Creep, relaxation and shrinkage of structural concrete

Autor(en): **Trost, Heinrich**

Objekttyp: **Article**

Zeitschrift: **IABSE reports = Rapports AIPC = IVBH Berichte**

Band (Jahr): **62 (1991)**

PDF erstellt am: **23.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-47621

Creep, Relaxation and Shrinkage of Structural Concrete

Fluage, relaxation et retrait du béton de construction

Kriechen, Relaxation und Schwinden von Konstruktionsbeton

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SUMMARY

The effects of time-dependent material properties of structural concrete are analysed on a common basis. Starting with the principle of superposition an algebraic σ-ε-τ relation containing a relaxation parameter or ageing coefficient χ is formulated and its application to the analysis and design of reinforced and prestressed concrete structures is illustrated: changes of forces due to imposed deformations and modifications of restraint conditions, stress redistributions between concrete and steel, time-dependent deformations.

RÉSUMÉ

Les effets différés affectant le béton de construction sont analysés sur une base commune: basée sur le principe de superposition, une relation σ-ε-τ contenant le paramètre de vieillissement χ est formulée. Son application dans l'analyse et la conception des structures armées et/ou précontraintes est illustrée dans le cas de la variation des efforts provenant de déformations imposées, de la modification des conditions de contrainte, des redistributions des efforts entre béton et acier et des déformations dues aux effets différés.

ZUSAMMENFASSUNG

Die Auswirkungen der zeitabhängigen Materialeigenschaften werden bei Konstruktionsbeton auf einer gemeinsamen Grundlage untersucht. Mit dem Superpositionsprinzip wird eine algebrische σ-ε-τ Beziehung unter Verwendung eines Relaxationskennwertes oder Alterungsbeiwertes χ formuliert und ihre Anwendung bei Berechnung und Entwurf von Konstruktionen in Stahl- und Spannbeton erläutert: Schnittgrössenänderungen bei erzwungenen Verformungen und Systemwechseln, Spannungsumlagerungen zwischen Beton und Stahl, zeitabhängige Verformungen.

1. Introduction

The performance of structural concrete at serviceability limit state depends mainly on a good design for quality and durability of the concrete construction. An important tool needed for this quality design is a consistent model and a sophisticated but not too complicated analytical model. This model should include the analysis of stress distribution, deflections and cracking. As creep and shrinkage of concrete have a strong influence on long-term damages of concrete structures, time-dependent effects should be taken into account for serviceability limit states and second order effects.

In order to describe the viscoelastic behaviour of concrete and reinforcement we are talking first of all about creep-problems and mean the time-dependent increase of deformation under known stress history and secondly about relaxation-problems and mean the time-dependent decrease of stresses under specific conditions of deformation; at last we consider shrinkage-problems and mean the shortening of non-loaded concrete members during the natural or artifical draining process.

The structural engineer has to take into consideration the relationship between stress, strain and time of concrete and reinforcement as well as the compatibility condition of the build-up cross-section, which is assumed to remain plain, and, if applicable, the conditions due to statical indeterminacy.

2. Time-dependent stress-strain relation of concrete

The theory of linear creep is based upon the principle of superposition stating that in viscoelastic materials various steps of stresses can be superposed considering their time of duration and their different age τ at loading or maturity of the concrete.

Using the principle of superposition, we may write for the total strain ϵ_t at any time t including shrinkage $\epsilon_s(t)$ according to [1,2]:

$$\epsilon(t) = \frac{\sigma(\tau_0)}{E(\tau_0)} \cdot [1+\phi(t,\tau_0)] + \int_{\tau=\tau_0}^{t} \frac{\partial\sigma(\tau)}{\partial\tau} \cdot \frac{1}{E(\tau)} \cdot [1+\phi(t,\tau)] \cdot d\tau + \epsilon_s(t) \quad (1)$$

and with a constant modulus of elasticity E and the well-known creep coefficient $\phi(t,\tau_0) = \phi_t$

$$\epsilon_t = \frac{\sigma_0}{E} \cdot (1+\phi_t) + \frac{\sigma_t - \sigma_0}{E} + \frac{1}{E} \cdot \int_{\tau=\tau_0}^{t} \frac{\partial\sigma(\tau)}{\partial\tau} \cdot \phi(t,\tau) \cdot d\tau + \epsilon_{s,t}. \quad (2)$$

With this principle of superposition one has to solve creep problems (i.e. calculation of deformations under a known stress history) as well as relaxation problems (i.e. determination of stresses under specified conditions of strain or deformation).

