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Simulation of the Mechanical Behaviour of Young Concrete
Simulation du comportement mécanique de béton jeune
Simulation des mechanischen Verhaltens von jungem Beton

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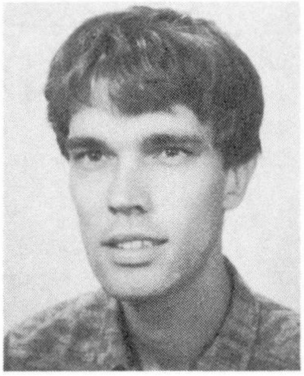

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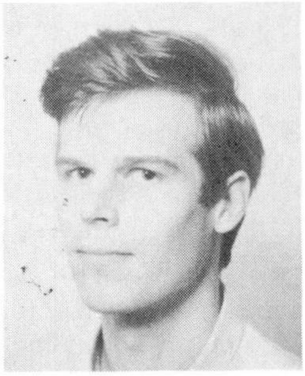

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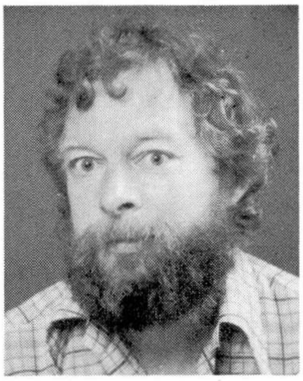

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SUMMARY
A model for the simulation of the time-dependent mechanical behaviour of concrete at an early age is outlined. Next, an efficient algorithm is derived for this model. The model is verified with experimental data. To demonstrate the applicability of the model, the stress is calculated as a function of time for an axisymmetric structure.

RÉSUMÉ
Un modèle pour la simulation du comportement mécanique du béton en fonction de l'âge est présenté pour un béton jeune. Un algorithme efficace est dérivé de ce modèle. Le modèle est vérifié à l'aide de données expérimentales. Afin de montrer la valeur du modèle, la contrainte est calculée comme une fonction du temps pour une structure axisymétrique.

ZUSAMMENFASSUNG
Ein Modell zur Simulation der zeitabhängigen Eigenschaften von jungem Beton wird beschrieben. Danach wird ein leistungsfähiger Rechenalgorithmus abgeleitet. Das Modell wird an Versuchsresultaten verifiziert. Die Anwendung wird an einem symmetrischen Betonbauteil demonstriert.

1. INTRODUCTION

The material properties in a hardening cement paste develop gradually during the hydratation process in which cement forms a compound with water. This chemical process produces heat and a consequence thereof we observe an increase in temperature in the structure. Stresses are introduced when thermal volume changes are prohibited. This happens especially in concrete members with a great volume compared with the surface.

In hardening cement paste we first observe an increase in temperature owing to the generated hydratation energy. Thereafter, a "cooling process" takes place during which crack formation is often observed. The temperature increase in the first period results in compressive stresses because the thermal expansion is suppressed by the surrounding concrete. These stresses, however, will not be significant since the stiffness of the young concrete is still low. The decrease in temperature in the second phase will cause higher tensile stresses because of the greater maturity and as a consequence thereof, the higher value for Young's modulus.

Basically, the hardening of the cement paste is a coupled thermomechanical problem. Because of the weak coupling in the present case, we can suffice by solving the separate problems of heat conduction and stress development. In the first part the temperature field and a so-called degree of maturity are calculated as a function of time taking account of the heat production by the cement paste [7,8]. Thereafter, the temperature field and the degree of maturity are used as input for the calculation of the stresses in the hardening cement paste.

In this paper a creep model is employed which has the form of a double power law (e.g., also [1]). The power law is developed in a Taylor series to make the model suitable for implementation in numerical programs. Experimental measurements are used to derive parameters for the model [6]. The model and the derived material parameters are subsequently employed in a finite element analysis of a cover element for a breakwater. Attention is also paid to the communication between the heat conduction analysis and the mechanical problem.

