Model for structural concrete members without transverse reinforcement

Model for Structural Concrete Members without Transverse Reinforcement

Modélisation d'un élément en béton dépourvu d'armatures transversales

Modell für Konstruktionsbauteile ohne Stegbewehrung

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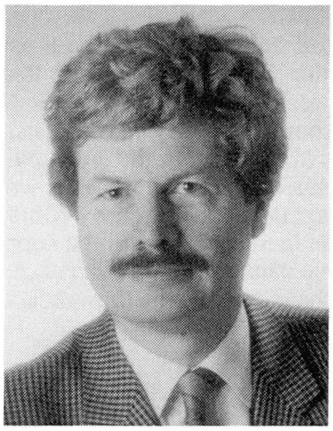

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SUMMARY

The presented mechanical model for structural concrete members without transverse reinforcement explains the structural behaviour from cracking until failure. The loads are transferred by an inclined biaxial tension-compression field between the cracks and this can be visualized by a simple truss. For calculating the failure load the discrete cracks must be examined and the load is mainly transferred in the tension zone of the member by friction of the crack-faces and by the dowel force of the longitudinal reinforcement. An explicit equation for the ultimate shear force is derived following clear mechanical principles. The theory explains the size effect on «shear»-failures as well as the influence of axial compression or prestress and of axial tension or restraints, and it also yields satisfactory results for lightweight concrete members.

RÉSUMÉ

Ce modèle présente le comportement d'une telle structure de la fissuration à la rupture. L'effort tranchant se transmet entre les fissures par une traction et une compression que l'on modélise par un treillis. Le calcul de la résistance doit tenir compte des fissures elles-mêmes et des effets fondamentaux de l'effort tranchant par rapport à la friction des surfaces des fissures ainsi que l'effet de goujon des barres d'armature. L'équation explicitant la résistance ultime au cisaillement est dérivée suivant des principes de mécanique clairs. La théorie explique l'influence de la taille de l'élément en béton sur la rupture par cisaillement. L'influence négative on positive sur la charge ultime des forces de compression ou de traction axiales, ainsi que de la précontrainte peut donc être décrite. L'utilisation de relations constitutives appropriées montre que le modèle examiné donne des résultats satisfaisants dans le cas d'éléments légers en béton.

ZUSAMMENFASSUNG

Das vorgestellte mechanische Modell für Konstruktionsbetonbauteile ohne Stegbewehrung erfasst das Tragverhalten von der Rissbildung bis zum Bruch. Die Querkraft wird durch ein zweiachsiges Zug-Druck-Spannungsfeld zwischen den Rissen übertragen und kann vereinfachend durch ein Fachwerk veranschaulicht werden. Zur Berechnung der Traglast müssen die diskreten Risse und die wesentlichen Querkrafttragwirkungen betrachtet werden: die Rissreibung und die Dübelwirkung der Längsbewehrung. Es wird eine explizite Beziehung für die Bruchquerkraft nach klaren mechanischen Prinzipien abgeleitet. Die Theorie erklärt den Massstabeinfluss bei «Schub»-brüchen, den Einfluss von Längsdruckkräften, der Vorspannung und von Längszugkräften oder Zwangsbehinderungen und sie liefert auch befriedigende Ergebnisse für Leichtbetonbauteile.

1. INTRODUCTION

In present codes members without transverse reinforcement like slabs are still designed with respect to resist shear forces with purely empirically derived formulae. Since such formulae are limited to the past experimental evidence (which often is not clearly described) they do not cover all "shear problems" like that of fully cracked members in silos or foundations with large depths and very low reinforcement ratios. Therefore a clear theory based on a model for the structural behaviour is needed so that practising engineers can solve the future tasks. Such a model for members without transverse reinforcement is also a prerequisite for a consistent design concept for structural concrete since these members form the link between members with transverse reinforcement and unreinforced concrete members. Since the latter obviously require the concrete tensile strength, a consistency in modelling strutural concrete can only be reached if its use is clearly acknowledged.

