Cracking and Deformation in Structural Concrete

Risse und Verformungen in Konstruktionsbeton

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SUMMARY

A modern conception of a consistent theory of serviceability limit state of structural concrete is reported. The common mechanical basis for cracking and deformation is the bond-stress-slip relationship and the according displacements within the transmission length. Formulae are given preferably for monotonic instantaneous loading and ribbed reinforcing bars. The extension to long-term or repeated loading is demonstrated.

ZUSAMMENFASSUNG

Es werden die neueren Auffassungen einer konsistenten Theorie für den Grenzzustand der Gebrauchstauglichkeit von Konstruktionsbeton mitgeteilt. Gemeinsame Grundlage für Rissebildung und Verformungen bildet dabei das Verbundgesetz sowie das daraus abgeleitete Verformungsverhalten in der Einleitungszone. Es werden die Zusammenhänge für den Fall monotoner Kurzzeitbelastung für gerippte Bewehrungsstäbe angegeben und die Möglichkeit der Erweiterung für Langzeitbelastungen oder wiederholter Belastungen dargestellt.

1. INTRODUCTION

The serviceability limit states (SLS) of structural concrete comprise the limitations of stresses, cracking, deformation and vibration. Cracking and deformation, however, are the most significant SLS for reinforced and prestressed members. Structures made of plain or slightly reinforced concrete are usually very rigid and therefore, deformation is not significant for design. Cracking in such structures should be considered under specific requirements, which are different from those in reinforced concrete: After cracking, the equilibrium state in concrete should be verified and no uncontrolled crack propagation should be permitted.

Deformation in reinforced or prestressed concrete is considerably increased when cracking occurs. Therefore, the calculation of deformation in the cracked state (State II) is of predominant interest. Cracking and deformation are thus closely linked to another and should be approached within a consistent theory of structural concrete on a sound mechanical basis.

2. CONSTITUTIVE LAWS IN TRANSMISSION LENGTH

The constitutive laws of cracking and deformation can be found by considering the forces and displacements between the reinforcement and the adjacent concrete near the crack. As a simple model, a member under pure tension may be suitable in indicating the relevant relationships (Fig. 1). In an uncracked state, concrete strain ε_{c1} and the steel strain ε_{s1} are equal due to the compatibility conditions of full bond. After a crack has occurred, tensile force F acting in the reinforcing steel is only ($F_s = F$). At a distance equal or greater than transmission length l_t, either side of the crack, the compatibility condition of the uncracked state is maintained. The forces acting in the concrete (F_c) and the steel (F_s) may be calculated using rigidity values as follows:

$$F_c = F.\ A_c/(A_c + \alpha_e A_s) = F/(1+\alpha_e.\rho) \qquad [1]$$

$$F_s = F.\ \alpha_e A_s/(A_c + \alpha_e A_s) = F.\alpha_e.\rho/(1+\alpha_e.\rho) \qquad [2]$$

$$F_c + F_s = F \qquad [3]$$

Concrete force F_c is transmitted from the steel to the concrete by bond forces for the distance given by transmission length l_t. As shown in Fig. 1, in this length steel and concrete strain diverge from each other within that distance. The difference between steel and concrete strain is the first derivative of local slip, i.e. the differential displacement between the steel and the concrete ($u_s - u_c$).

$$ds/dx = \varepsilon_s - \varepsilon_c \qquad [4]$$

$$s = \int(\varepsilon_s - \varepsilon_c)dx = u_s - u_c \qquad [5]$$

Due to the slip s, bond stresses are generated according to the bond stress slip relationship, $\tau = \tau(s)$. This relationship depends predominantly on the surface of the steel, concrete strength fc, the position of the reinforcing steel during concreting (bond condition), the type of loading and whether the concrete is confined or not.

Various bond stress-slip-relationships have been proposed by different researchers. A synthesis has been provided in the very general law given in CEB Model Code 1990. This general law may be specified when one keeps in mind that ribbed reinforcing bars are preferable for crack control. In the case of unconfined concrete and for monotonic loading, the diagram shown in Fig. 2 may be applied.