2. HEAT PRODUCTION IN HARDENING CEMENT PASTE

The hydratation process can be characterized by different parameters (e.g., [4,7,8]). In this paper Reinhardt [7,8] will be followed, and the degree of maturity $r(t)$ is employed, which is defined as the quotient of the accumulated heat production $Q(t)$ at time t and the total heat production at infinity Q_∞:

$$r(t) = \frac{Q(t)}{Q_\infty} \tag{1}$$

The actual heat production q $[J/m^3 s]$ at time t is calculated using [8]:

$$q = q_{max}\, f(r) = a\, e^{-b/T} f(r) \tag{2}$$

with a and b material constants, T the absolute temperature $[K]$ and $f(r)$ the heat production characteristic. $f(r)$ is a function of the degree of maturity and must be determined experimentally.

Next, an evolution formula for Young's modulus can be developed based upon the known temperature $T(t)$ and the degree of maturity $r(t)$. In this paper, the expression proposed by Reinhardt, Blaauwendraad and Jongedijk [8] is utilized:

$$E(t) = E_0 \int_0^{r(t)} \left\{\frac{T}{273}\right\}^7 \left\{1 - \exp\left[\frac{-\beta\,(t-\tau)}{(T/273)^6}\right]\right\} dr(t). \tag{3}$$

In eq. (3), τ is the time at which the increment dr is added, E_0 is the initial value of Young's modulus, which depends on the amount of cement, and β is a "delay" factor. In the example which will be discussed in a subsequent section, the values E_0=20000N/mm^2 and β=0.075 have been employed.

3. VISCO-ELASTIC MODEL

The creep behavior of young concrete will be described with a visco-elastic model. As point of departure the creep formulation

$$\varepsilon_{ij}(t) = C_{ijkl}\int_0^t \frac{1}{E(\tau)} J(t-\tau)\,\dot{\sigma}_{kl}(\tau)\,d\tau, \tag{4}$$

is used. In eq. (4), $\dot{\sigma}_{kl}(\tau)$ is the the stress rate tensor, $E(\tau)$ is Young's modulus at $t=\tau$, $J(t-\tau)$ is a dimensionless function which describes the creep behavior and the summation convention is employed for repeated subscripts. The compliance term $1/E(\tau)$ is usually combined with $J(t-\tau)$ in a creep function, but this convention is not followed here because of the important role of the time dependent Young's modulus. Furthermore, the fourth-order tensor C_{ijkl} is defined by:

$$C_{ijkl} = \tfrac{1}{2}(1+\nu)[\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}] - \nu\delta_{ij}\delta_{kl} \tag{5}$$

with ν Poisson's ratio, which is assumed to be independent of the degree of maturity.

The function $J(t-\tau)$ has been assumed to have the following form [1,4]:

$$J(t-\tau) = 1 + \alpha\,\tau^{-d}\,(t-\tau)^p \tag{6}$$

with α, p and d material constants. The factor τ^{-d} represents the influence of the time of the application of the load on the creep behavior, while $(t-\tau)^p$ constitutes the load duration.

For $d=0$ and $p=1$ the power law of eq. (6) degenerates to the same mechanical model as a single Maxwell unit. This case as well as other integer values of p are interesting since analytical solutions can then be obtained, which permits verification of the numerical algorithm to be discussed in the next section.

4. NUMERICAL ALGORITHMS

When we define $J_c(t-\tau)=\alpha\tau^{-d}(t-\tau)^p$ and substitute eq. (6), we can rewrite eq. (4) as

$$\varepsilon_{ij}(t) = C_{ijkl}\int_0^t \frac{1}{E(\tau)}(1+J_c(t-\tau))\,\dot{\sigma}_{kl}(\tau)\,d\tau, \tag{7}$$

Differentiating eq. (7), using Leibniz' rule, assuming that $J_c(0)$ vanishes, and rearranging gives:

$$\dot{\sigma}_{ij}(t) = D_{ijkl}E(t)\dot{\varepsilon}_{kl}(t) - \int_0^t J'_c(t-\tau)\,\dot{\sigma}_{ij}(\tau)\,d\tau \tag{8}$$