The only load transfer of a point load on a member without using the concrete tensile strength (apart from the anchorage) are direct struts from the load to the supports tied together by the non-staggered tension chord. This model complies with the lower bound theorem of the theory of plasticity. It yields the full bending capacity at midspan, but this is contradicted by many tests, as e.g. Kani /1/, Leonhardt/ Walther /2/ and Bresler/Scordelis /3/. The reason is obvious from the crack pattern: the assumed compression strut is very flat and crosses the widely opening failure crack. The model is wrongly applied and the theory of plasticity is not valid for this case, as was again recently confirmed by Muttoni /4/. The direct transfer of the total load by inclined struts is only possible if the load is so near to the supports, that the struts are situated over the cracks. This also means, that the shear force cannot only be transferred in the compression zone as pretended e.g. by Kotsovos /5/, but that there is a shear transfer in the cracked tension zone of members without transverse reinforcement.

2. MODEL

2.1 Assumptions

The failure of members without transverse reinforcement is characterized by a single crack propagating into the compression zone and therefore the pattern of the discrete cracks is modelled as shown in Fig. 1 for the well-known test beam with a point load. The B-region with shear force then consists of the solid concrete "teeth" between the cracks as established by the works of Kani /1/, Fenwick /6/ and Taylor /7/. In each tooth the decrease ΔT of the force in the tension chord is equilibrated by an equivalent force in the compression chord and the following shear carrying actions:

- dowel force V_d of longitudinal reinforcement.
- friction along the cracks with the vertical component V_f; the term friction is used instead of "aggregate interlock" since it applies to all concrete types.

Both actions are combined with a shear force component in the compression zone, but this does not indicate an inclined compression chord. All assumptions are further explained in /9/, where the model is derived and justified and the results are compared with tests.

2.2 Equilibrium

The load is transferred by all the shear carrying actions (Fig.1a):

$$V = V_c + V_f + V_d \tag{1}$$

From the equilibrium of moments of a tooth (Fig.1b) as well as of the isolated compression zone alone, the following equation may be derived for the shear force component in the compression chord (notation see Fig.1):

$$V_c = \frac{2}{3} \frac{c}{z} V \tag{2}$$

From that follows that only a small proportion of the total shear force is transferred in the compression chord. Consequently the load is mainly carried in the cracked tension zone. With Eq.(2) the term V_c can be replaced in Eq.(1) and with $z = d - c / 3$ the vertical equilibrium is given by:

$$V = \frac{z}{d-c} \left(V_f + V_d\right) \tag{3}$$

The moment equilibrium of the free-body diagram in Fig.1a yields an equation for the force in the longitudinal steel depending on the moment and the shear carrying actions. With Eq.(3) the force V_f may be replaced and the following equation for the strain in the longitudinal steel be derived:

$$\epsilon_s = \frac{1}{E_s A_s z}\left[V\,(x + \Delta x) + N\,z_c - V_d \frac{z}{ta}\right] \quad \text{with}: \Delta x = \frac{d-c}{ta}\left(1 + \frac{2}{3}\frac{c}{z}\right) \quad ; \; ta = \tan\beta_r \tag{4}$$

Similarily the force and the strain in the compression chord may be determined.