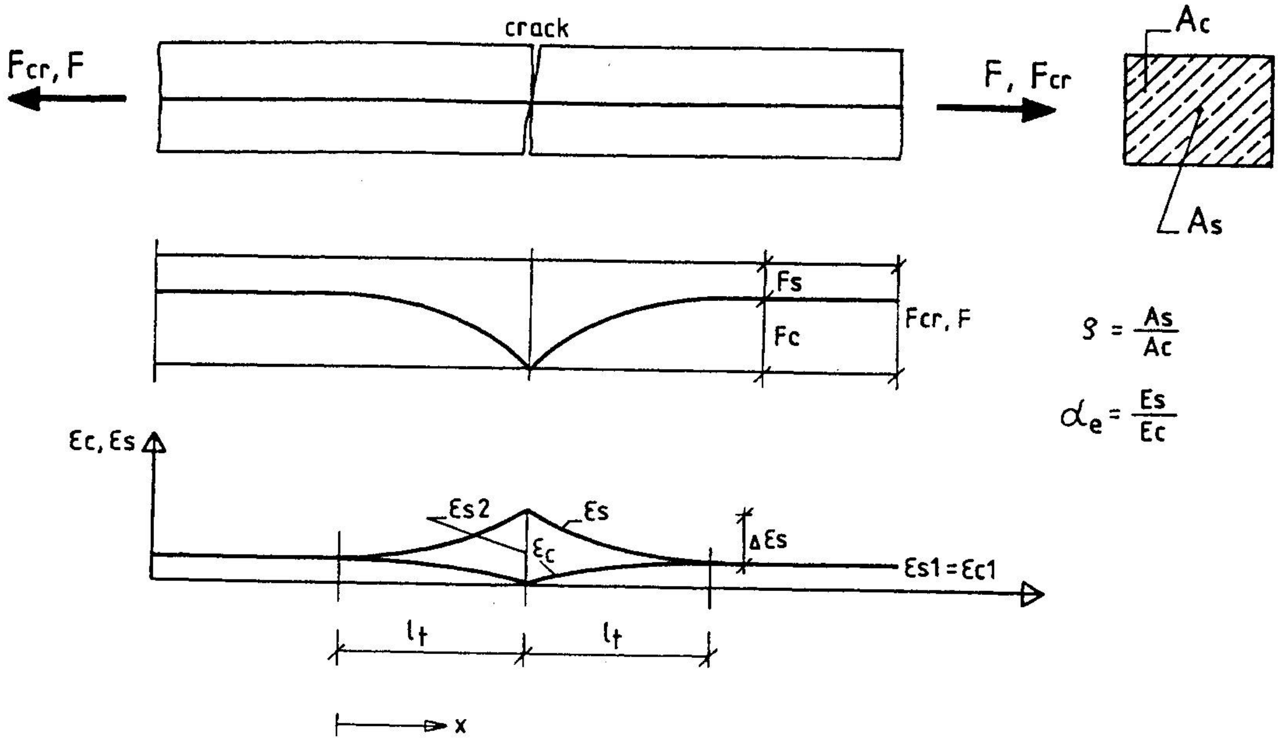

Fig. 1: Pure Tension, Forces and Strains near a Crack

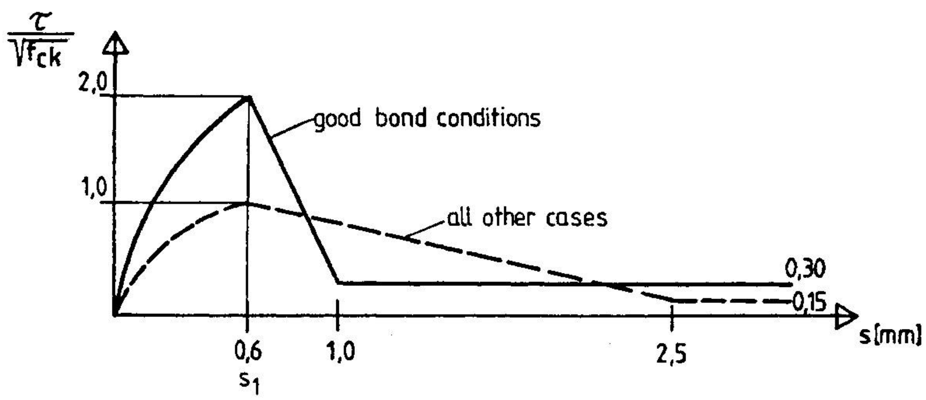

Fig. 2: Bond-Stress-Slip-Relationship of Ribbed Reinforcing Bars under Monotonic Short-Term-Loading