In eq. (8), $J'_c(t-\tau)$ is the first derivative of J_c with respect to $(t-\tau)$. When assume that $\dot{\sigma}_{ij}(t)\Delta t=\Delta\sigma_{ij}(t)$, with Δt the time step, and define $\Delta\varepsilon_{kl}(t)$ in a similar fashion, we get for time increment n:

$$\Delta\sigma_{ij}(t) = D_{ijkl}\,E(t)\Delta\varepsilon_{kl}(t) - \Delta t\left[\sum_{q=0}^{n-1} J'_c(t-\tau_q)\,\Delta\sigma_{ij}(\tau_q)\right] \tag{9}$$

so that the stress increment $\Delta\sigma_{ij}(t)$ from t to $t+\Delta t$ can be calculated from the strain increment $\Delta\varepsilon_{ij}(t)$ of the current time step and the stress increments $\Delta\sigma_{ij}(\tau_q)$ of all previous time steps. The product of an additional strain increment $\Delta\varepsilon_{ij}(t)$ and the actual Young's modulus $E(t)$ increases the current stress, while the second term in eq. (9) describes the relaxation process. The disadvantages of algorithm (9) are the fact that the entire load history of each material point needs to be stored and that the computational times explode as the period increases which has to be analyzed. Even for modern computer equipment this is an impossible requirement if realistic engineering structures have to be analyzed.

To derive an algorithm which is more suitable for use in numerical programs, $J'_c(t-\tau)$ is developed in a Taylor series at time $t=t_0$ [5]. Alternatively, $J'_c(t-\tau)$ can be developed in a Dirichlet series (e.g., [1,3]). Indeed, long time periods can be described better using a Dirichlet

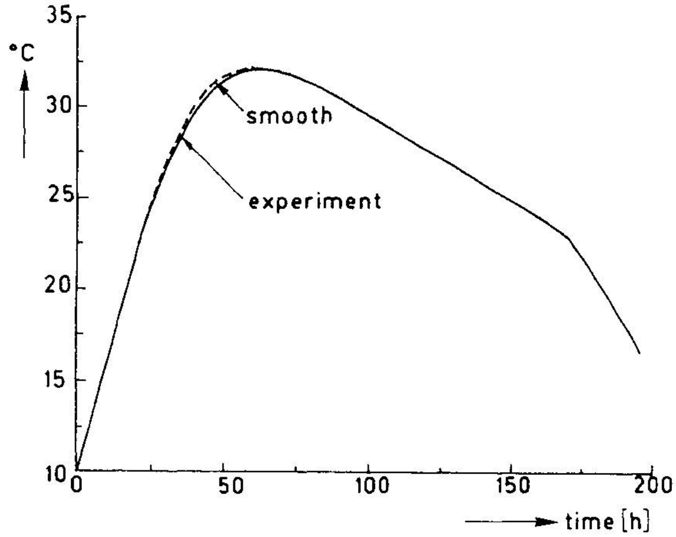

Fig. 1. Temperature functions from [6] and smooth input.

series, but for short time periods which are of interest in this study, a Taylor series is probably quite accurate. Developing $J'_c(t-\tau)$ at $t=t_0$ gives

$$J'_c(t-\tau) = J'_c(t_0) + J''_c(t_0)(t-\tau-t_0) + \frac{1}{2!}J'''_c(t_0)(t-\tau-t_0)^2 + \frac{1}{3!}J_c^{IV}(t_0)(t-\tau-t_0)^3 + \cdots \tag{10}$$

When we substitute $J_c(t-\tau)=\alpha\tau^{-d}(t-\tau)^p$, eq. (10) can be rewritten as

$$J'_c(t-\tau) = \alpha\, p\ \tau^{-d}\left[a_0 + a_1(t-\tau-t_0) + a_2(t-\tau-t_0)^2 + a_3(t-\tau-t_0)^3 + \cdots\right] \tag{11}$$

a_0, a_1 etc. are functions of the power p and of t_0. Collecting terms with the same power of τ yields:

$$J'_c(t-\tau) = \alpha\, p\ \tau^{-d}\ \sum_{r=0}^{m} h_r(t,t_0)\tau^r \tag{12}$$

with m the number of series used in the Taylor expansion. When $m=3$, $h_0(t,t_0)$ is defined by

