Degree of safety in mixed structural systems

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Degré de sécurité dans les systèmes à structures mixtes

Sicherheitsgrad im Mischbauwerksystem

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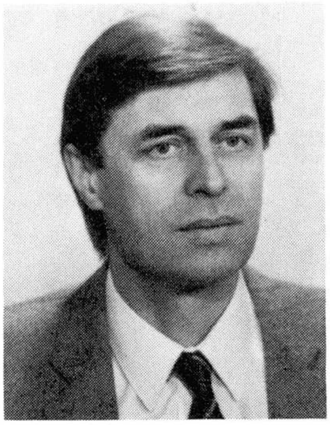

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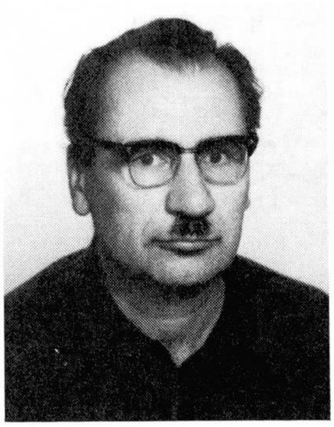

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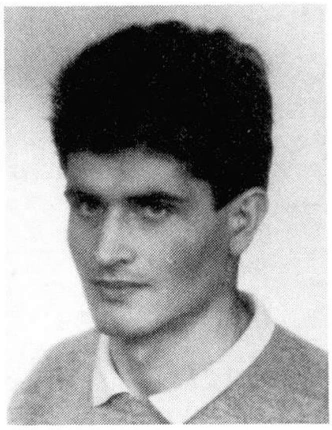

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SUMMARY
A probabilistic analysis of mixed structural elements can be used to equalize the degree of safety of several layers in various limit states. The method has been illustrated by means of a practical example for the manufacturer of these elements.

RÉSUMÉ
L'analyse probabilistique des éléments de structures mixtes peut être utilisée pour égaliser le degré de sécurité de plusieurs couches dans les divers états limites. La méthode est présentée à l'aide d'un exemple pratique par les fabricants de ces éléments.

ZUSAMMENFASSUNG
Mit der probabilistischen Analyse von Verbundkonstruktionselementen ist es möglich, die Sicherheitsgrade verschiedener Schichten in den Grenzzuständen auszugleichen. Die Methode wird an einem praktischen Beispiel dargestellt.

1. INTRODUCTION

The calculation of internal forces in elements composed of several layers of different materials is based on the theory elaborated in [1]. The appearance of new materials has greatly increased the choice of mixed layered elements used in civil engineering.

In this, the materials in the various layers have very different mechanical properties and modes of bearing capacity loss. This points to the fact that safety verification based on present methods cannot include the problems of equalizing the safety of individual layers.

Since the standards are inadequate, it is not possible to prescribe the bearing capacity of equalized safety degree according to the chosen safety indices . As a result, the composite elements are not safe enough or economical.

The problem of practical application is analyses for the manufacturer and safety equalization of different layers in composite elements, bringing the damage or failure risk in individual layers to the levels prescribed by the society and required by the regulations.

2. LIMIT STATES EQUATIONS

The differential equation of the deflection line caused by flexural and shear deformations in a multi-layered element loaded with a transversal action is presented in [1]. The solution consists of the homogeneous part w_h and the particular part w_p:

$$w_h = C_1 + C_2 \cdot x + C_3 \cdot e\exp\left[(x-l)\sqrt{\frac{B \cdot A}{B_s \cdot B_d}}\right] + C_4 \cdot e\exp\left[-x\sqrt{\frac{B \cdot A}{B_s \cdot B_d}}\right] + M_L\left(\frac{x^3}{6 \cdot B \cdot l} - \frac{x^2}{2}\right) - M_R\,\frac{x^3}{6 \cdot B \cdot l} \quad (1)$$

$$w_p = \frac{q}{2 \cdot B}\left[\frac{x^4}{12} - \frac{x^3 \cdot l}{6} + \frac{x^2\, B_s}{A}\left(\frac{B_d}{B} - 1\right)\right] - \frac{x^2 \cdot B_s \cdot \vartheta}{2 \cdot B} \quad (2)$$

