Energy dissipation analysis for creep in heated concrete

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Analyse de la dissipation d'énergie par fluage dans les bétons chauffés

Analyse der Energiedissipation für Kriechen in erwärmtem Beton

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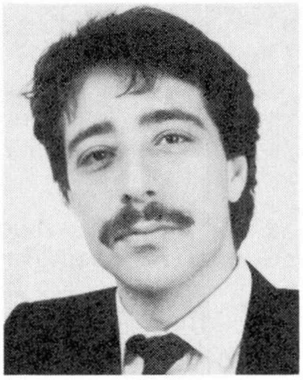
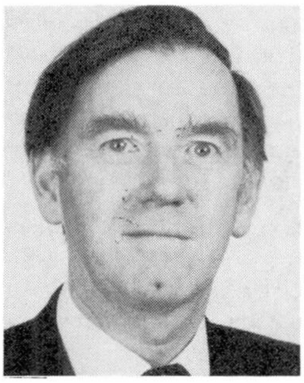


Khaldoun Zeitouni obtained his B.Sc (Eng) degree in civil engineering in 1983 and since that time has been researching into the time- and temperature-dependent behaviour of concrete structures undergoing creep; with a particular interest in their mathematical description. This work will lead to a Ph.D. thesis.



As a member of the academic staff of King's College George England obtained his B.Sc (Eng) in 1975; became Reader in Engineering Mechanics and now holds a personal Chair as Professor of Mechanics and Structures. His consultancy activities centre around structural problems involving time dependency.

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SUMMARY

The paper highlights the need for creep analyses of non-uniformly heated concrete structures. The energy dissipation rate from the structure as a whole uniquely defines a given problem, and is used to establish fundamental theorems relating to the nature of the energy-dissipation-rate/time curve for a structure and to derive a variational technique based on power concepts to evaluate the time-dependent stresses. One outcome of the work is the identification of a simple direct approach to the calculation of the limiting (in time) steady-state stresses. An illustrative example is presented.

RÉSUMÉ

L'article montre le besoin d'analyse de fluage sur des structures en béton chauffées de manière non-uniforme. Le taux de dissipation d'énergie de la structure dans sa totalité, qui définit à lui seul et entièrement un problème donné, est utilisé pour l'établissement de théorèmes fondamentaux liés à la nature des courbes du taux de la dissipation d'énergie dans le temps pour une structure; il en découle une technique de formulation variable basée sur des concepts de puissance pour évaluer les contraintes en fonction du temps. Un résultat des travaux est l'identification de la simple approche directe pour le calcul des contraintes de l'état constant limité dans le temps. Un exemple d'application est présenté.

ZUSAMMENFASSUNG

Dieses Referat wirft ein Licht auf die Notwendigkeit von Kriech-Analysen der ungleichmässig geheizten Betonstrukturen. Die Rate der Energiedissipation der Konstruktion als Ganzes beschreibt einzig und allein ein bestimmtes Problem und wird benutzt, um fundamentale Theoreme festzustellen, die mit der Art der Kurve der Energiedissipationsrate/Zeit einer Konstruktion zusammenhängen, und um eine Variations-Methode abzuleiten, die auf Kraftkonzeptionen basiert, um die zeitbedingten Belastungen auszuwerten. Eines der Ergebnisse dieser Arbeit ist die Identifikation eines einfachen direkten Ansatzes zur Berechnung der zeitlich limitierenden Belastungen. Ein illustratives Beispiel wird vorgelegt.

1. INTRODUCTION

Creep in concrete structures causes displacements to change with time and stresses to undergo redistribution whenever non-homogeneous properties are present. Because creep is strongly temperature dependent non-uniformly heated structures often exhibit major redistribution of internal stresses - during time-invariant mechanical loading - and with attendant changes to external supporting reactions.

Understanding the behaviour of heated concrete structures is of importance to the Engineer, and he must be provided with design rules which allow him to predict structural performance from specified conditions of loading, temperature, etc.