The total strain $\epsilon(t)$ can be determined by using simple quadrature rules in case the stress distribution $\sigma(t)$ is known; on the other hand, when $\epsilon(t)$ is known, equation (1) is an integral equation and the stress distribution $\sigma(t)$ is the unknown function. Under general conditions a complete analytical solution is impossible, because the creep-function $\phi(t,\tau)$ has to be determined with the help of experiments or according to standards and has a complicated mathematical form. To avoid such difficulties, it is possible in order to compute the stress distribution due to external or internal restraints to change the integral equation (1) into a simpler algebraic stress-strain relation and then solve the relaxation-problems using well-known and simple mathematical procedures (see fig.1). The following algebraic equation is obtained by modification of the integral equation (2)

$$\epsilon_t = \frac{\sigma_0}{E} \cdot (1+\phi_t) + \frac{\sigma_t - \sigma_0}{E} \cdot [1+\chi_t \cdot \phi_t] + \epsilon_{s,t} \quad (3)$$

where the parameter χ_t can be calculated and is identified as relaxation parameter [2,3] or aging coefficient [1,4] with

$$\chi_t = \frac{\int_{\tau_0}^{t} \frac{\partial\sigma(\tau)}{\partial\tau} \cdot \phi(t,\tau) \cdot d\tau}{(\sigma_t - \sigma_0) \cdot \phi_t} = \frac{\sum_{\tau_i} \Delta\sigma(\tau_i) \cdot \phi(t,\tau_i)}{(\sigma_t - \sigma_0) \cdot \phi_t} \,. \quad (4)$$

Now introducing the strain condition for relaxation $\epsilon_t=\epsilon_0=\text{const}$, equation (3) leads without ϵ_s to the following relation by using the known aging coefficient χ_t and the condition $E_c \cdot \epsilon_o - \sigma_0 = 0$:

$$0 = \sigma_0 \cdot \phi_t + (\sigma_t - \sigma_0) \cdot (1 + \chi_t \cdot \phi_t) \quad \text{or} \quad \frac{\sigma_t}{\sigma_0} = 1 - \frac{\phi_t}{1 + \chi_t \cdot \phi_t} \,. \tag{5}$$

After introducing the relaxation coefficient ψ_t analogue to the creep coefficient

$$\phi_t = \frac{\epsilon_t - \epsilon_o}{\epsilon_o} \quad \text{by} \quad \psi_t = -\frac{\sigma_t - \sigma_o}{\sigma_o} = 1 - \frac{\sigma_t}{\sigma_o} \tag{6}$$

one can also compute the exact time-dependent aging coefficient χ_t and proof by experiments [5] with

$$\chi_t = \frac{1}{\psi_t} - \frac{1}{\phi_t} = -\frac{\sigma_0}{\sum_{\tau_i} \Delta\sigma(\tau_i)} - \frac{1}{\phi_t} \,. \tag{7}$$

Fig. 1 explains the necessary mathematical background as well as some values for the aging coefficient χ_t, which oscillates between 1.0 and 0.6 with a mean value of $\bar{\chi} = 0.8$.