The symbols are taken from [1]. Equations (1) and (2) contain an indefinite vector $k^T\{C_1, C_2, C_3, C_4, M_L, M_R\}$ for which the equations with six known border conditions have to be determined. If the element consisting of n continuous fields is observed, where the field is a part of an element with an uninterrupted function for continuous transversal loading, $6 \cdot n$ border conditions or rather continuity conditions for internal characteristic points have to be defined. This yields a system consisting of $6 \cdot n$ linear equations accompanied by vector k with $6 \cdot n$ unknown constants. The system in the matrix form is:

$$\{w\} = [z] \cdot \{k\} + \{s\} \quad (3)$$

where: $\{w\}$ - displacement vector , $[z]$ - coefficients matrix,
$\{k\}$ - unknown coefficients vector , $\{s\}$ - load vector.

Further analyses are made if the statical system of the element is a field with two supports. The cross-section of the element consists of three mixed layers. The element is used for the facade, and its resistance to wind was tested.

From the safety aspect, i.e. the probability of bearing capacity failure, there is a margin between the safety and non-safety zone in an n-dimensional vector space, and this margin is expressed with a limit state equation:

$$Z = G\ (X,K) = 0 \qquad (4)$$

where: Z - safety margin, G - bearing capacity value function, X - basic variables vector, K - deterministic parameters vector.

In an element mixed of three layers, four limit state equation can be written:

1. Ultimate limit state of the compressive face:

$$Z_1 = X_6 \sqrt[3]{K_3 \cdot X_4 \cdot X_5} - \frac{X_8 \cdot K_1^2}{8 \cdot X_1 \cdot X_2} = 0 \qquad (5)$$

2. Ultimate limit state of the tensile face:

$$Z_2 = X_3 - \frac{X_8 \cdot K_1^2}{8 \cdot X_1 \cdot X_2} = 0 \qquad (6)$$

3. Ultimate limit state of the core:

$$Z_3 = X_7 - \frac{X_8 \cdot K_1}{2 \cdot X_2 K_2} = 0 \qquad (7)$$

4. Serviceability limit state of the element:

$$Z_4 = \frac{K_1}{K_6} - \frac{5 \cdot X_8 \cdot K_1^4}{192 \cdot K_3 \cdot X_1 \cdot X_2^2} - \frac{X_8 \cdot K_1^2}{8 \cdot X_5 \cdot X_2 \cdot K_2} - \frac{K_4 \cdot K_5 \cdot K_1^2}{8 \cdot X_2} = 0 \qquad (8)$$

The symbols and meanings of the basic variables and of the deterministic parameters are shown in Table 1.

3. TEST SPECIMENS AND STATISTICAL DATA

Statistical data of the mechanical properties of the layers and of geometrical characteristics of the element are obtained by tests and measurements on random samples from the manufacture, and the result can be considered to represent the real situation in the manufacture. The results are shown by histograms in Fig.1. Statistical values of the basic variables and deterministic parameter values are comprehensively presented in Table 1.

