^2 N_1'^2 + G_{II} N_1^2)\, dv^{(2)}$	$\int_I (E_I S_I^2 N_2'^2 + G_I N_2^2)\, dv + \int_{II} G_{II} N_2^2\, dv^{(1)} +$ $+ \int_I (E_I S_I^2 N_1'^2 + G_I N_1^2)\, dv + \int_{II} G_{II} N_1^2\, dv^{(2)}$
$K_{10,10}$	0	$\int_{II} E_{II} N_2'\, dv^{(1)} + \int_{II} E_{II} N_1^2\, dv^{(2)}$

Table 2 - Timoshenko formulation: some terms of the stiffness matrix (I layer, 1 joint, (1) element)