In order to develop sound analytical and design procedures it is essential to understand the basic principles which govern the way in which a structure will perform during creep. The first part of this paper therefore addresses some philosophical aspects of structural performance in a mathematical context. These lead to a useful predictive analysis for the time-varying stresses which is based on a power variational formulation. The analysis is then recognised in the context that,

> **"The total Virtual Power of a structure or body that is undergoing creep, is zero when the strain rates (and support displacement rates) are at all times compatible, and the structure/body is subjected to an equilibrium variation to the internal stresses (and support reactions)".**

A three-dimensional analysis is then formulated and some observations relating to its predictive capacity are made and compared with experimental performance for justification. In the last section a simple example is presented for a once redundant pin-jointed structure, to illustrate the solution technique.

Reference also is made to other uses of the theory in two- and three-dimensional finite element analysis.

2. THEORETICAL DEVELOPMENT

The derivation of the Virtual Power equation of section 4, Eq.(23), is assisted here by the adoption of a specific constitutive creep law in which the strain rates, $d\varepsilon/dt$, are related to the current stresses, σ, and stress rates, $d\sigma/dt$, only. Eq(1) is the one-dimensional relationship which is suitable for representing the temperature-dependent flow component of creep for concrete.

$$\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \sigma\phi(T)\frac{dc}{dt} \tag{1}$$

In this equation c represents a normalised creep strain, with respect to stress, σ, and a temperature function, $\phi(T)$. Rearrangement of Eq.(1) gives,

$$\frac{d\varepsilon}{dc} = \frac{1}{E}\frac{d\sigma}{dc} + \sigma\phi(T) \tag{2}$$

Comparison of Eqs.(1) and (2) reveals a simple time transformation such that Eq.(2) represents a linear Maxwell law for which the normal viscosity, η, is replaced by the reciprocal of $\phi(T)$ and time is replaced by the normalised creep parameter itself, c. This parameter is a psuedo-time variable in the analysis which follows, and all 'dotted' terms relate to differentiations with respect to pseudo-time, c. In the engineering problem conversion to real time, t, is effected at the end of the analysis by reference to the c-t curve for the appropriate material. Eq.(2) may now be rewritten,

$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \sigma\phi(T) \tag{3}$$

In the creeping material subjected to external mechanical loading the work balance equation takes the form,

$$\dot{W} = \dot{D} + \dot{U} \tag{4}$$

where $\dot{W}$ is the rate of working of the external loads

$\dot{D}$ is the rate at which energy is dissipated in creep

$\dot{U}$ is the rate of change of internal strain energy

Hence $$U = \tfrac{1}{2}\int \{\sigma\}^T\{\varepsilon_e\}dv \tag{5}$$

$$\dot{D} = \int \{\sigma\}^T\{\dot{\varepsilon}_c\}dv \tag{6}$$

where $\{\sigma\}$ and $\{\varepsilon_e\}$ are the conventional six component elastic stress and strain vectors and $\{\dot{\varepsilon}_c\}$ (derived from experimental creep data) have the form,

$$\{\dot{\varepsilon}_c\} = \begin{Bmatrix} \dot{\varepsilon}_{c,x} \\ \dot{\varepsilon}_{c,y} \\ \dot{\varepsilon}_{c,z} \\ \dot{\varepsilon}_{c,xy} \\ \dot{\varepsilon}_{c,yz} \\ \dot{\varepsilon}_{c,zx} \end{Bmatrix} = \phi(T) \begin{bmatrix} 1 & -\nu_c & -\nu_c & 0 & 0 & 0 \\ -\nu_c & 1 & -\nu_c & 0 & 0 & 0 \\ -\nu_c & -\nu_c & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu_c) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu_c) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu_c) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} \tag{7}$$

Also $\{\varepsilon_e\}$ are related to the stresses in a similar manner to Eq.(7), after ν_c is replaced by ν_e the elastic Poisson's ratio, and $1/\phi(T)$ is replaced by the elastic modulus, E.