The remaining values of the stresses for $t \to \infty$ are of particular interest, i.e. σ_∞ after completion of creep and relaxation. The result of extensive investigations is the following statement: Assuming a usual load carrying age of 3 days $< \tau_0 <$ 90 days and typical values for the final creep coefficient $1 < \phi_\infty < 4$ of standard concrete one can determine a general aging coefficient of $\chi \approx 0.8$. This leads to the simple form of the stress-relaxation

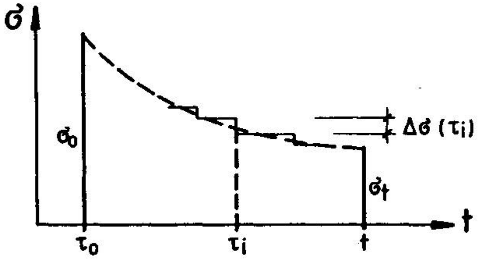
$$\frac{\sigma_t}{\sigma_0} = 1 - \frac{\phi_t}{1 + 0.8 \cdot \phi_t} \,. \tag{8}$$

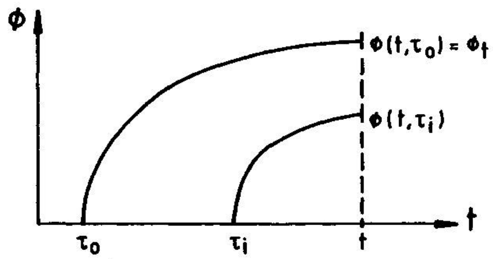
Fig. 2 explains these relations and shows all evaluations for an average final creep coefficient $\phi_\infty = 2.5$.

$$\varepsilon(t) = \frac{\sigma(\tau_0)}{E(\tau_0)}\,[1 + \phi(t,\tau_0)] + \int_{\tau=\tau_0}^{t} \frac{\partial \sigma(\tau)}{\partial \tau}\,\frac{1}{E(\tau)}\,[1 + \phi(t,\tau)]\,d\tau + \varepsilon_s(t)$$

with $E(\tau) = E_c$; $\phi(t,\tau_0) = \phi_t$ and

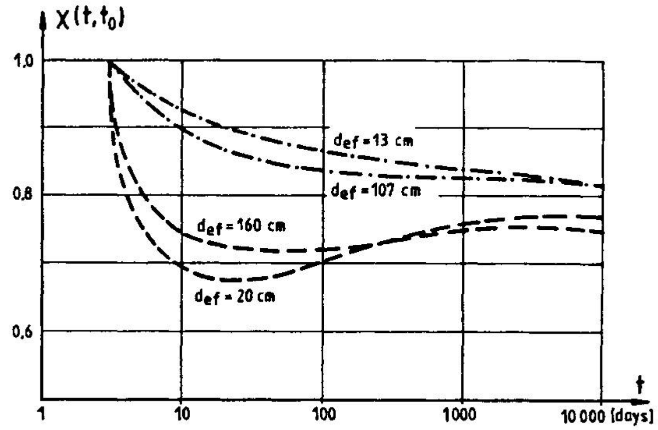
aging coefficient $\chi_t = \dfrac{\sum \Delta\sigma(\tau_i)\cdot\phi(t,\tau_i)}{(\sigma_t - \sigma_0)\cdot\phi_t}$



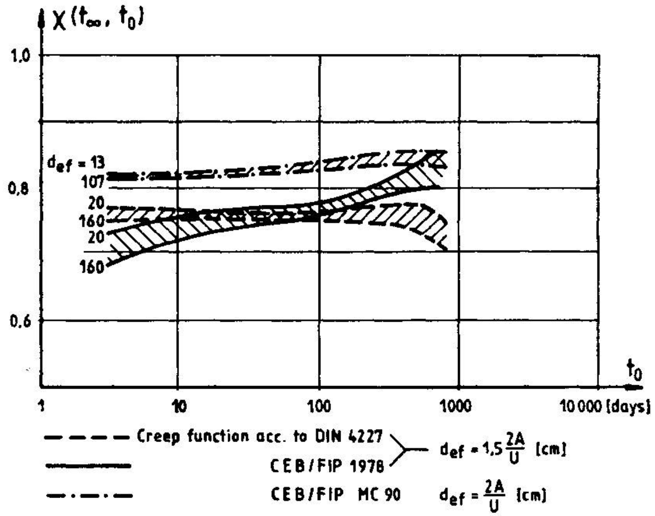


$\sigma_t - \varepsilon_t$ – relation:

$$\varepsilon_t = \frac{\sigma_0}{E_c}\,(1 + \phi_t) + \frac{\sigma_t - \sigma_0}{E_c}\,(1 + \chi_t \cdot \phi_t) + \varepsilon_{s,t}$$



development for t_0 = 3 days



end values $\chi_\infty(t_0)$

AGING COEFFICIENT X

Fig. 1: Relation between stress, strain and time due to the application of the principle of superposition and values of the aging coefficient χ (see [5])