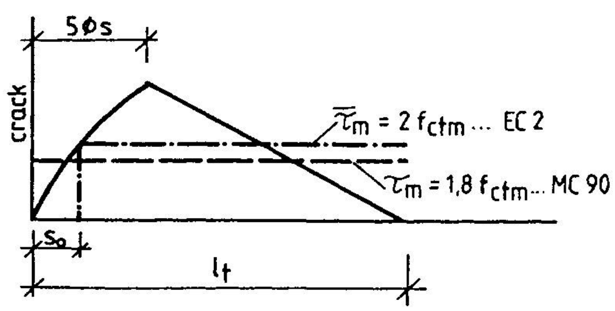

Fig. 3: Distribution of Bond Stress in the Transmission Length

Under service load conditions, crack width should be limited by a value of approximately $w_k = 2s \leq 0.5$ mm. Obviously, all calculations of the serviceability limit state use the first branch of the diagram only. Similarly, this may be predicted for confined concrete as the limit for the first branch (s_1) is increased from 0.6 to 1.0 mm. The constitutive law of this branch may be given by

$$\tau = \tau_{max} \, (s/s_1)^{\alpha} \qquad [6]$$

In the literature, actual values for exponent α are given in a range from 0.22 to 0.40.

In the vicinity of a transverse crack, a zone of reduced bond between the reinforcement bar and the surrounding concrete can be observed. For ribbed bars, the length of this zone depends on bar diameter Φ_s, concrete cover c, and the spacing of the ribs. Using formula [6] it is suggested that bond stress τ and slip s be reduced to within a distance of $x \leq 5\Phi_s$ from that crack by a factor λ, where

$$\lambda = x/5\Phi_s \leq 1 \qquad [7]$$

The differential equation for sliding bond,

$$d^2s/dx^2 = k.\,\tau \qquad [8]$$

with the bond-slip-relationship [6] reads as follows:

$$d^2s/dx^2 = k.\tau_{max}.\,(s/s_1)^{\alpha} \qquad [9]$$

This is a homogeneous non-linear differential equation of the second order, where the solution for the slip s(x) can be given as

$$s(x) = k_s \cdot x^{\frac{2}{1-\alpha}} \qquad [10a]$$

From this solution for slip the distribution of steel stress σ_s and bond stress τ can be found with the following derivatives:

$$\sigma_s(x) = k_\sigma \cdot x^{\frac{1+\alpha}{1-\alpha}} \qquad [10b]$$

$$\tau(x) = k_\tau \cdot x^{\frac{2\alpha}{1-\alpha}} \qquad [10c]$$

Equation [10] describes the distribution of the forces and displacements along transmission length l_t and hereby fulfills the requirements stated at the beginning of this chapter.

The previous considerations are only valid for monotonic short-term loading. Under long term loading (t) or repeated loading (n) the slip will increase. A simple way to describe this increased slip $s_{n,t}$ is by using Equation [11]

$$s_{n,t} = s.\,(1+k_{n,t}) \qquad [11a]$$

The displacement factor k_t for a permanent load can be calculated according to equation [11b]

$$k_t = (1+10t)^{0.080} - 1 \qquad [11b]$$

where t is load duration in hours.

For repeated loading the displacement factor k_n can be determined with Equation [11c]

$$k_n = (1+n)^{0.107} - 1 \qquad [11c]$$

where n is the number of load cycles.