Eq.(5) now has the form,

$$U = \tfrac{1}{2}\int \frac{1}{E}\{\sigma\}^T[V_e]\{\sigma\}dv \tag{8}$$

and $$\dot{U} = \int \frac{1}{E}\{\sigma\}^T[V_e]\{\dot{\sigma}\}dv \tag{9}$$

Also $$\dot{D} = \int \phi(T)\{\sigma\}^T[V_c]\{\sigma\}dv \tag{10}$$

Experiments reveal that $\nu_e \doteq \nu_c$; hence writing $[V_e] = [V_c] = [V]$ the external work rates are,

$$\begin{aligned} \dot{W} &= \int \frac{1}{E}\{\sigma\}^T[V]\{\dot{\sigma}\}dv + \int \phi(T)\{\sigma\}^T[V]\{\sigma\}dv \\ &= \int \{\sigma\}^T\left(\frac{d}{E} + \phi(T)\right)[V]\{\sigma\}dv \end{aligned} \tag{11}$$

where $d = d(\ )/dc$

Because $$\dot{W} = \int\{\sigma\}^T\{\dot{\varepsilon}\}dv \tag{12}$$

it follows that the three-dimensional stress/total strain relationship is,

$$\{\dot{\varepsilon}\} = \left(\frac{d}{E} + \phi(T)\right)[V]\{\sigma\} \tag{13}$$

Observations of Eq.(9) and (10) suggest that if a steady state of stress exists then $\dot{U} \to 0$ and $\dot{D}$ tends to a constant positive value at large times. The nature of these time-dependent variations of $\dot{U}$ and $\dot{D}$ however are not revealed; though numerical computations have for some time indicated that the variation of $\dot{D}$ with time (pseudo-time) is a monotonically declining function which approaches asymptotically a minimum value[1].

A formal proof of this behaviour is now given and used to derive the variational equations from which a solution for the time-varying stresses is obtained.

From Eqs.(6) and (10) the incremental change in $\dot{D}$, $\Delta\dot{D}$, over a time interval, Δ_c is,

$$\dot{D} + \Delta\dot{D} = \int\{\sigma+\Delta\sigma\}^T[V]\{\sigma+\Delta\sigma\}\phi(T)dv$$

but $\Delta\dot{D} = \dot{D}(c+\Delta c) - \dot{D}(c)$, hence,

$$\Delta\dot{D} = \int\{(\{\sigma\}^T[V] + \{\Delta\sigma\}^T[V])(\{\sigma\} + \{\Delta\sigma\}) - \{\sigma\}^T[V]\{\sigma\}\}\phi(T)dv$$

Ignoring second order terms in $\Delta\sigma$ leads to,

$$\Delta\dot{D} = \int\{\{\Delta\sigma\}^T[V]\{\sigma\} + \{\sigma\}^T[V]\{\Delta\sigma\}\}\phi(T)dv \tag{14}$$

From Eq.(13)

$$\{\dot{\varepsilon}\} = (\frac{d}{E} + \phi(T))[V]\{\sigma\}$$

Multiplying both sides by $[V]^{-1}$ gives,

$$[V]^{-1}\{\dot{\varepsilon}\} = (\frac{d}{E} + \phi(T))\{\sigma\}$$

$$\therefore \quad [V]^{-1}\{\dot{\varepsilon}\} = \frac{1}{E}\{\dot{\sigma}\} + \phi(T)\{\sigma\} \tag{15}$$

or

$$\phi(T)\{\sigma\} = [V]^{-1}\{\dot{\varepsilon}\} - \frac{1}{E}\{\dot{\sigma}\} \tag{16}$$

Substituting $\phi(T)\{\sigma\}$ from Eq.(16) into Eq.(14) yields,

$$\Delta\dot{D} = \int(\{\Delta\sigma\}^T[V]\{[V]^{-1}\{\dot{\varepsilon}\} - \frac{1}{E}\{\dot{\sigma}\}\} + \{[V]^{-1}\{\dot{\varepsilon}\} - \frac{1}{E}\{\dot{\sigma}\}\}^T[V]\{\Delta\sigma\})dv$$

