Composite beams with elastic steel-concrete connection

Composite Beams with Elastic Steel-Concrete Connection

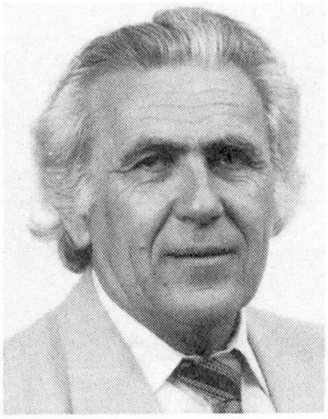
Poutres mixtes avec liaison élastique acier-béton

Verbundträger mit elastischer Stahl-Beton-Verbindung

Roman FEDA
Assist. Prof.
Czech Technical University
Prague, Czechoslovakia

Roman Feda, born 1964, received his Civil Eng. degree at the Czech Technical University in Prague in 1987. Now is Assistant Professor, Department for Steel Structures and bridges, CTU Prague.

Jiří PECHAR
Prof.
Czech Technical University
Prague, Czechoslovakia



Jiří Pechar, born 1930, Professor of steel structures, since 1986 Director of Bulding Research Institute in Prague.

SUMMARY
The paper deals with strength and rigidity of stud shear connectors determined from push-out tests. The tests results are compared with results from other papers. Finite elements method solution is used for composite beam with partial interaction. Basic equations and a functional of the problem are presented. Calculation of the stiffnesss matrix, normal stresses and displacements are shown.

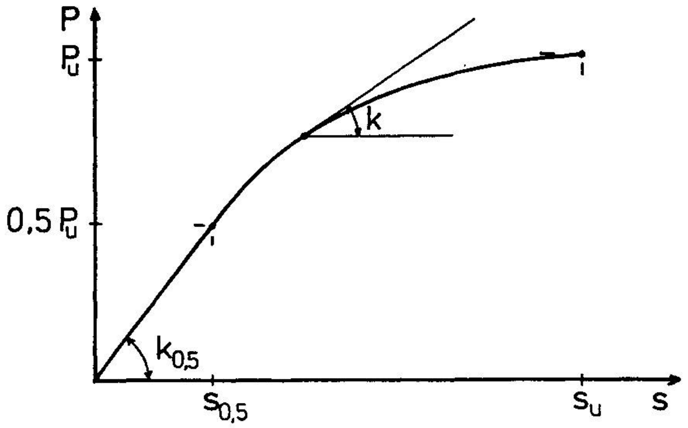
RÉSUMÉ
L'étude traite des caractéristiques de déformation des connecteurs à cisaillement résultant d'essais à la presse. Ces résultats sont comparés avec ceux acquis à l'étranger. La méthode des éléments finis est appliquée à l'analyse ce la poutre mixte acier-béton avec liaison souple. On y mentionne des équations fondamentales; on y présente le problème fonctionnel et le calcul de la matrice de rigidité de l'élément ainsi que celui des contraintes et des déformations normales.

ZUSAMMENFASSUNG
Der Aufsatz berichtet über die experimentelle Untersuchung des Kopfbolzendübelverbundes. Die resultierende Scherversuchswerte werden mit anderen Versuchsreihen verglichen. Für die Analyse des Verbundträgers mit nachgiebigem Verbund wurde die Finitelementmethode angewandt und im Aufsatz beschrieben.

1. SHEAR CONNECTORS

1.1 Basic characteristics

The basic characteristics of shear connectors are usually determined by push-out tests (see fig. 1). The values of P_u, $k_{0,5}$, k, s_u are necessary for the analysis of a composite beams.



The maximum force P_u is the dominating parameter for the fixing of the ultimate bearing capacity of shear connectors. The additional parameters $k_{0,5}$ and stiffness k shall be used for the analysis of beams with elastic steel-concrete connection.

The ductility s shows the stud´s shear resistance.

Fig. 1 Typical push-out characteristic

1.2 The stud resistance

The stud connectors are for the long time already a predominated type of shear connectors. Their normative strength in Czechoslovakia given in Czechoslovak specifications was many years ago based on the unadequate ultimate strength of bolts steel as well on the lack of experiences with stud connectors. Therefore the further investigations were necessary. Fortunately the technology of bolts fabrication increases the bolts ultimate strength so that the values given in EC 4 [1] are fully justified.