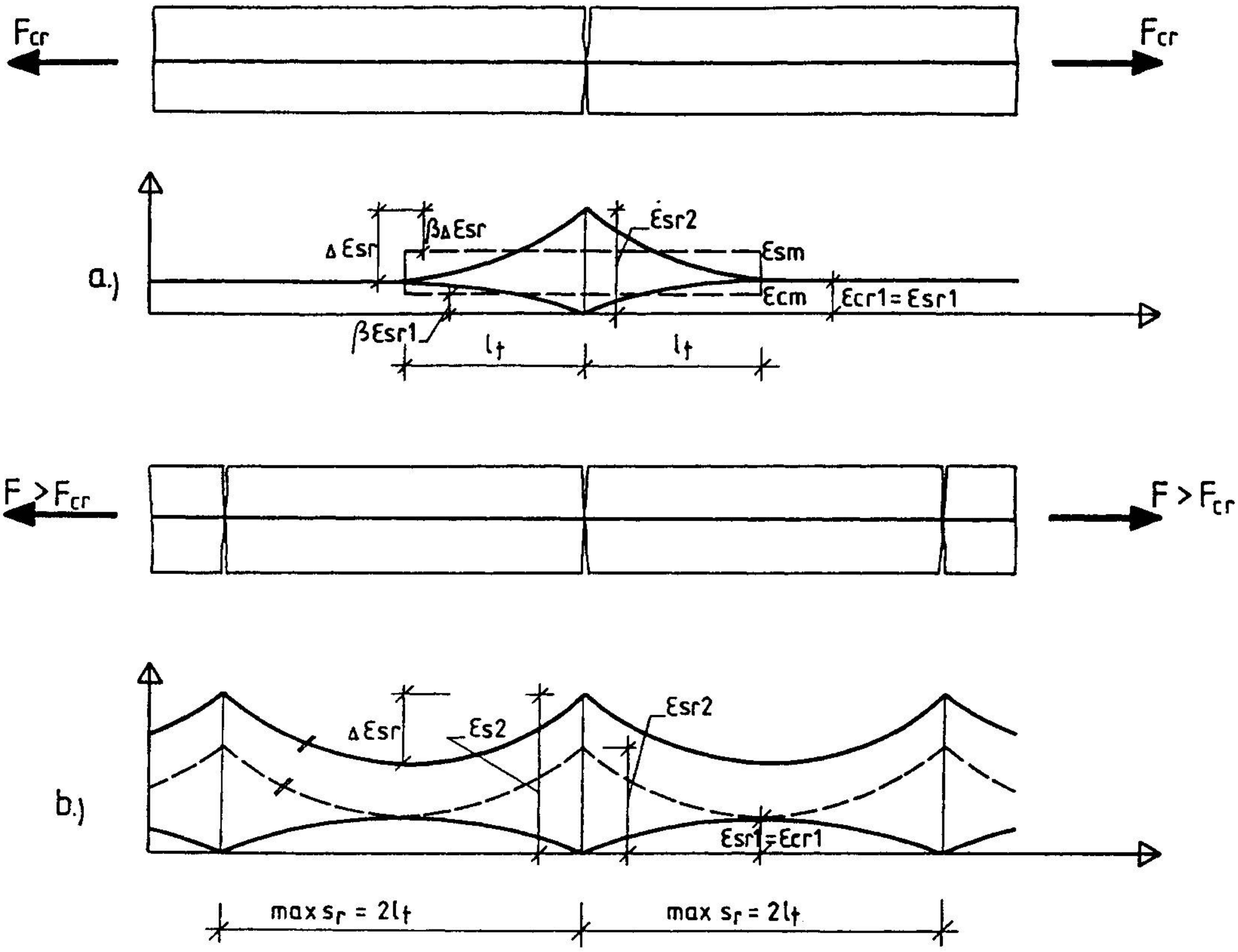

Fig. 4: Strains a) Near a Single Crack
b) For maximum Crack Spacing max s_r

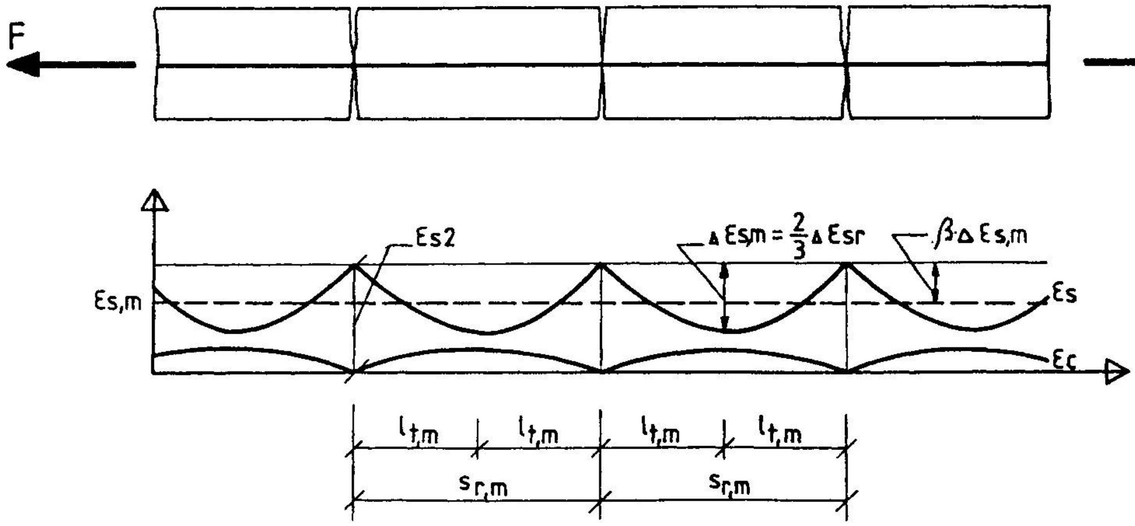

Fig. 5: Strains for average Crack Spacing $s_{r,m}$

3. APPROPRIATE SIMPLIFICATIONS UNDER SERVICE LOAD CONDITIONS

The distribution of bond stress over transmission length under short-term monotonic loading is plotted as a solid line in Fig. 3. For practical application, it is suitable to use mean bond stress τ_m over transmission length. The zone of reduced bond close to the crack may be approximated by bond less length s_o as it is done in EC 2 or by reducing mean bond stress along the MC 90 line. The values for mean bond stress in these two codes can be derived from the mean tensile stress of concrete f_{ctm} and equal to 2.0 f_{ctm} and 1.8 f_{ctm} respectively.

Using uniformly distributed bond stress simplified equations for transmission length, crack width and mean steel strain can be established.

4. STAGES OF CRACKING

For serviceability limit problems, the distinction between the uncracked concrete, crack formation phase and stabilized cracking stages is helpful in estimating crack width and deformation.

The prerequisite for the uncracked stage is that no crack has occurred. This will normally be fulfilled where concrete tensile stress σ_c is limited to the lower fractile of the tensile strength. A more conservative criterion is not to allow any tensile stresses.

$$\sigma_c \leq f_{ctk,min} \text{ or } \leq 0 \qquad [12]$$

When calculating the tensile stress, all actions, loads, and imposed or hindered deformation should be considered.

After the first crack has appeared, one crack after another occurs during the crack formation phase. In this phase, single cracks play an important role (Fig. 1). The distance between single cracks is greater than twice the transmission length, and no transmission length overlapping of adjacent cracks takes place. There is no influence of one crack on the crack width of the next.