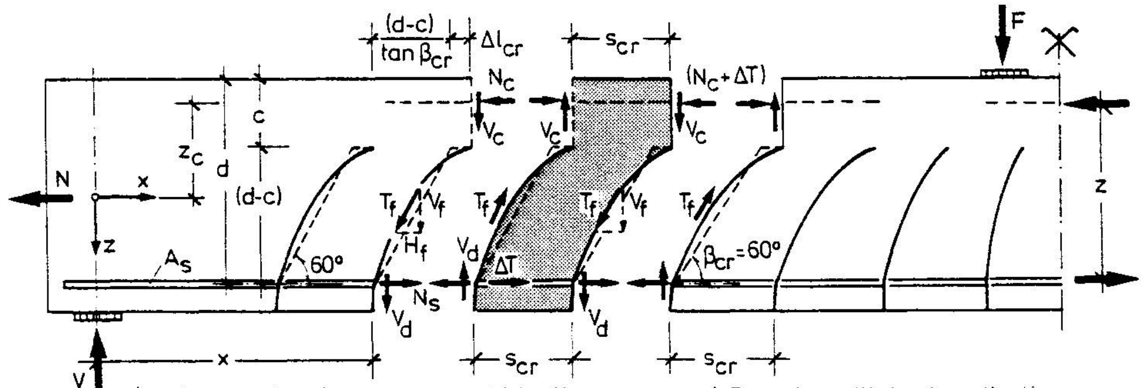

Fig.1: Member with tooth-elements and its forces

2.3 Stress Fields in the Tension Zone

The distribution of the friction stresses along the crack depends on the crack shape as well as on the proportion between the shear carrying actions, whereby both parameters are interrelated with each other. Instead of iteratively determining an "exact" distribution for a defined crack shape, a simple assumption is made for a statically admissable stress field in the tension zone.

The distribution of the friction stresses τ_f is made up by a constant part τ_{f1} (Fig.2a) and a parobolic distribution τ_{f2} (Fig.2b). The latter considers that the dowel action reduces the slip in the lower part of the crack and therefore its value is related to the dowel force. The equilibrium of the shear stresses along the neutral axis (Fig.2b) yields the condition:

$$\tau_{f2} = 2 \cdot v_{n,d} \quad \text{with} \quad v_{n,d} = V_d / b_w (d-c) \tag{5}$$

With the given τ_f-distribution the shear force V_f in Eq.(3) may be expressed by the representative shear stress τ_f at the mid-height of the crack:

$$V = b_w \cdot z \cdot \tau_f + \frac{3}{4} \frac{z}{d-c} V_d \tag{6}$$

The shear force may now be determined for any load stage after cracking if constitutive laws are formulated for the resistances τ_f and V_d as done in /9/.

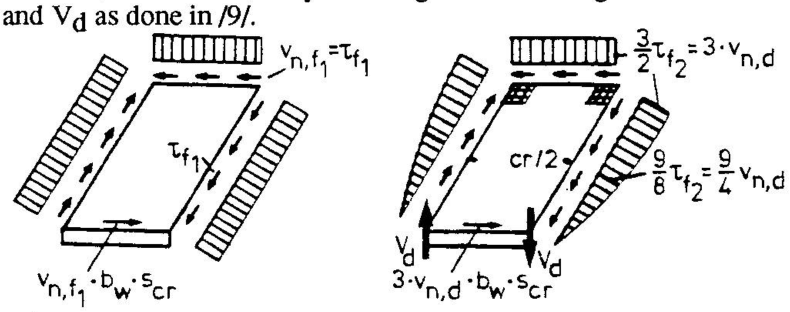

Fig.2: Distribution of friction and shear stresses in tension zone of tooth /9/

However, the model is not complete if not also the stress field in the tooth between the cracks is known so that "the structural action may be visualized" (Breen /8/). Since the stresses involved are small the linear elastic theory may be applied. The constant friction stresses τ_{f1} result in an inclined biaxial compression tension field (Fig.3a). The stress field for the dowel action with the combined friction stresses τ_{f2} is not elementary and therefore a strut-and-tie model is derived (Fig.3b). The concentrated tension tie for the dowel force spreads out and is equilibrated by the biaxial compression-tension field in the concrete resulting from the friction stresses.

2.4 Truss Model

The stress field between the cracks is mainly characterized by the inclined biaxial tension-compression field, which was first proposed in /10/. It is therefore obvious, that the simple truss shown in Fig.4 represents well the main feature of the structural action of members without transverse reinforcement. With that the concept of strut-and-tie models can also be extended to members or parts of members which are unreinforced. It must, however, be pointed out that the failure cannot be assumed by simply limiting the capacity of the tensile struts to the axial tensile strength of the concrete, but that the discrete crack and the shear carrying mechanisms must be looked at.