Noting that

$$[[V]^{-1}]^T = [V]^{-1}$$

$$\Delta\dot{D} = \int(\{\Delta\sigma\}^T\{\dot{\varepsilon}\} - \frac{1}{E}\{\Delta\sigma\}^T[V]\{\dot{\sigma}\} + \{\dot{\varepsilon}\}^T\{\Delta\sigma\} - \frac{1}{E}\{\dot{\sigma}\}^T[V]\{\Delta\sigma\})dv$$

The first and third terms of this equation vanish by reason of $\{\dot{\varepsilon}\}$ being compatible and $\{\Delta\sigma\}$ being an equilibrium set of stresses; hence,

$$\Delta\dot{D} = -\int\frac{1}{E}(\{\Delta\sigma\}^T[V]\{\dot{\sigma}\} + \{\dot{\sigma}\}^T[V]\{\Delta\sigma\})dv$$

Dividing through by $\Delta c>0$ and taking the limit as $\Delta c \to 0$,

$$\frac{d\dot{D}}{dc} = -\int\frac{1}{E}(\{\dot{\sigma}\}^T[V]\{\dot{\sigma}\} + \{\dot{\sigma}\}^T[V]\{\dot{\sigma}\})dv$$

$$= -2\int\frac{1}{E}\{\dot{\sigma}\}^T[V]\{\dot{\sigma}\}dv \tag{17}$$

The scalar quantity under the integral sign can only be positive if the matrix [V] is positive definite. For a compressible material, $\nu<\frac{1}{2}$, the eigenvalues of [V] are all positive; hence [V] is positive definite. It then follows that the slope of the $\dot{D}$ curve is always negative, Eq.(17). This statement is necessary but not sufficient to define the monotonic behaviour of the $\dot{D}$ curve. Additionally it is necessary to establish a positive curvature of the curve at all times. Thus by differentiating Eq.(17) with respect to pseudo-time, c, gives,

$$\frac{d^2\dot{D}}{dc^2} = -2\int\frac{1}{E}(\{\ddot{\sigma}\}^T[V]\{\dot{\sigma}\} + \{\dot{\sigma}\}^T[V]\{\ddot{\sigma}\})dv \tag{18}$$

From Eq.(15)

$$[V]^{-1}\{\ddot{\varepsilon}\} = \frac{1}{E}\{\ddot{\sigma}\} + \phi(T)\{\dot{\sigma}\}$$

Substituting this into Eq.(18) gives,

$$\frac{d^2\dot{D}}{dc^2} = -2\int(\{\ddot{\varepsilon}\}^T[V]^{-1}\{\dot{\sigma}\}-\phi(T)\{\dot{\sigma}\}^T[V]\{\dot{\sigma}\}+\{\dot{\sigma}\}^T[V][V]^{-1}\{\ddot{\varepsilon}\}-\phi(T)\{\dot{\sigma}\}^T[V]\{\dot{\sigma}\})dv$$

The first and third terms of this equation vanish because of compatibility and equilibrium. Hence the equation reduces to,

$$\frac{d^2\dot{D}}{dc^2} = 4\int\phi(T)\{\dot{\sigma}\}^T[V]\{\dot{\sigma}\}dv \geqq 0 \tag{19}$$

Thus Eqs. (17) and (19) together, do establish that the $\dot{D}$ curve is of a monotonically declining shape leading to a steady-state value at infinite time.

3. STRUCTURAL PERFORMANCE

The theory of the previous section was based on the belief that the initial stresses change during creep and tend to limiting or steady-state values at large times. This hypothesis has been tested in a number of experiments and the findings are given in this section.

Imposed support displacements in statically indeterminate structures cause immediate changes to the internal moments and stresses, and to external supporting reactions. It has been long recognised that when these displacements are sustained creep tends to return the structure to the original stress, moment and force state, when the mechanical loading is sustained and the material behaviour of the structure is homogeneous. In this case the steady state is identical with the initial elastic state.

In structures heated non-uniformly and with sustained temperature gradients, the steady state is distinct from the initial elastic state [2,3]. Prestress, when it exists, acts simply as a component of the mechanically applied loading on the structure and therefore influences the steady-state solution.