At the end of the crack formation phase, the final primary crack pattern has been estabilished. The distance between the cracks is greater than the transmission length but less than the doubled value. The remaining tensile force in the concrete is too low for further cracks to occur, except for sporadic secondary cracking. Increasing load or imposed deformation will induce an increase in crack width, but no further cracks will appear.

5. CALCULATION OF CRACK WIDTH

The process of cracking discussed above allows calculating crack width for both stages of cracking on a uniform basis. Crack width w can be estimated as w = 2s where s is slip on both sides of the crack according to Equation [5]. By introducing average steel and concrete strain to transmission length ε_{sm} and ε_{cm} according to Fig. 4a, crack width can be calculated as

$$w = 2l_t.(\varepsilon_{sm} - \varepsilon_{cm}) \qquad [13]$$

Transmission length can be estimated with the simplifications presented in Chapter 3, using the equilibrium between steel force and bond force:

$$F\text{-}F_s = F_c = T = l_t.\Phi_s.\Pi.\tau_m$$

$$A_s.\sigma_{s2}/(1+\alpha_e.\rho) = l_t.\Phi_s.\Pi.\tau_m$$

$$l_t = (\Phi_s/4).(\sigma_{s2}/\tau_m).[1/(1+\alpha_e.\rho)] \qquad [14]$$

where σ_{s2} is steel stress at the crack and ε_{s2}, the corresponding strain.

Average steel strain and concrete strain can be given as:

$\varepsilon_{sm} = \varepsilon_{s2} - \beta\Delta\varepsilon_{sr}$ and [15a]

$\varepsilon_{cm} = \beta\varepsilon_{cr1} = \beta\varepsilon_{sr1}$ [15b]

β is an integration factor for strain distribution in transmission length, taking into account the simplification of the uniformly distributed bond stress. Here, it could be considered with a value: $\beta = 0.6$.