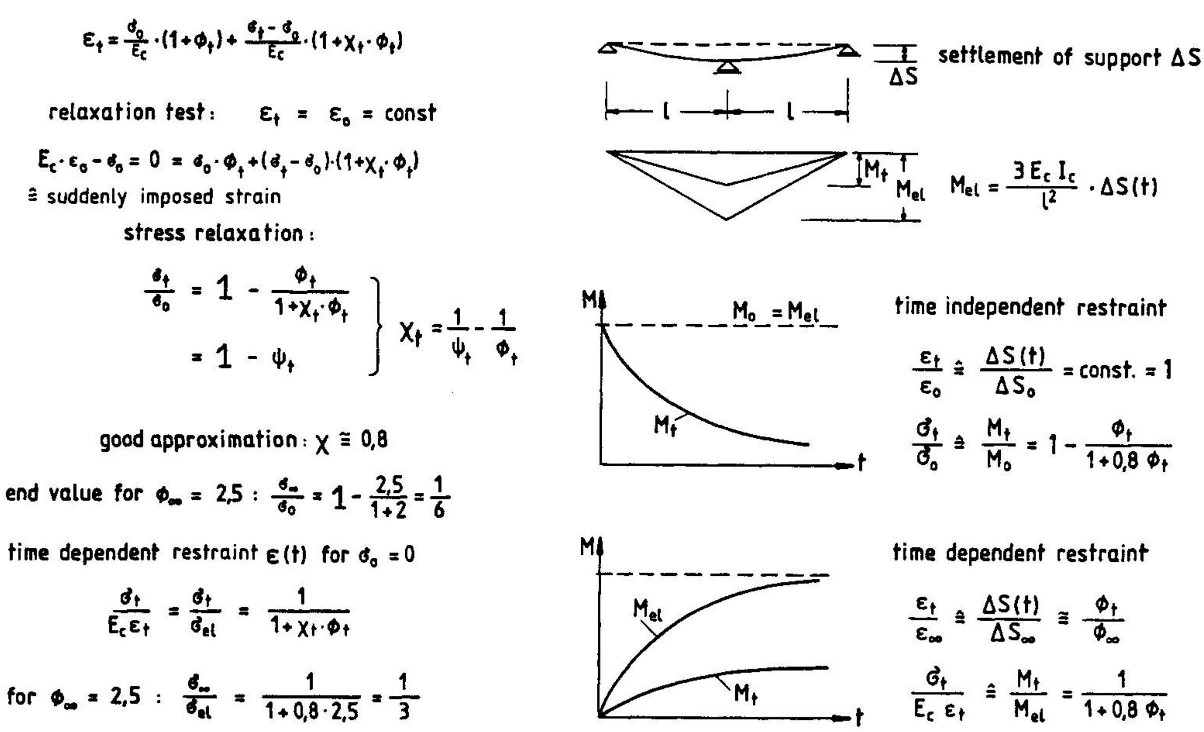


Fig. 2: Development of stress during relaxation test and sectional forces in continuous beams due to sudden or gradual settlement of support

Fig. 15 and fig. 16 of Journal No.295 of DAfStb [5] proof that the theoretical model can be applied successfully to practical problems. The Journal indicates creep and relaxation tests for very old concrete and shows a good agreement of computed and measured stress relaxation by using the measured creep coefficients.

3. Change of reaction forces and sectional forces

In statical indeterminate systems imposed deformations lead to sectional forces and stresses; those values will decrease according to the relaxation process when considering a sudden restraint and will increase to only a fraction of the elastic values if a gradual time dependent restraint is occuring.