The assessment of several series of push-out tests [2], [3], [4] is given in fig. 2. Different bolts in different tests were unified to the bolt with diameter 18,2 mm and ultimate strength R_u = 460 MPa, using transformation formula (1) [5]

$$P_u = 5 \frac{\pi d^2}{4} R_u^{0,65} \left(\frac{E_c}{E_a} \right)^{0,4} R_c^{0,35} \qquad (1)$$

R_u ...... ultimate strength of steel

d ...... bolt´s diameter

E_c ...... Young modulus of concrete

E_a ...... " " of steel

R_c ...... ultimate strength of concrete

From fig. 2 can be derived, that the measured bolts resistances are in good conformity. Therefore for further analysis the formulas (2) and (3) from |1| were accepted.

$$P_d = 0,56\, R_u \frac{\pi d^2}{4} \qquad (2) \qquad\qquad P_d = 0,21\, d^2 \sqrt{R_c E_c} \qquad (3)$$

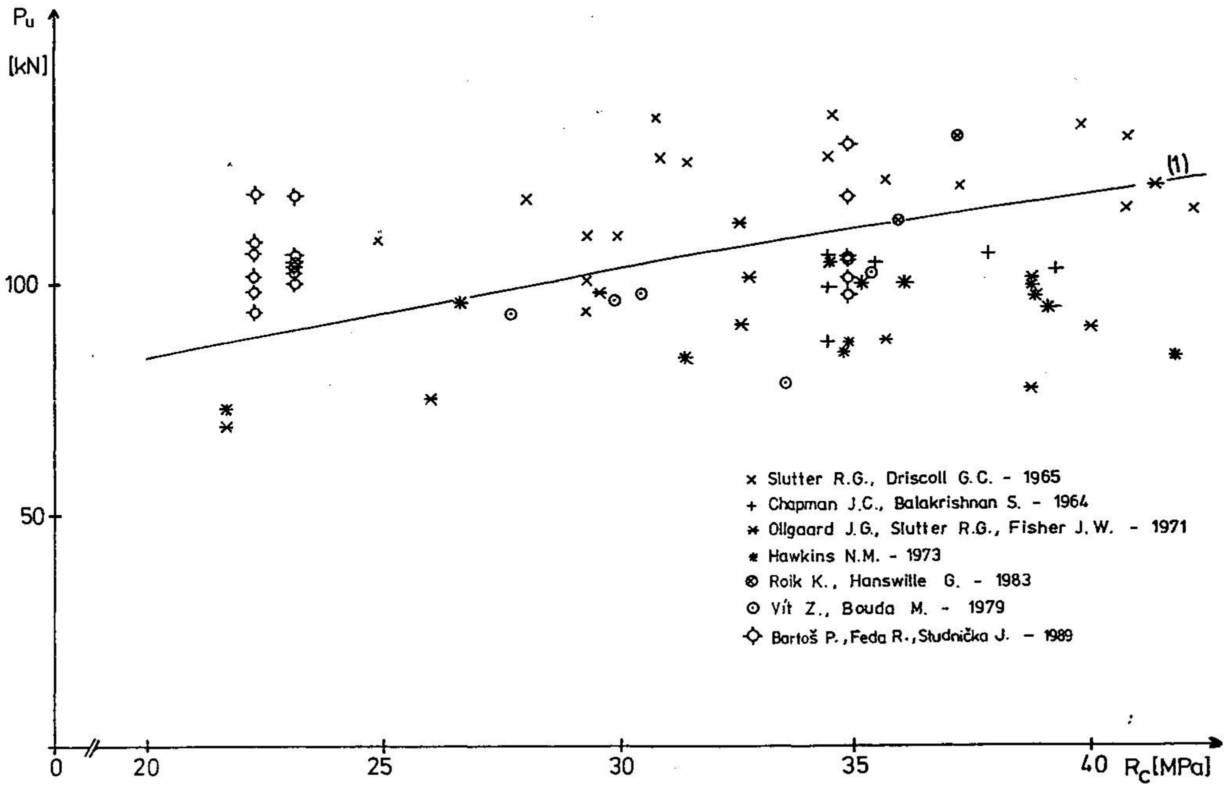


Fig. 2 Strength of stud shear connectors - standard push-out tests.