$$h_0(t,t_0) = a_0 + a_1(t-t_0) + a_2(t-t_0)^2 + a_3(t-t_0)^3 \tag{13}$$

and $h_1(t,t_0)$, $h_2(t,t_0)$ are defined in a similar way. In principle, there is no restriction on the value of m, but significant gains in accuracy are usually not made by using more than five terms. Eq. (12) can be substituted in eq. (9), whereupon we get after rearranging:

$$\Delta\sigma_{ij}(t) = D_{ijkl}\ E(t)\Delta\varepsilon_{kl}(t) - \Delta t\sum_{r=0}^{m}\alpha p h_r(t,t_0)\left[\sum_{q=0}^{n-1}\tau_q^{-d}\ \tau_q^r\ \Delta\sigma_{ij}(\tau_q)\right] \tag{14}$$

Expression (14) has the desired properties, since the actual time t no longer appears in the sum over $n-1$ time steps. Every time a load step is executed the sum $\tau_q^{-d}\ \tau_q^r\Delta\sigma_{ij}(\tau_q)$ can be updated. In this way only m values need to be stored for each material point, and the calculation time will not grow with an increasing number of time steps.

As stated before, an analytical solution can be obtained for $d=0$ and for integer values of p. It appeared [2] that for small time steps the numerical algorithm yielded a solution which could hardly be distinguished from the analytical solution. An analytical solution could also be constructed for the case of $d=0$ and $p=\frac{1}{2}$. However, the agreement between the numerical solution and the analytical solution, which involves an error-function [2], was not as good as in the cases with the integer values for p, especially in the first hours of the simulation. This is not surprising, since a Taylor series already consists of polynomials. The error in the first hours can be made smaller by taking a smaller value for t_0, but the consequence of doing so is that the Taylor series cannot capture the creep function accurately at later stages of the simulation. In the examples shown in this paper t_0 has been elected in the middle of the simulation period [2].

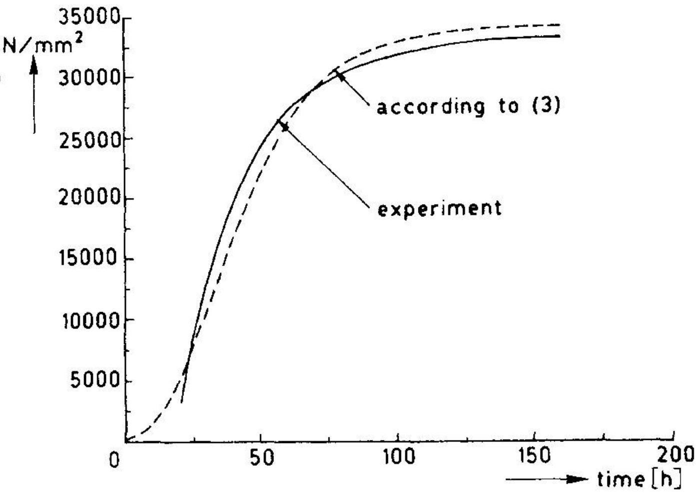

Fig. 2. Development of the stiffness $E(t)$ according to [6] and eq. (3).

5. PARAMETER DETERMINATION

The application of the power law model to the prediction of the creep and relaxation behavior of hardening cement paste, needs realistic values of the parameters α, d and p. A direct use of values which are found in the literature is rather difficult, for similar parameters are often used in a slightly different form [1,4]. In this study the values for d range from 0.30 to 0.40, while the range for p is from 0.25 to 0.35. The specific values depend on the hardening conditions (humidity, temperature etc.) and the amount and type of cement. The shrinkage caused by drying of the hardening cement paste is not taken into account in this calculations. Incorporation of this phenomenon would result in different values of α, d and p.