The effects of the time-dependent behaviour of concrete on the sectional forces due to these types of restraints are explained in fig. 2 by means of the settlement of support of a two span beam.

Of course the results are valid for any continuous beam. Using well-known methods of structural analysis one can determine the moment distribution M_{el} due to $\Delta s(t)$ according to the theory of elasticity. Usually one can neglect the contribution of the reinforcement to the stiffness of the build-up cross-section but it is also possible to take this influence into consideration by applying procedures described in [6]. In reality one often observes a combination of sudden and gradual settlement of support.

A typical example for the change of sectional forces is resulting from the modification of the restraint conditions after the application of loads due to a change of the structural system. If precast concrete beams are connected in a continuous manner (see fig. 3) or the whole structure is built in some field stages, the corresponding rotation will be preserved, which is the condition of a constant strain or rotation. According to fig. 3 one can compute the initial internal moment M_A at time t=0 as a sum of the moment M_{el}, which is obtained from an analysis of the structure as a whole, and a restraint moment M_A-M_{el}; this illustrates the substantial change of internal forces due to external permanent loads and to the process of creep and relaxation. More investigations about the building of continuous beams in several stages can be found in [7].

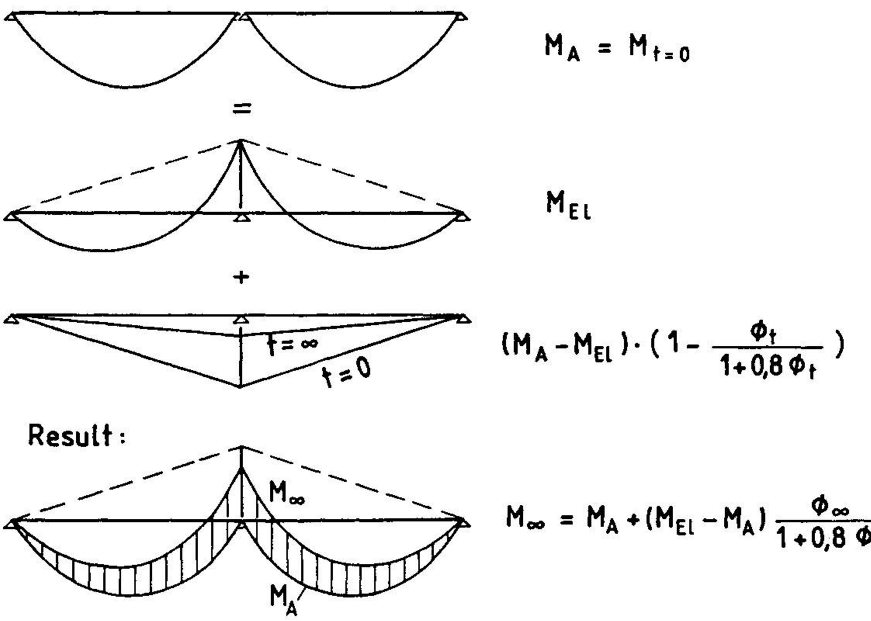


Fig.3: Redistribution of sectional forces due to a change of system

4. Redistribution of internal stresses and forces in cross-sections

This redistribution of internal stresses under external permanent loads and prestressing forces is caused by the amount and distribution of bonded reinforcement in the cross-section - independent if the reinforcement is prestressed or not. This restraining effect of the bonded reinforcement on creep and shrinkage of unreinforced concrete is described with the redistribution-parameter λ as ratio of the real to the free deformation in the steel fiber. These time-dependent stresses in steel and concrete can easily be derived on the basis of equilibrium and compatibility of strains in a section.