The comparison of different national standards is given in fig. 3.

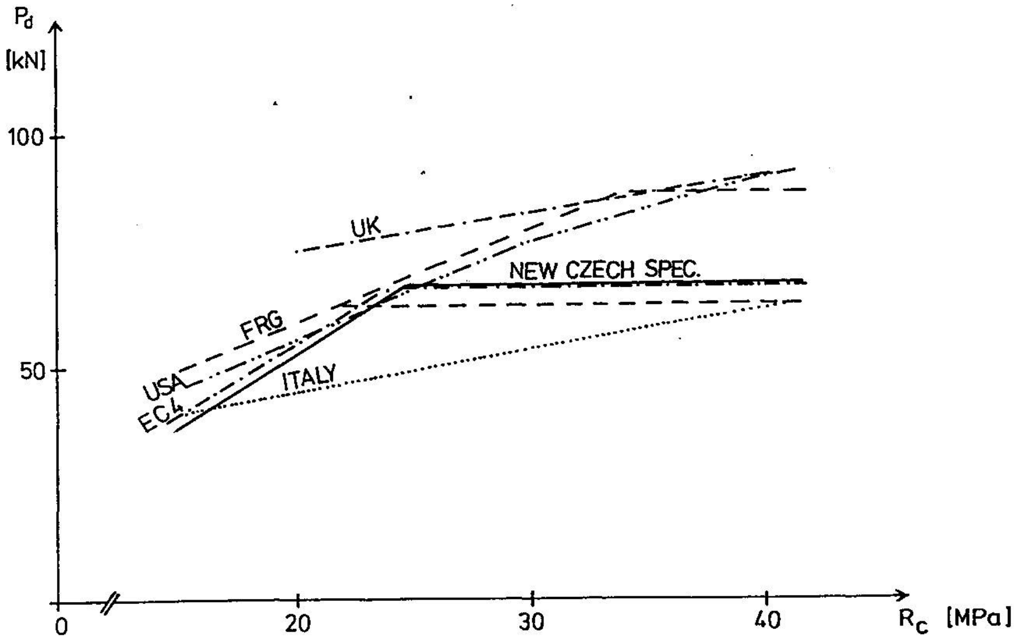


Fig. 3 Comparison of standards.

1.3 Connectors´rigidity

The connectors´rigidity is the most important factor for both linear and nonlinear analysis of composite beams with slip between steel and concrete. Its value can be taken from standard push-out tests, in spite of the fact, that both the stress distribution in the concrete slab and the shear force in the slab-steel connection are different of those in a composite beam. If necessary, the modified push-out-stress respecting the dominated effect of those two mentioned above can be used. The evaluation of rigidity tests made in ČSSR and abroad [6] is in fig. 4. For bolts with diameter 18,2 mm and for concrete $R_c = 20 \div 35$ MPa the bolt´s rigidity varies between 40 - 60 $kNmm^{-1}$.