The existence of a steady state in a structure is often masked because the creep of concrete as a material exhibits a declining rate with increasing time under stress. This masking feature was overcome in experiments carried out at King's College in which selective support displacements were imposed at appropriate times to cause the structure to approach its steady state alternately from below and above [4]. Figure 1 illustrates the behaviour as observed and provides conclusive evidence of the existence of a preferred or steady state.

The theory of Section 2 indicates that during any period of sustained loading and temperature the rate at which energy is dissipated during creep is always decreasing and tending to a minimum value in the steady state. Figure 2 (upper curve) shows the typical $\dot{D}$/c behaviour. This implies that whenever a change of state is imposed on the structure, e.g. by a change of loading or temperature, or by the imposition of displacements,

(a) there will exist a new limiting value of $\dot{D}_{SS}$ defined by the new conditions of load and/or temperature,

and (b) the current value of $\dot{D}$ will be such that $\dot{D} \geqq \dot{D}_{SS}$. Figure 3 illustrates this behaviour.

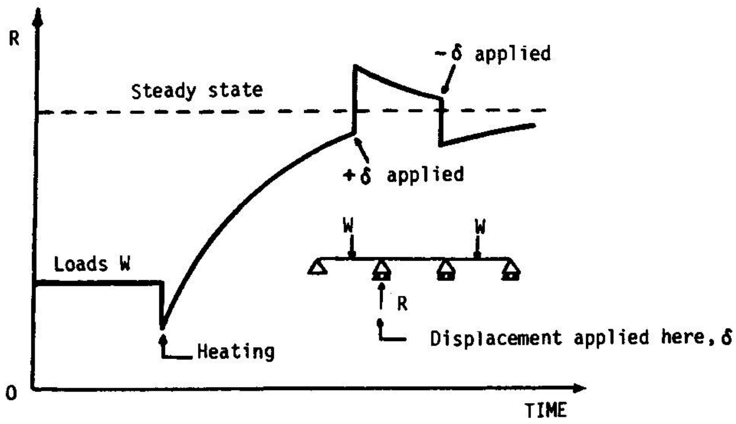

Fig. 1. Influence of creep, temperature and imposed displacement on beam reaction, R.

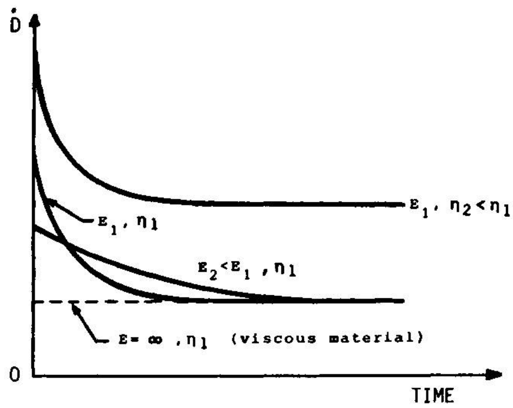

Fig. 2. Dependence of Energy Dissipation Rate on elastic and viscous parameters of Maxwell model. Note: for concrete, $\eta = 1/\phi(T)$.

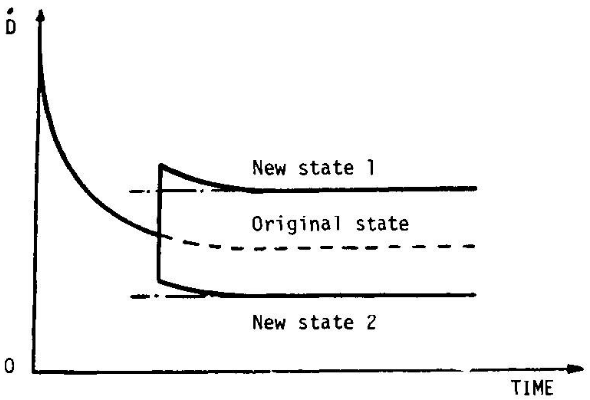

Fig. 3. Changes in Energy Dissipation Rate due to changes of state.