From $\Delta\varepsilon_{sr} = \varepsilon_{sr2} - \varepsilon_{sr1}$, it follows that

$\varepsilon_{sm} - \varepsilon_{cm} = \varepsilon_{s2} - \beta\varepsilon_{sr2}$ [16]

When tensile force is equal to cracking force as plotted in Fig 4a, ε_{s2} in Equation [16] reads ε_{sr2}. Consequently, Equation [16] can be transformed to

$\varepsilon_{sm} - \varepsilon_{cm} = (1-\beta)\,\varepsilon_{sr2} = 0.4\,\varepsilon_{sr2}$ [17]

Maximum crack width, max w in the final crack pattern can be correlated to maximum crack spacing being twice the transmission length.

$\max s_r = 2.l_t$ [18]

The greatest crack width can be found when crack spacing on both sides of the crack under consideration is equal to maximum crack spacing (Fig 4b). In general, tensile force is higher than cracking force and the differential average strain values for steel and concrete can by calculated according to Equation [16].

A general formula for max w can be derived from Equations [13], [14] and [16] introducing tensile force F and cracking force F_{cr} as indicated in Fig 4.

$\max w = 2.(\Phi_s/4).[F_{cr}/(A_s.\tau_m)].[1/(1+\alpha_e.\rho)](F - \beta F_{cr})/A_s.E_s$ [19]

which can be transformed into

$$A_s = \sqrt{\frac{\Phi_s \cdot F_{cr}\,(F - \beta \cdot F_{cr})}{2\tau_m \cdot E_s \cdot \max w \cdot (1 + \alpha_e \rho)}} \qquad [20]$$

From Equation [20], the necessary amount of reinforcement required for crack width max w and a selected bar diameter Φ_s may be calculated directly.

The calculation of crack width under long-term or repeated loading may be based on equivalent considerations taking into account the relevant losses of bond stress. This can be achieved by using the appropriate values for average bond stress τ_m and integration factor β.

Crack control in thick members needs specific consideration. When calculating the cracking force due to imposed deformation, it should be taken into account that concrete stresses are not on the thickness of the cross section. When the first crack has appeared, cracking force at the crack is acting in the reinforcing bars near the surface. The cracking force is transmitted to the adjacent concrete by bond forces. Further cracks will appear, when tensile strength in the effective concrete zone $A_{c,ef}$ near the surface is exceeded. This might occur before full cracking force is transmitted to the concrete. Thus, "brush cracking" can occur. $A_{c,ef}$ depends on bar diameter and thickness of concrete cover c and may be estimated as

$A_{c,ef} = 2.5\ (c+\Phi_s/2)$ [21]

Equation [20] may be applied to thick members by replacing ρ with ρ_{ef}, which is the reinforced percentage of the effective concrete area.

$\rho_{ef} = A_s / A_{c,ef}$ [22]

6. CRACK WIDTH CONTROL WITHOUT CALCULATION

For specific values of max w, a correlation between steel stress σ_s and necessary bar diameter Φ_s can be found. Such relationships are shown on Table 7.4.3 in MC 90, where the values for reinforced concrete are based on a crack width of w = 0.3 mm, and these for prestressed concrete of w = 0.2 mm respectively.

Steel stress [MPa]	Maximum bar diameter [mm]	
	Reinforced sections	Prestressed sections
160	32	25
200	25	16
240	20	12
280	14	8
320	10	6
360	8	5

7. DEFORMATION DUE TO TENSION

The elongation of a member under tension, a tensile chord of a beam or a tie can calculated with

$$\Delta l = \varepsilon_{s,m} \cdot l \qquad [23]$$

Average steel strain, $\varepsilon_{s,m}$, indicates the average elongation in the full member. this value differs from ε_{sm} in the transmission length of a single crack or between two cracks with maximum crack spacing expressed in Equation [15a]. The difference between these two values is caused by differential crack spacing. Mean steel strain ε_{sm} is associated with the maximum crack spacing. Actual crack spacing s_r in a member with a greater total length, however, varies in transmission length to twice that value

$$l_t \le s_r \le 2.l_t \qquad [24]$$

Crack spacing s_r along the axis of the member is arbitrarily distributed according to the probabilistic distribution of cracking resistance $A_c.f_{ct}$ or $A_{c,ef}.f_{ct}$ respectively. Average crack spacing $s_{r,m}$ can be calculated for an assumed probability distribution. For various realistic distributions, average crack spacing can is found to be close to 4/3 l_t. Thus, this value would be a reasonable approximation.