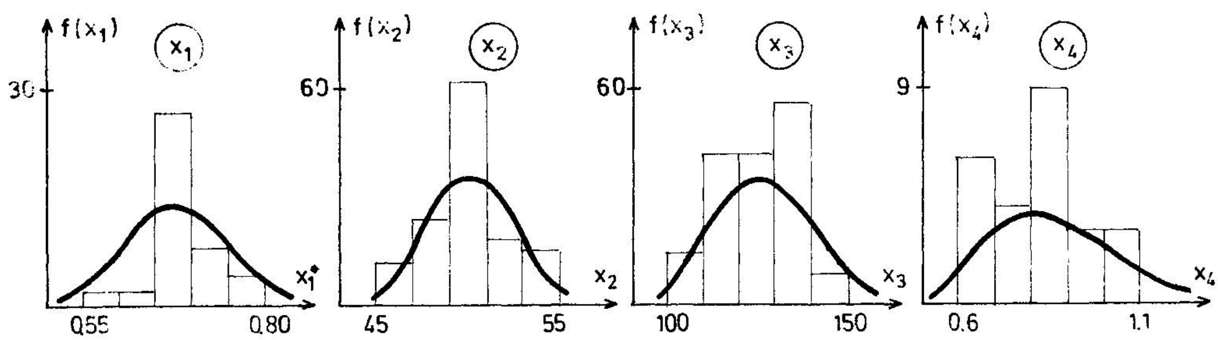


Fig.1a Histograms of basic variables X_1, X_2, X_3, X_4

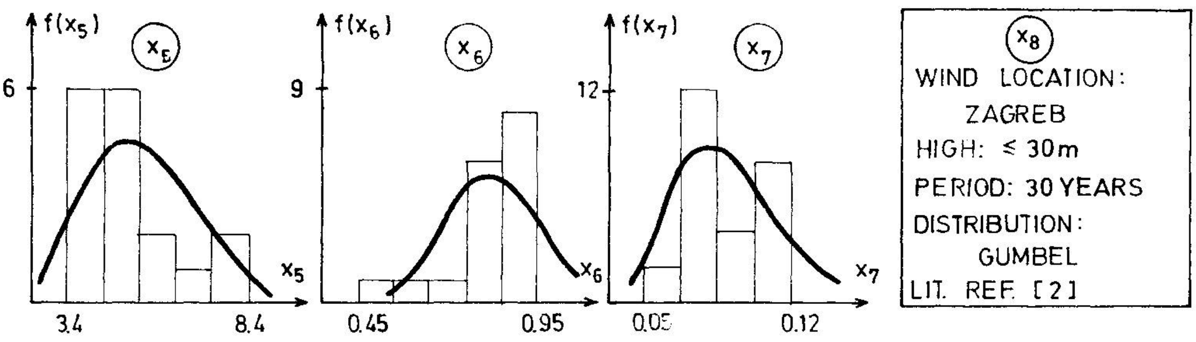


Fig.1b Histograms of basic variables X_5, X_6, X_7, X_8

BASIC VARIABLES $[\vec{X}]$				
VARIABLES	MEAN VALUES	C.O.V.	DISTRIBUTION	
X_1	720 mm^2	0.07	NORMAL	METAL FACE AREA ($X_1 = X_1^{\bullet} \cdot K_2^{\bullet}$)
X_2	51 mm	0.03	NORMAL	CENTROID DISTANCE THE FACES
X_3	126 N/mm^2	0.08	NORMAL	YIELD STRENGTH
X_4	8.1 N/mm^2	0.20	LOGNORMAL	ELASTICITY MODULUS OF THE CORE
X_5	5.0 N/mm^2	0.24	LOGNORMAL	SHEAR MODULUS OF THE CORE
X_6	0.812	0.15	LOGNORMAL	FACE BUCKLING COEFFICIENT
X_7	0.085 N/mm^2	0.24	LOGNORMAL	CORE SHEAR STRENGTH
X_8	850 N/m^2	0.28	GUMBEL	UNIFORMLY DISTRIBUTED LOAD

DETERMINISTIC PARAMETERS $[\vec{K}]$	
$K_1 = l$ SPAN OF ELEMENT	$K_2 = b$ WIDTH OF ELEMENT
$K_3 = 70\,000\ N/mm^2$ FACE MODULUS OF ELASTICITY	$K_4 = \alpha_T$ TEMP. COEFFICIENT OF FACES
$K_5 = \Delta T$ TEMP. DIFFERENCE OF FACES	$K_6 = k$ DEFLECTION LIMIT

Table 1 Data for limit states equations

The value of k^* presents the element face length.

4. EQUALIZATION OF SAFETY DEGREE

If statistical values of the basic variables and deterministic parameters from Table 1 are inserted in the limit states equations, safety indices for ultimate limit states and serviceability limit state for various displacement limits are obtained, as shown in Fig.4 and Fig.5.
If the required design value, i.e. safety index for the ultimate limit state $\beta_f = 4.2$ and for the serviceability limit state $\beta_f = 2.0$, the safety equalization is achieved by looking for adequate spans and displacement limits satisfing the required criteria, according to Fig. 2 and 3:
a/ for the ultimate limit state :

$$\beta_1, \beta_2, \beta_3 \Rightarrow \beta_{min} \geq \beta_f = 4.2 \qquad (9)$$

b/ for the serviceability limit state :

$$\beta_4 \geq \beta_f = 2.0 \quad ; \quad \beta_4 = f(K_6) \qquad (10)$$

For example, the chosen span $l = 2.0$ m, $\beta_{min} = \beta_3 = 4.33 > \beta_f = 4.2$ and the permissible displacement limit 1/150, yield $\beta_4 = 3.61 > \beta_f = 2.0$. Besides safety equalization to the safety index design values, the remaining two β_i in expression (9) can be equalized to the minimum one. This can be done by selecting the materials of lower qualities, by decreasing the cross-section of the layers, etc. Sensibility coefficients α_i show immediately which changes of the basic variables will yield effective results.