Problems associated with cyclic temperature changes represent another class for which the 'state' is changing repeatedly, and hence $\dot{D} >> \dot{D}_{ss}$ always, Figure 4. Such problems may be formulated in terms of a single parameter, the **average weighted energy dissipation rate** over a complete temperature cycle, $\dot{D}_{av}$. For this representation the $\dot{D}_{av}$ curve is again of monotonic shape tending to a non-zero steady-state value*. Eqs.(17) and (19) then have counterparts for which $\dot{D}$ is replaced by $\dot{D}_{av}$.

Figure 4 illustrates also that although the average energy dissipation rate declines montonically with time, and the dissipation rate during any constant temperature period has a similar form; on a cycle-by-cycle basis the energy dissipation rate in the lower temperature state, T_1, is seen paradoxically to rise as the steady state* is approached.

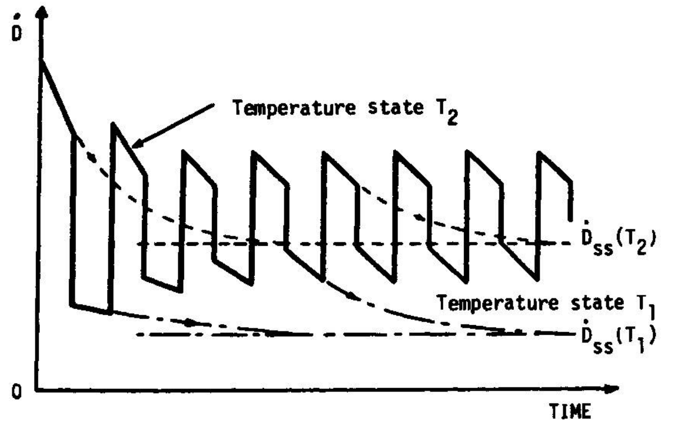

Fig. 4. Energy Dissipation Rate - Time behaviour under cyclically changing temperature states.

4. TIME-DEPENDENT STRESS ANALYSIS

The theory of Section 2 is used here to develop an analysis procedure for the determination of the time-varying stresses during sustained loading and temperature conditions. For the continuum problem it is approximate but may be exact for problems containing a finite number of redundancies.

Equating the right hand side of Eq.(17) to Eq.(10) after differentiation leads to:

$$\int \phi(T)\{\sigma\}^T[V]\{\dot{\sigma}\}dv = -\int \frac{1}{E}\{\dot{\sigma}\}^T[V]\{\dot{\sigma}\}dv \tag{20}$$

i.e.

$$\int \left(\phi(T) + \frac{d}{E}\right)\{\sigma\}^T[V]\{\dot{\sigma}\}dv = 0 \tag{21}$$

Employing Eq.(13) then reduces this equation to the statement,

$$\int \{\dot{\varepsilon}\}^T\{\dot{\sigma}\}dv = 0 \tag{22}$$

Expressing $\{\dot{\sigma}\}$ as $\{\frac{d\sigma}{dc}\} \doteq \{\frac{\delta\sigma}{\delta c}\}$ allows Eq. (22) to be written,

$$\int \{\dot{\varepsilon}\}^T\{\delta\sigma\}dv = 0 \quad \text{for } \delta c \neq 0 \tag{23}$$

Eq.(23) is valid in a more general sense than the specific derivation might suggest. It represents the same Virtual Power statement as given in Section 1 for the case of no imposed boundary displacement rates. The requirements of Eq.(23) are therefore:

*This state is not a true steady state but is simply a repeating cyclic state.

(a) $\{\dot{\varepsilon}\}$ represent a set of compatible internal strain rates.
(b) $\{\delta\sigma\}$ represent any set of equilibrium stresses and not necessarily those relating to the actual changes over the time interval δc.