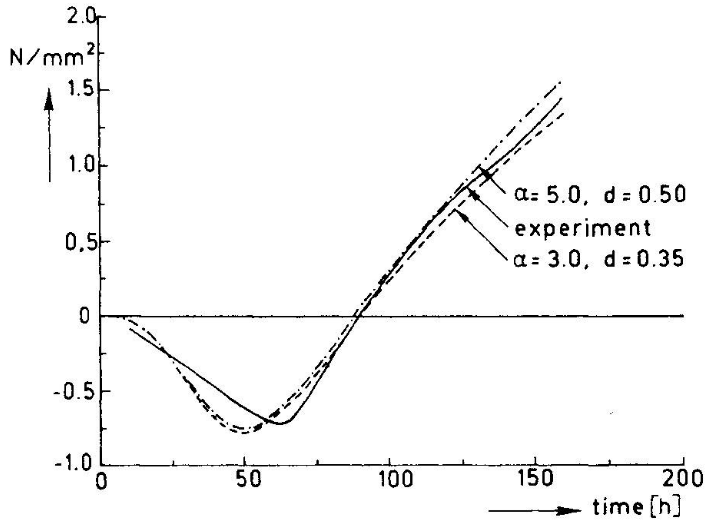

Fig. 3. Calculated stresses with $p = 0.30$, $t_0 = 80$h., $\Delta t = 1.0$h.

Van Heyningen and Boon [6] have measured the development of the temperature, the stiffness and the stresses in concrete cubes in a laboratory environment. In Fig. 1 two temperature functions are plotted as a function of time, with the dashed line a smooth approximation. A smooth approximation of the measured time-temperature curve is used since a slight disturbance in the temperature results in a pronounced variation in the stress. The smooth temperature curve of Fig. 1 is subsequently used to determine Young's modulus with aid of eq. (3) and is compared with experimental data in Fig. 2.

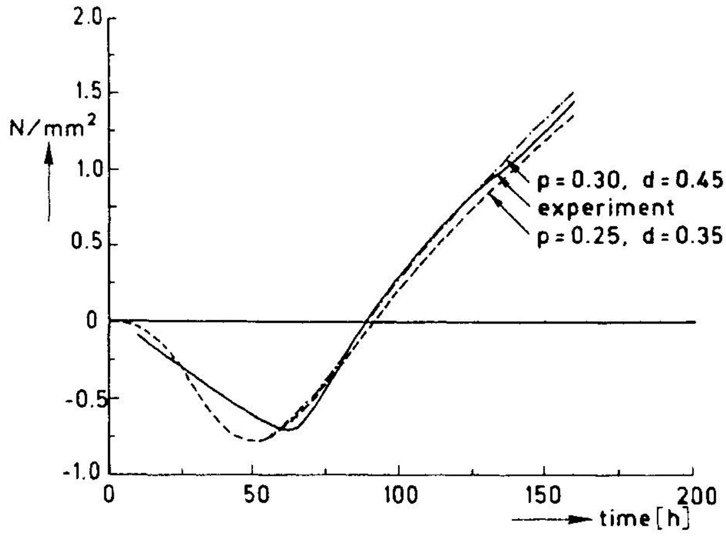

Fig. 4. Calculated stresses with $\alpha = 4.0$, $t_0 = 80$h., $\Delta t = 1.0$h.

The calculation of the stresses is performed on one element with a coefficient of thermal expansion of $11 \times 10^{-6} m/mK$ and a step size $\Delta t = 1$h. is used. Figs. 3 and 4 show the development of the stress in the first 7 days for different combinations of the parameters. Unfortunately, different combinations of α, d and p result in nearly the same stress history. In particular, the influence of the parameters on the peak stress is virtually the same. An increase of α yields a smaller peak stress, but the same effect can be established by increasing p or decreasing d. Moreover, an increase of α or p and a decrease of d all cause the same shift of the peak stress to a later time. To improve the stress simulation in the early hours and to control the time at which the peak stress occurs, it is necessary to extend the power law model with an additional function. When the possibility of crack formation is the primary goal of the analysis, the inaccuracies in the first 2 days are less important, since crack formation usually takes place during the cooling process. It is finally noted that the range of parameter values which is employed here, reasonably compares with values quoted by other investigators [1,4], especially d and p.