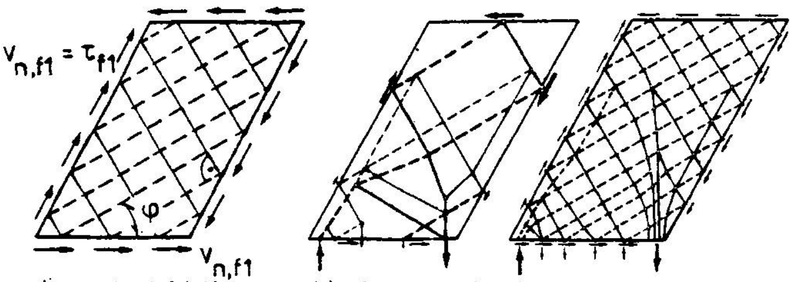

Fig.3: Stress fields between cracks

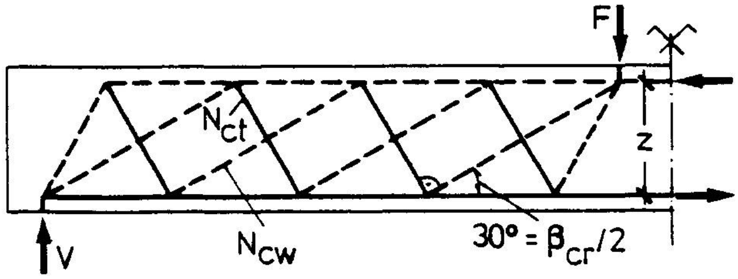

Fig.4: Truss model for members without transverse reinforcement

3. CONSTITUTIVE RELATIONS

For the steel and the concrete in compression bi-linear stress-strain relations can be assumed. The axial tensile strength of the concrete of

$$f_{ct} = 0,246 \cdot f_c^{2/3} \tag{7}$$

is an average value determined from the relative few tests where the tensile strength was actually determined with control specimens. With that the ultimate dowel force of the longitudinal reinforcement can be determined according to the proposal by Baumann /11/ or Vintzeleou and Tassios /12/.

The transfer of forces in cracks depends not only on the tensile strength of the concrete, but also on the roughness as well as on the crack widthΔn and the slipΔs increase as clearly described by Walraven /13/. It was assumed that the limiting friction stress is that transferrable without normal stress on the crack surface:

$$\tau_{fu} = 0,45 \cdot f_{ct} \left(1 - \frac{\Delta n}{\Delta n_u}\right) \quad \text{with} \quad \Delta n_u = 0,9 \text{ mm} \tag{8a}$$

With this value a critical slip of the crack faces is reached:

$$\Delta s_u = 0,336 \ \Delta n + 0,01 \ [\text{mm}] \tag{8b}$$

4. ULTIMATE CAPACITY

After the limiting friction stress τ_{fu} and the critical slip Δs_u acc. to Eq.(8) are reached, the tooth rotates and separates from the compression zone. The crack width at the mid-height of the crack can be derived from the horizontal crack opening Δu due to the elongation of the longitudinal steel and the slip (Fig. 5) giving the failure criterion:

$$\Delta n = 0,71 \cdot \varepsilon_s \cdot s_{cr} \tag{9}$$

With that the friction stress τ_{fu} acc. to Eq.(8a) is known and may be inserted in Eq.(6) for the total shear force. If then the Eq.(4) is used to replace the steel strain the following explicit equation for the ultimate load may be derived in the critical section with x_u=(a-1,5d):

$$V_u = \frac{b_w \ d \ 0,4 f_{ct} - 0,16 \frac{f_{ct}}{f_c} \lambda \frac{z_c}{d} N + V_{du}}{\left[1 + 0,16 \frac{f_{ct}}{f_c} \lambda \left(\frac{a}{d} - 1\right)\right]} \tag{10}$$