$$s_{r,m} = 4/3\ l_t \qquad [25]$$

Accordingly, transmission length on both sides of the crack is reduced to

$$l_{t,m} = 0.5\ s_{rm} = 2/3\ l_t \qquad [26]$$

Then, transferred bond force is reduced corresponding to the reduced transmission length

$$T = 2/3.l_t.\Phi_s.\Pi.\tau_m = 2/3\ A_s.E_s.\ \Delta\varepsilon_{sr} \qquad [27]$$

The reduction of steel strain at the crack to the midpoint between the cracks is then given by

$$\Delta\varepsilon_{s,m} = 2/3\ \Delta\varepsilon_{sr} \qquad [28]$$

The mean strain throughout the entire member may thus be taken as:

$$\varepsilon_{s,m} = \varepsilon_{s2} - \beta.\ \Delta\varepsilon_{s,m} = \varepsilon_{s2} - \beta.2/3.\ \Delta\varepsilon_{sr} \qquad [29]$$

For instantaneous loading, the integration factor may be assumed as being $\beta = 0.6$, as demonstrated in Chapter 5. Thus, average steel strain in the total member can be given as

$$\varepsilon_{s,m} = \varepsilon_{s2} - 0.6.2/3.\ \Delta\varepsilon_{sr} = \varepsilon_{s2} - 0.4.\ \Delta\varepsilon_{sr} \quad [30]$$

The simplified stress-strain-diagram of a member under pure tension thorughout the full cracking stage range relevant for serviceability limit states is shown in Fig 6.

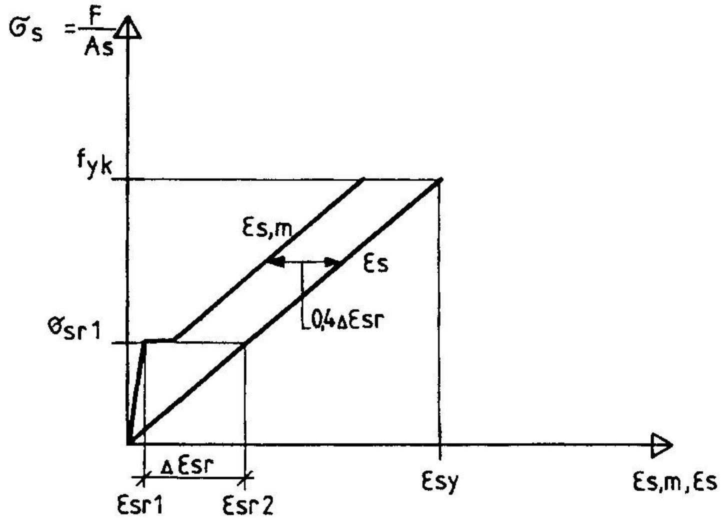

Fig. 6: Simplified Stress-Strain-Relationship of Embedded Reinforcing Steel under Monotonic Instanteneous Loading

8. RANGE OF APPLICATION

The principles and application rules reported above may be applied to any type of structural concrete with reinforcement, i.e. normally reinforced or prestressed concrete. Prestressing may be applied with any type of tendon: internal or external tendons, bonded or unbonded prestressing steel. For serviceability limit states, prestressing force is taken into account as an external force acting on the reinforced member. Stresses due to direct and indirect actions are combined with prestressing stresses in accordance with the relevant combination rule. Thus, the combination of action effects which cause cracking can be analysed.

When crack control is required, it is preferable that ribbed reinforcing steel has applied. Prestressing steel and smooth bars are of minor advantage with respect to crack control. Excessive deformation, in particular, can be controlled with prestressed tendons.

As stated above the relationships shown are valid for service load conditions. The parameters are predominantly reported for instantaneous loading. They may be adapted to long-term and repeated loading. Especially mean bond strength τ_m and integration factor β should be modified accordingly.

The analysis of a structure at ultimate limit state may be carried out on the basis of a strut and tie model. If this model is orientated on a stress field according a linear analysis it may also be applied to serviceability limit state. Cracking and deformation of ties can be calculated with the models reported above.