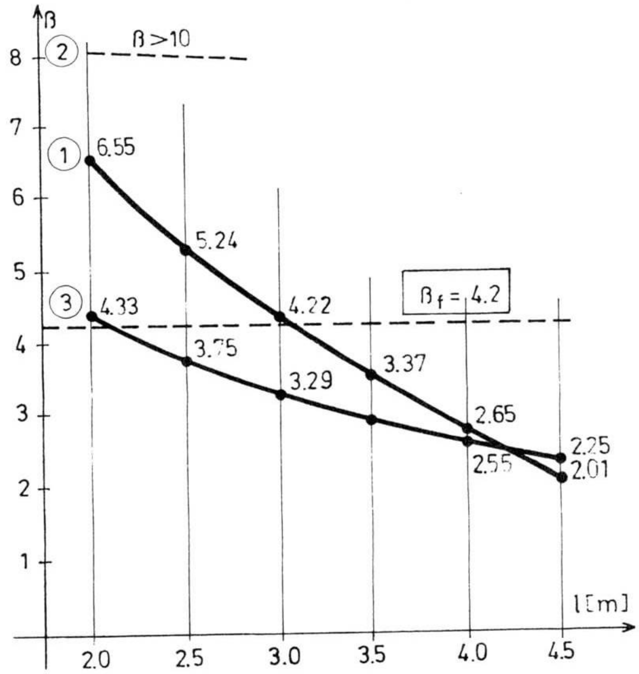


Fig.2 Safety index for ultimate limit states

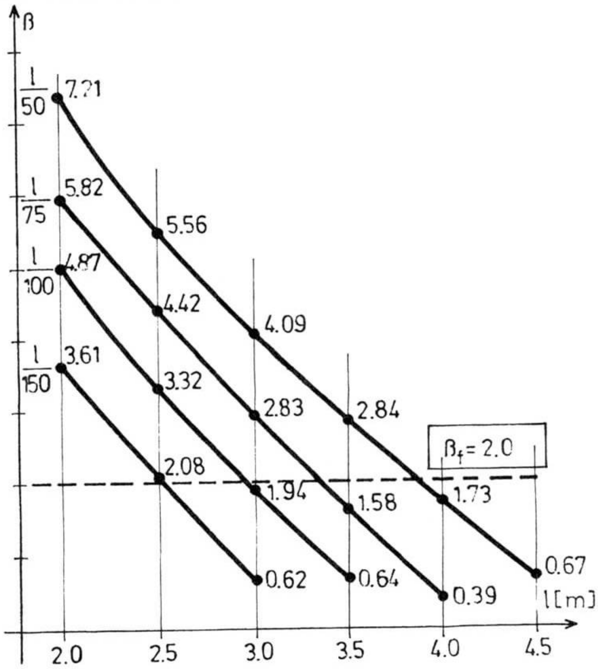


Fig.3 Safety index for serviceability limit state

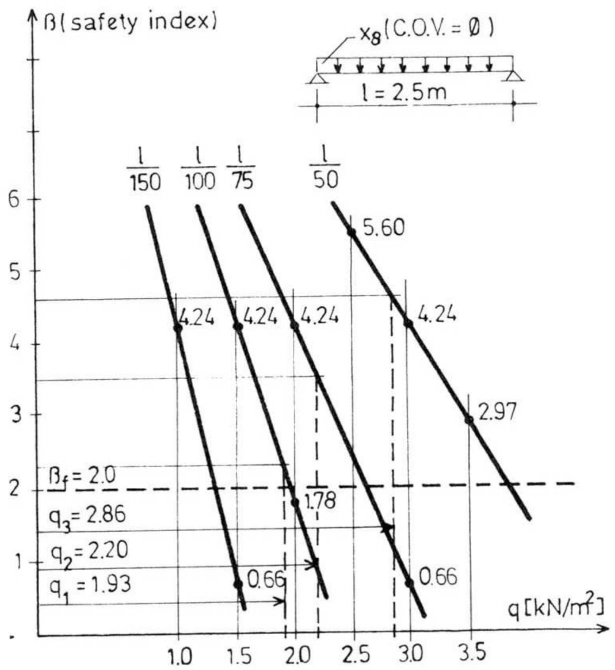
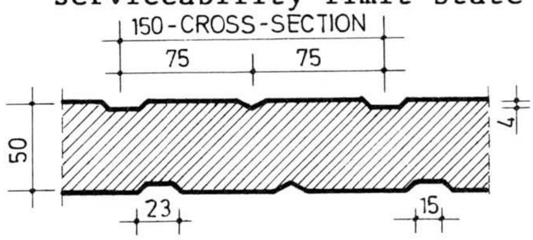


Fig.4 Serviceability limit state for displacement limit



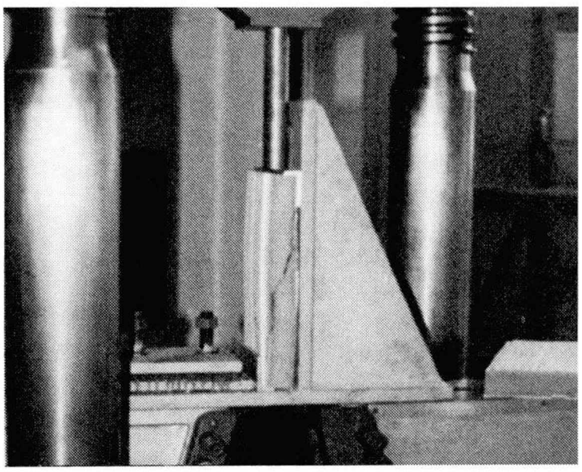


Fig.5 Laboratory testing for shear

Tests on three elements with a span of 2.5 m have shown that, at failure, their loads were q_1, q_2 and q_3. They satisfied the displacement limits 1/110, 1/90 and 1/70 in relation to $\beta_f = 2.0$ (Fig. 4). Fig. 5 shows laboratory testing of shear characteristic of the specimen.

5. DISCUSSION

The results obtained by laboratory testing and by numerical analyses can be discussed regarding the following statements:

The safety indices for tensile face β_2 obtained from the limit state equation (6) are very high as expected.

Safety equalization of individual faces is done in two steps. The first step is with regard to the design values β_f and the second is by decreasing the required basic variable values.
Note: when equalizing the safety degree for all the three layers to the same safety index for the ultimate limit state, we should keep in mind that the probabilities of bearing capacity failure in all the layers should be added, and the common safety index could thus be below the required one. In this case, however, this is only theoretically significant because in only three composite materials the probability of failure does not essentially change.

If the bearing capacity is to be increased to a 4 m span considering the required β_f design value, a core with greater shear strength should be provided, and a span above 3 m should have greater face buckling values (Fig. 2).

An issue still open is adopting the design values β_f. Decision should also be made whether a definite safety index should be required for each of the four limit states, or the ultimate limit state and serviceability limit state should be distinguished as usual.

We consider that the control of composite elements during manufacture to ensure the guaranteed safety can be done by controlling small specimens with a statistical processing of the basic variables data.

6. CONCLUSIONS

This paper presents a probabilistic approach to the safety of composite elements. Since the safety verification of these elements is not yet definitely codified, it can still be discussed, but the results should serve as guidelines for the manufacturer. Equalization of safety is proposed for individual layers as well as for design values, based on probabilistic approach where the probability of failure is expressed with the safety index.

REFERENCES

1. STAMM K., WITTE H., Sandwichkonstruktionen. Springer Verlag, Wien 1974. 337 p.
2. MILCIC V., PAUSE Z., TRUPCEVIC D., A probabilistic approach to defining a wind load, Gradevinar, Vol. 37, No.3.,1985, pp 101-107.