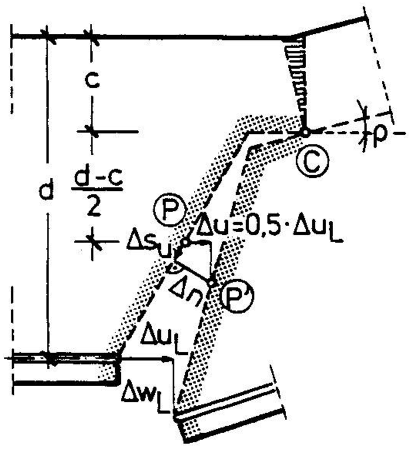

Fig.5: Kinematic consideration for crack width

Thereby the parameter λ is a dimension-free value for the crack width:

$$\lambda = \frac{\varepsilon_{sy} \cdot d}{\omega \cdot \Delta n_u} = \frac{f_c}{E_s \cdot \rho} \cdot \frac{d}{\Delta n_u} \quad \text{with} \quad \omega = \rho \frac{f_y}{f_c} \tag{11}$$

It comprises the well-known influence of the longitudinal reinforcement ratio as well as that of the depth of the member. With that a simple dimensioning diagram for the dimension-free ultimate shear force can be presented (Fig.6).

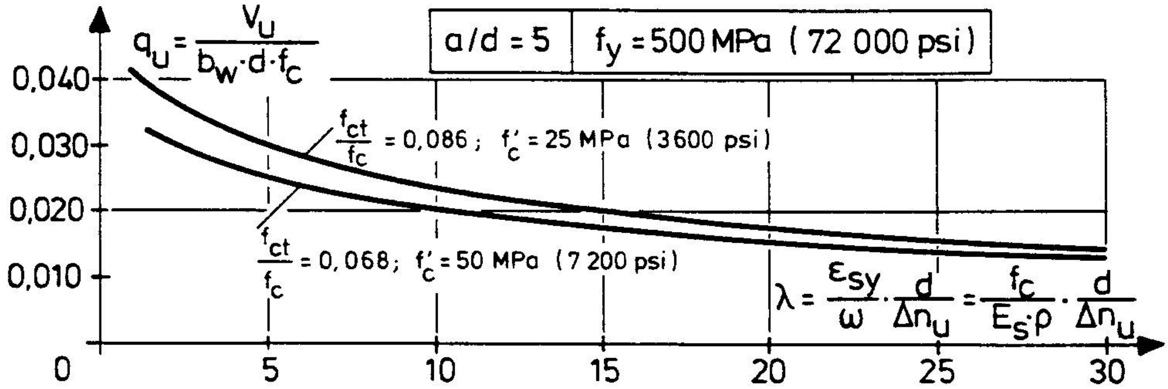

Fig.6: Dimensioning diagram for the ultimate shear force or load

5. DISCUSSION

5.1 Size Effect

The influence of the absolute depth of the member on the ultimate shear force is now obvious: the crack width increases proportionally with the crack height or the depth of the member (Fig.5), and consequently acc. to Eq.(8) the friction stress as the dominant shear carrying capacity is reduced. A further smaller size effect is also due to the dowel action /9/.

The proposed model is conservative if the crack width is very small as for thin members. Then the total length of the crack may be within the fracture zone near the crack-tip and small tensile stresses can be transferred up to a maximum crack width of about 0,15mm (Hillerborg and König /8/). This is shown in Fig.7 where the distributions of these tensile stresses along the crack are plotted for different test beams from Walraven /14/: the crack width increases with increasing depths and the extent of the fracture zone decreases. Since the model already relies on these tensile stresses within the curved part of the crack near its top (see Fig.3 and /9/), only the shaded area in Fig.7 remains for an additional contribution to the capacity of the member. For all members with larger depths the vertical component of the resultant T_{ct} is small and contributes only 5 to 7 % to the ultimate dimension-free shear force; only in case of the beam A1 it increases to 20 % since the crack widths remain very small. However, such members with small depths normally fail in "bending" (like this beam A1), unless they are extremely heavily reinforced.