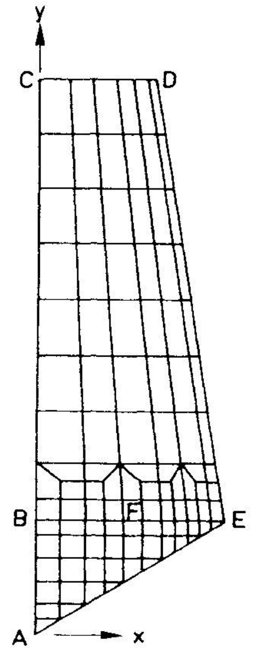

Fig. 5. Element mesh of one leg of a tetrapod.

6. STRESS CALCULATION IN A TETRAPOD

An example is now presented. It involves the calculation of the stresses during the hardening process in a tetrapod, which is used as a cover element for a breakwater. The element mesh is shown in Fig. 5. The boundary conditions of the heat conduction are a prescribed temperature of 15 $^\circ C$ along the edges CD and DE, while along the axis of axi-symmetry AC and edge AE no heat flow is permitted. For sake of simplicity the day-night cycle of the temperature has not been modeled.

The resulting time-temperature and time versus the degree of maturity curves are plotted respectively in Fig. 6 and Fig. 7 for some characteristic points in the tetrapod. The temperature as well as the degree of maturity are subsequently stored in a tabular form, whereby the time increments have been chosen smaller in the first 24 hours than afterwards.

Based upon the temperature and the degree of maturity as a function of time, the evolution of Young's modulus can be calculated using eq. (3). This results in the curves of Fig. 8, which give the development of Young's modulus in the corners of the tetrapod. A typical value for Young's modulus is in the order of 35000 N/mm^2. Fig. 8, however, shows a value of more than 55000 N/mm^2 near point A. This high value is caused by the fact that eq. (3) is rather sensitive for high temperatures, whereby it is noted that the temperature is again a function of the degree of maturity.

The fact that scalar variables (temperature) as well as vectorial quantities (displacements) enter the calculation has some consequences for the degree of interpolation and for the finite element discretization. Use of quadrilateral four-noded elements in a heat conduction analysis for instance results in a bilinear temperature distribution. When the result of the heat conduction analysis is subsequently used in the stress analysis, a linear temperature distribution also results in a linear thermal strain distribution. This strain distribution can only be described by a second-order displacement field, which necessitates the use of a quadratic element for the stress analysis. More general, we need an element for the stress analysis with an interpolation polynomial which is one order higher than the interpolation polynomial used in the heat conduction problem.

A related issue is the choice of the element mesh. The heat conduction problem and the stress analysis usually impose different requirements on the spatial discretization. If different meshes are employed for each analysis, the temperatures and the degrees of maturity which result from the first calculation, must be interpolated to give nodal values in the mesh to be used in the stress analysis. Since the temperatures are primary variables in the heat conduction analysis, this interpolation is straightforward and results in a unique value of the temperature in each node of the mesh for the stress analysis. The calculation of the degree of maturity in the nodes of the new mesh is more complicated, for the degree of maturity is calculated in the integration points during the heat conduction analysis. To obtain a value for the degree of maturity in the nodes, the values in the integration points are extrapolated. This method gives a non-unique value of the degree of maturity in each node. If the same element mesh is used again for the stress analysis, this is not a major problem, since the interpolation to the (possibly new) integration points can be carried out using the old extrapolated nodal values of that particular element. When different meshes are employed in both analyses, it may be more correct to directly interpolate the degree of maturity in the new integration point from the integration points of the heat conduction analysis.

A third issue is the calculation of Young's modulus. This quantity must be known in the integration points of mesh for the stress analysis. To this end the temperature and the degree of maturity must be interpolated from the nodes to the integration points. Thereafter, Young's modulus can be computed as a function of time with aid of eq. (3).

Figs. 9, 10 and 11 show the development of the principal stresses in some points of the cross section BE in the course of time. In these figures σ_1 is the principal stress which is oriented approximately parallel to the x-axis, and σ_2 is the principal stress which is oriented approximately parallel to the y-axis.