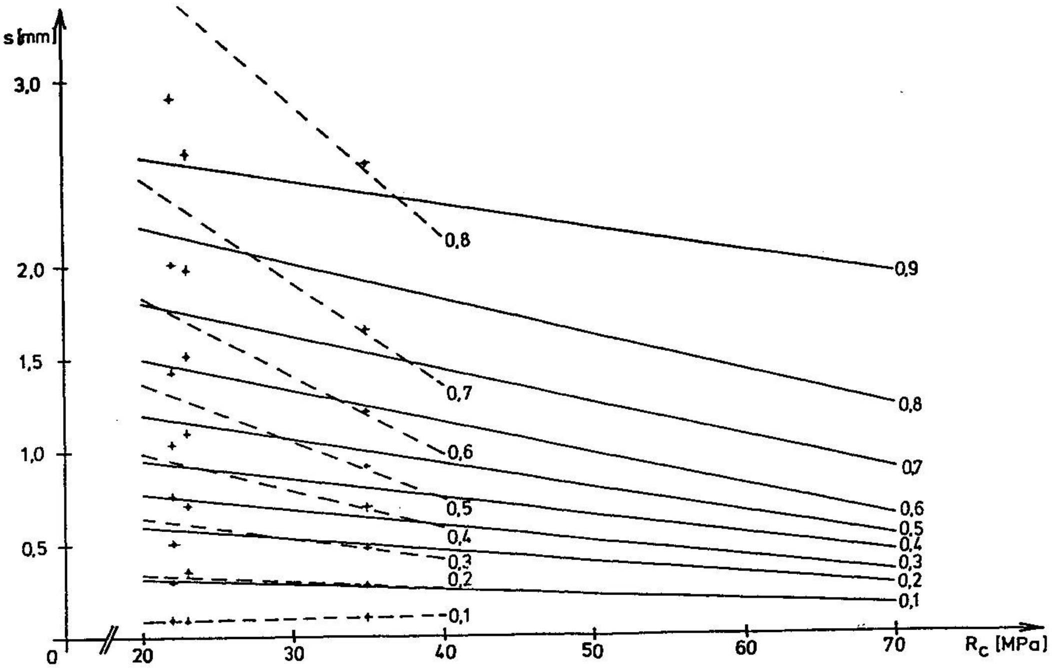


Fig. 4 s - P/P_u - R_c relationships; - [6], - - - Czech tests

1.4 Connectors´ductility

The slip corresponding to the bolts´ defect was measured in many tests. It was found out, that the measured values vary in large range. Anyway the bolts´ductility is almost in all cases bigger than necessary for practical use of composite beams.

2. FINITE ELEMENT ANALYSIS OF COMPOSITE BEAMS WITH ELASTIC STEEL-CONCRETE CONNECTION

2.1 Problem Formulation

Finite Element Method (FEM) was used for the theoretical analysis of a prismatic composite beam loaded by vertical load p and horizontal load q . Composite cross section is described by its geometrical characteristics : cross section area A, moment of inertia I, gravity centre distance of each cross section part from the steel-concrete contact line Z and by physical data : Young modulus E, slip resistance (shear connectors´ rigidity) k_s , uplift resistance k_w . All mentioned characteristics with suffix "a" belong to the steel elements, with suffix "c" to the concrete elements (fig. 5).

In the coordinating system the horizontal axis x is paralel to the longitudinal axis of the beam and the axis z is vertical. Displacements u, w are measured along axis x, z. For solving the functional representing the total potential energy of analysed system it is necessary to derive corresponding both physical and geometrical-deformative equations.

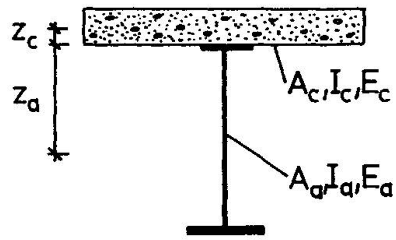


Fig. 5 Composite beam

Geometrical-deformative equations can be reduced to four relations:

steel: $\varepsilon_a = \frac{du_a}{dx} - z\frac{d^2 w_a}{dx^2}$ (4) concrete: $\varepsilon_c = \frac{du_c}{dx} - z\frac{d^2 w_c}{dx^2}$ (5)

and for steel-concrete connections:

slip: $u_{ac} = u_c - z_c\frac{dw_c}{dx} - u_a - z_a\frac{dw_a}{dx}$ (6) uplift: $w_{ac} = w_a - w_c$ (7)

Physical equations:

steel: $\sigma_a = E_a \varepsilon_a$ (8) concrete: $\sigma_c = E_c \varepsilon_c$ (9)

slip: $t_s = k_s u_{ac}$ (10) uplift: $t_u = k_u w_{ac}$ (11)

Using equations (4), (5), (8), (9), (10), (11) the functional is

$$W = \frac{1}{2}E_a\int_{V_a}\varepsilon_a^2 dV_a + \frac{1}{2}E_c\int_{V_c}\varepsilon_c^2 dV_c + k_s\int_L u_{ac}^2 dx + k_u\int_L w_{ac}^2 dx + \int_L p_a w_a dx + \int_L p_c w_c dx + \int_L q_a u_a dx + \int_L q_c u_c dx \quad (12)$$

In the first a second member of eq. 12 the integration through the beam volume V can be made separately along the axis x and z.