4.1 Solution procedure

Equilibrium at any time is chosen to be represented by the following state of stress,

$$\{\sigma\} = \{\sigma_0\} + a_1\{\sigma_1\} + \ldots\ldots\ldots + a_n\{\sigma_n\} \qquad (24)$$

in which $\{\sigma_0\}$ is any set of stresses satisfying equilibrium of the boundary loading. $\{\sigma_1\}$ to $\{\sigma_n\}$ are sets of internal self-equilibrating stresses, and a_1 to a_n are time-dependent weighting parameters.

It may be observed from the form of Eq.(24) that $\{\delta\sigma\}$ of Eq.(23) take the form,

$$\{\delta\sigma\}_i = \frac{\partial}{\partial a_i}\{\sigma\} = \{\sigma_i\} \qquad (25)$$

The stress solution is now confined to the determination of the n values of a_i. Eq.(23) is used to generate n independent equations from which the a_i parameters may be evaluated.

Eq.(24) may be expressed as,

$$\{\sigma\} = \{\sigma_0\} + [J]\{a\} \qquad (26)$$

where $[J] = \left[\{\delta\sigma\}_1 \ldots\ldots\ldots \{\delta\sigma\}_n\right]$

Employing Eq.(13), with the inclusion of $\{\sigma\}$ as defined in Eq.(26), $\{\dot{\varepsilon}\}^T$ of Eq.(23) is defined in terms of the stress components (viz. Eq.(24) representation) and weighting parameters, a_i. The introduction of $\{\delta\sigma\}$ into Eq.(23) then leads to a set of first order differential equations in time from which a_i may be evaluated. Thus, Eq.(23) becomes,

$$0 = \int\left(\frac{1}{E}[J]^T[V][J]\{\dot{a}\} + \phi(T)[J]^T[V][J]\{a\} + \phi(T)[J]^T[V]\{\sigma_0\}\right)dv$$

This reduces to the following set of equations.

$$[A]\{\dot{a}\} + [B]\{a\} + [C] = 0 \qquad (28)$$

The general terms of the matrices in Eq.(28) are,

$$A_{r,s} = \int\frac{1}{E}\{\sigma_s\}^T[V]\{\sigma_r\}dv$$
$$B_{r,s} = \int\phi(T)\{\sigma_s\}^T[V]\{\sigma_r\}dv \qquad (29)$$
$$C_r = \int\phi(T)\{\sigma_0\}^T[V]\{\sigma_r\}dv$$

5. ILLUSTRATIVE EXAMPLE

5.1 Statement of the Problem

The inset diagram of Figure 5 shows a symmetrical pin-jointed structure of three members (each of length, L, and the same cross-sectional area, A) supporting a single concentrated load, W, acting in the line of the central member. The outer members, inclined at angle θ to the horizontal, are at temperatures, $T_1<T_2$. It is assumed that the elastic modulus, E, is uniform throughout and that the Maxwell creep law of Eq.(2) is applicable; with $\phi(T) = T$.

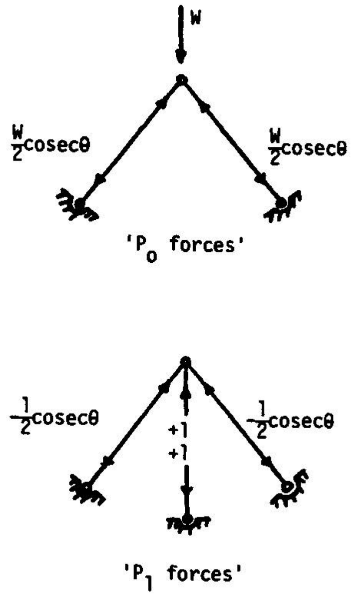

Fig. 5. Loadings on statically determinate released structure.

5.2 Solution Procedure

The equivalent one-dimensional statement to Eq.(23) is,

$$\int \dot{\varepsilon}\delta\sigma dv = 0$$

which becomes for a set of pin-jointed members, subjected to axial forces only,

$$\sum_{\text{members}} \dot{\varepsilon}\delta\sigma AL = 0 \tag{30}$$

For the once redundant problem the state of stress is represented everywhere by the expression,

$$\sigma = \sigma_0 + a_1\sigma_1 \tag{31}$$

where σ_0 and σ_1 are defined by the stresses created by the actual loading and a unit internal force applied separately to a statically determinate released structure as detailed in Figure 5.