7. CONCLUDING REMARKS

The development of a power law model in a Taylor series is an efficient way to simulate the creep behavior of concrete at early ages. A good simulation of the

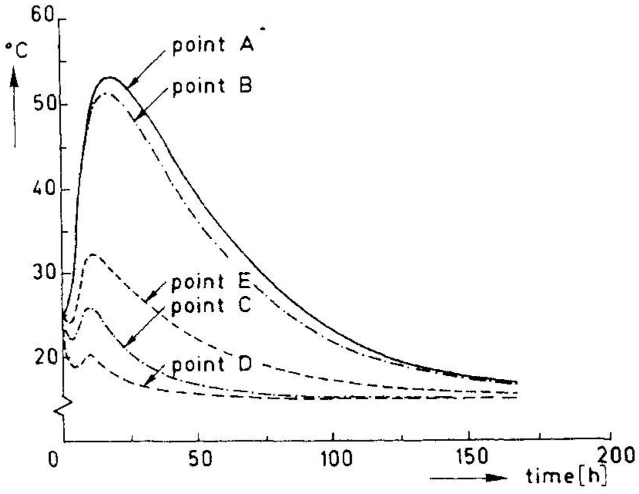

Fig. 6. Temperature as a function of time in the corners of the tetrapod.

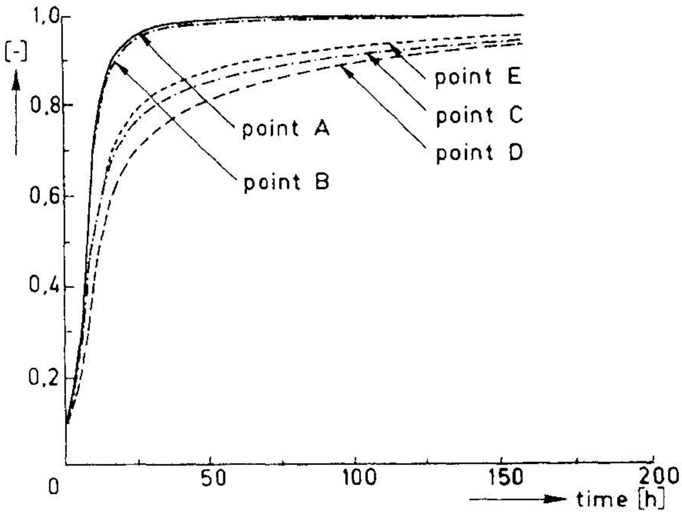

Fig. 7. Development of the maturity $r(t)$ in the corners of the tetrapod.

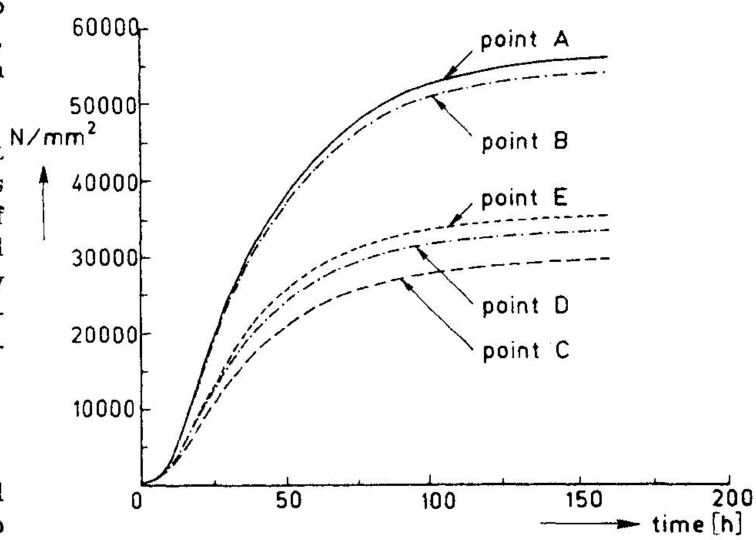

Fig. 8. Development of the stiffness $E(t)$ in the corners of the tetrapod.

stress development in a young concrete member can be made with the power law model after the first two days, while an additional function is needed for a better prediction in the initial phase. A good qualitative agreement between model prediction and experimental data could be achieved for some sets of the parameters α, p and d.