Using equations (6), (7) the final form of the functional W and unknown function u_a, u_c, w_a, w_c is

$$W = \int_L \Big[\frac{1}{2}E_a A_a\left(\frac{du_a}{dx}\right)^2 + \frac{1}{2}E_a I_a\left(\frac{d^2 w_a}{dx^2}\right)^2 + \frac{1}{2}E_c A_c\left(\frac{du_c}{dx}\right)^2 + \frac{1}{2}E_c I_c\left(\frac{d^2 w_c}{dx^2}\right)^2 +$$

$$+ k_s\left(u_c - z_c\frac{dw_c}{dx} - u_a - z_a\frac{dw_a}{dx}\right) + k_u\left(w_a - w_c\right)^2\Big]dx + \int_L\left(p_a w_a + p_c w_c + q_a u_a + q_c w_c\right)dx \quad (13)$$

2.2 Stiffness matrix

The stiffness matrix of an element is calculated as usually. The u-function approximation is made by a linear function, the displacement-w approximation is made by a cubical function. After set-up of individual matrices B, D, the stiffness matrix can be calculated by well-known formula

$$K = \int_0^l B^T . D . B dx \qquad (14)$$

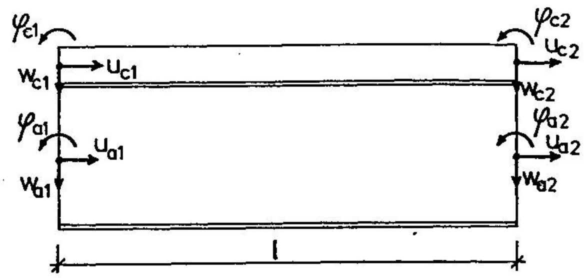


Fig. 6 Finite element of composite beam

2.3 Slip, uplift and normal stresses

Slip and uplift are calculated by equations (6), (7) in which the corresponding derivation of function w has been introduced.

Then

$$u_{ac} = u_a + z_a \varphi_a - u_c + z_c \varphi_c \qquad (15) \qquad\qquad w_{ac} = w_a - w_c \qquad (16)$$

Stress are calculated by eq. (17), (18), (19), (20), where again the corresponding derivation of function u, w are introduced.

$$\sigma_{ch} = \left(\frac{du_c}{dx} + z_c \frac{d^2 w_c}{dx^2} \right) E_c \qquad (17) \qquad\qquad \sigma_{cd} = \left(\frac{du_c}{dx} - z_c \frac{d^2 w_c}{dx^2} \right) E_c \qquad (18)$$

$$\sigma_{ah} = \left(\frac{du_a}{dx} + z_a \frac{d^2 w_a}{dx^2} \right) E_a \qquad (19) \qquad\qquad \sigma_{ad} = \left(\frac{du_a}{dx} - (h_a - z_a) \frac{d^2 w_a}{dx^2} \right) E_a \qquad (20)$$

2.4 Conclusion

It was proved that FEM is suitable for the analysis of composite steel-concrete beams with both rigid and elastic connection. The stiffness matrix of an element K(12, 12) was derived and the calculation of slip, uplift and normal stresses is shown. Different variations of beam supporting (continuous beams) and load applications both on concrete slab as well on steel girder is possible.

REFERENCES

1. EUROCODE No. 4, 1985
2. PHAM L., Design Strength of Stud Shear Connectors. Australian road research, April 1979.
3. OLLGAARD J.G., SLUTTER R.G., FISHER J.W., Shear Strength of Stud Connectors in leight and normal-density concrete. Eng. Journal AISC, April 1971.
4. ROIK K., HANSWILLE G., Beitrag zur Bestimmung der Tragfähigkeit von Kopfbolzendubeln. Der Stahlbau, November 1983.
5. OEHLERS D.J., JOHNSON R.P., The Strength of Stud Shear Connection in Composite Beams. The Structural Engineer, June 1987.
6. OEHLERS D.J., COUGHLAN C.G., The Shear Stiffness of Stud Shear Connections in Composite Beams. Journal CSR, June 1986.