For the pin-jointed structure Eq.(30) reduces to the form,

$$\sum\left(\frac{\dot{P}}{AE} + \frac{PT}{A}\right)\frac{\delta P}{A}AL = 0 \tag{32}$$

where the summation is taken over all the members, and $P = \sigma A$, etc. Further substitution leads to,

$$\sum\left\{\frac{\dot{a}_1 P_1}{AE} + (P_0 + a_1 P_1)\frac{T}{A}\right\}P_1 L = 0$$

or
$$\dot{a}_1 \sum \frac{P_1{}^2 L}{AE} + a_1 \sum \frac{P_1{}^2 TL}{A} + \sum \frac{P_0 P_1 LT}{A} = 0 \tag{33}$$

Eq.(33) is then written,

$$A^*\dot{a}_1 + B^*a_1 + C^* = 0 \qquad (34)$$

where,

$$A^* = \frac{1}{E}(1+2\sin^2\theta)$$

$$B^* = T_1 + 2T_2 \sin^2\theta$$

$$C^* = -WT_1$$

The solution of Eq.(34) for a_1 is,

$$a_1 = e^{-B^*c/A^*} G^* - C^*/B^*$$

Here, the constant, G^*, is obtained from the initial condition at t=0, i.e. from the elastic solution; and the product $-C^*/B^*$ represents the steady-state solution to which the creep solution tends at large times. Figure 6 shows the way in which the member stresses (and forces) change during creep for this non-uniform temperature problem.

Eq.(23) has also been used successfully in three-dimensional analysis in a prestressed concrete nuclear reactor containment vessel [5].

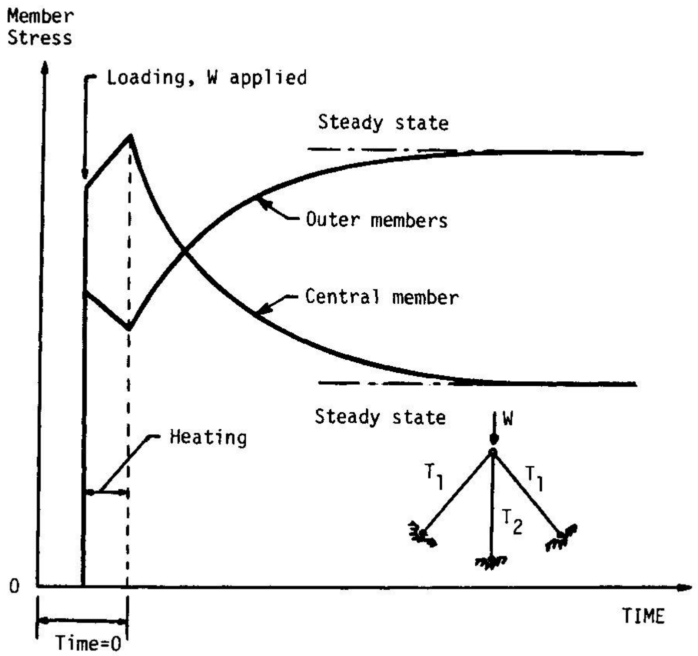

Fig. 6. Variation of member stresses with time in heated pin-jointed structure

6. CONCLUDING REMARKS

Section 2 has established a formal proof that the energy dissipation rate during creep (for the structure as a whole)† is a monotonically declining quantity leading to a minimum value in the steady state.

The theoretical existence of a steady state for which stresses become time-invariant but strains continue to change is in accord with experimental findings.

A general theory derived from energy dissipation concepts is presented in Section 4 for the evaluation of time-dependent stresses under sustained loads and temperatures during non-homogeneous creep.

†Non-homogeneous creep where the isotropic behaviour is Maxwell.

An application of the theory to a simple example has demonstrated the capabilities of the procedures and the nature of the creep problem in concrete at non-uniform temperatures.

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