A further size effect is on the cracking moment and thereby on the steel strain due to the tension stiffening effect of the concrete between cracks. However, this is only worth considering for members with extremely high values λ, like e.g. foundation slabs with 2m depth and very low reinforcing ratios /9/.

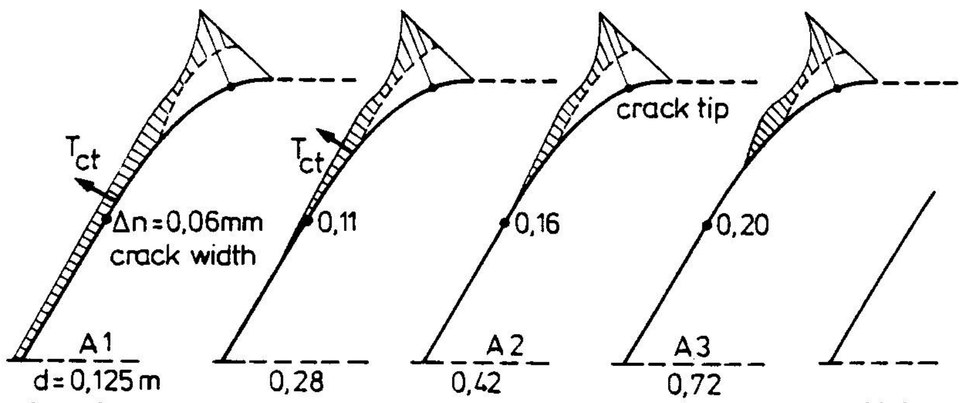

Fig.7: Distribution of tensile stresses in the fracture zone for some test beams with increasing depth

5.2 Axial Forces and Prestress

Axial forces were considered in the equilibrium equation for determining the force in the tension chord leading to Eq.(6), and Fig.8 gives an examples of its influence on the ultimate shear force. Axial compressive forces increase and tensile forces decrease the capacity as qualitatively already known. This is the more pronounced the higher the depth of the member is, since then the crack width is larger and more sensitive towards changes in the steel strain.

The proposed model and theory can also consistently be extended to fully cracked members due to high axial tension (Fig.8) or even restraints /9/. In such a case the shear force can only be transferred by friction along the crack and by the dowel forces of both reinforcement layers. Thereby the amount of the top-reinforcement, as expressed by the parameter η in Fig.8, is of course decisive for the crack width and consequently the possible frictional resistance.

The structural model for a prestressed member is the same as given in Fig.4, apart that the inclinations of the compression struts may be slightly flatter due to the flatter crack inclinations. The influence of prestress on the ultimate load is best explained by looking at the stiffness of the tension chord (which is a prestressed tie), since its strain governs the width of the failure crack. The tension chord of a p.c.-member with the same mechanical reinforcing ratio is different in comparison to a r.c.-member:
- the strain will normally be smaller, but can also be larger if almost the yield strength is reached;
- the dowel force is smaller, because less steel area is needed and smaller diameters are used.

This is reflected in the comparison of ultimate capacities in Fig.9. The p.c.-member exhibits larger failure loads for large depths than the corresponding r.c.-member, but for small depths there is almost no difference.

5.3 Lightweight Concrete

The proposed model is also valid for lightweight concrete (LC) and only the constitutive relations must be adapted. The concrete tensile strength is lower than for normal concrete and a reduction to 66 % can be assumed /9/. The friction capacity acc. to Eq.(8a) is also reduced due to the lower tensile strength, but furthermore higher values for the critical slip occur than given in Eq.(8b):

$$\Delta s_u = 0{,}38 \cdot \Delta n + 0{,}01 \ [mm] \quad (12)$$

With these minor modifications the proposed theory gives quite satisfactory results in comparison with an empirical formulae by Walraven /14/ as shown in Fig. 10.

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