Special attention has to be paid to the communication between the heat conduction analysis and the stress analysis. When both calculations are carried out with the same element mesh, a good compromise must be found to describe flow and peak stresses in a proper way.

In regions with high temperatures the applied empirical model for the determination of Young's modulus results in unrealistic high values. This may be caused by the fact that the employed empirical functional for the determination of Young's modulus is rather sensitive for high temperatures.

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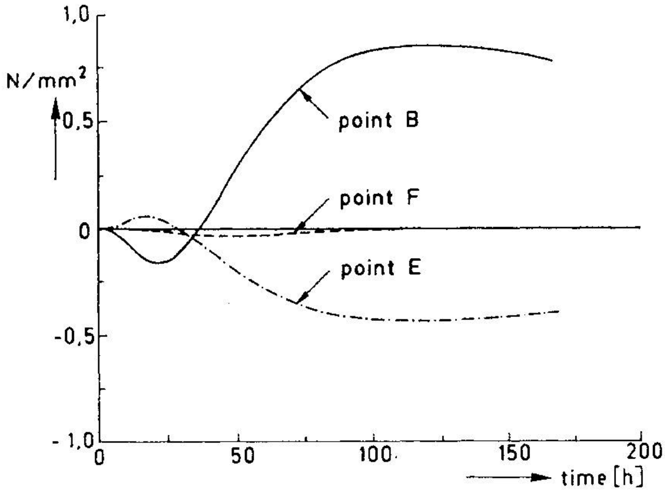

Fig. 9. Stress development σ_1 in some points in cross section BE

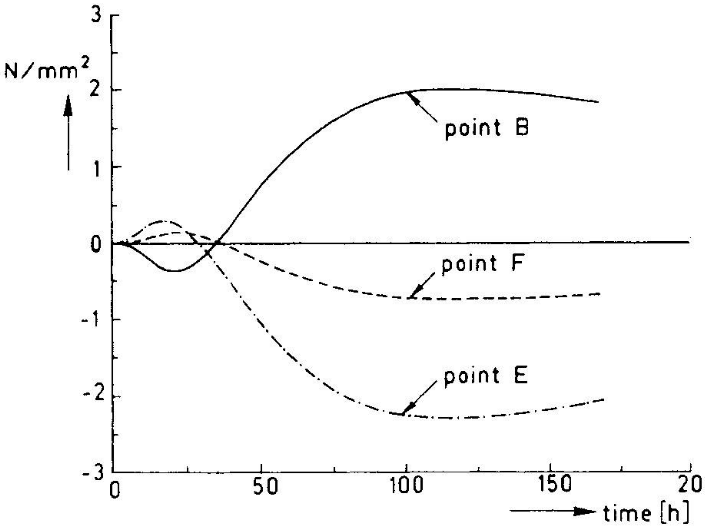

Fig. 10. Stress development σ_2 in some points in cross section BE

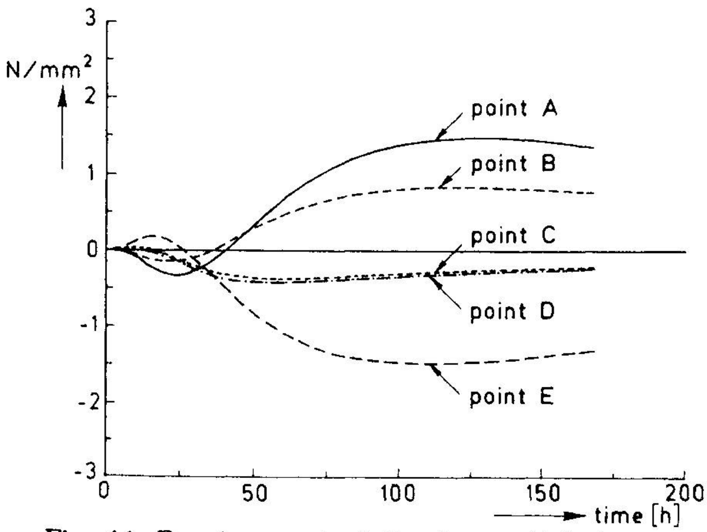

Fig. 11. Development of the tangential stress σ_t near the corners

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