Fig. 4 explains the change of stress and deformation in a prestressed member with only one layer of prestressing steel. If the bond between steel and concrete is removed the time-dependent deformation would be the sum of free creep strain $\phi_t \cdot \epsilon_{c,o}$ and shrinkage strain ϵ_s. Using φ as abbreviation Index for the effects of creep, shrinkage and relaxation, the compatibility condition in the fibre of the bonded prestressing tendon is expressed by

$$\epsilon_{p,\varphi} = \epsilon_{cp,\varphi} = \lambda(\phi_t \cdot \epsilon_{cp,g+P} + \epsilon_s) = \lambda(\phi_t \cdot \epsilon_{cp,o} + \epsilon_s) \qquad (9)$$

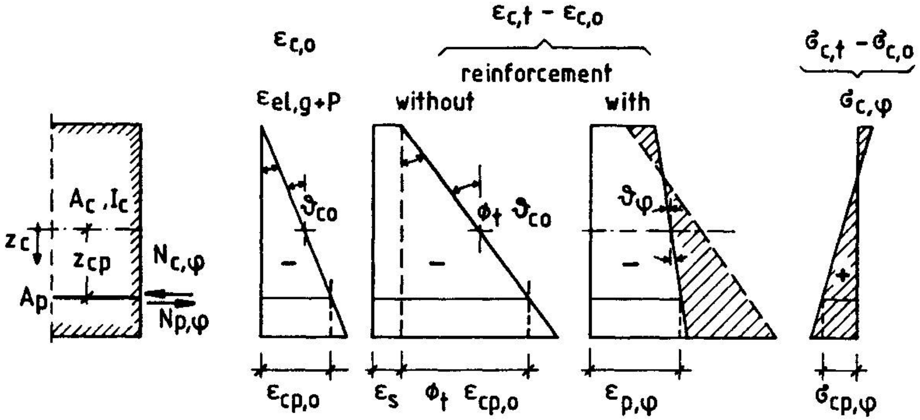
One can obtain with eq. (3) and the equilibrium for the state of Eigenstresses $N_{p,\varphi} = -N_{c,\varphi}$ the value of the redistribution-parameter λ by

$$1/\lambda = 1 + n \cdot \frac{A_p}{A_c} \left(1 + \frac{A_c}{I_c} \cdot z_{cp}^2\right)(1 + \chi \cdot \phi_t) \qquad (10)$$

with the maximum value $\lambda = 1$ for concrete without any reinforcement or by neglecting the restraining effects of the bonded steel. Then the so-called loss of prestress (see fig. 5 with all definitions, given in the EUROCODE 2 [11]) is calculated by

$$\Delta\sigma_{p,c+s+r} = \sigma_{p,\varphi} = \frac{N_{p,\varphi}}{A_p} = \lambda \cdot [n \cdot \phi_t \cdot \sigma_{cp,o} + E_p \cdot \epsilon_s + \Delta\sigma_{p,r}], \qquad (11)$$

where in addition the steel relaxation is introduced with $\Delta\sigma_{p,r}$. This derivation (see [3,6]) is demonstrated in fig. 4 and illustrated for a typical example, where the value of $\lambda = 0.55$ reduces the loss of prestress from 210 to 110 N/mm^2.



$$\varepsilon_{p,\varphi} \equiv \varepsilon_{cp,\varphi} = \lambda\ (\phi_t \cdot \varepsilon_{cp,o} + \varepsilon_s) \wedge \text{Compatib.}$$

$$\frac{\sigma_{p,\varphi} - \Delta\sigma_{p,r}}{E_p} = \frac{\sigma_{cp,o}}{E_c}\ \phi_t + \varepsilon_s + \frac{\sigma_{cp,\varphi}}{E_c}\ (1 + \chi\ \phi_t)$$

$$\frac{\sigma_{p,\varphi}}{E_p}\left[1 - \frac{E_p}{E_c}\ \frac{\sigma_{cp,\varphi}}{\sigma_{p,\varphi}}\ (1 + \chi\ \phi_t)\right] = \phi_t \cdot \varepsilon_{cp,o} + \varepsilon_s + \frac{\Delta\sigma_{p,r}}{E_p}$$

$$\text{Mit } [\ ] = \frac{1}{\lambda} \text{ und } \frac{\sigma_{cp,\varphi}}{\sigma_{p,\varphi}} = -\frac{N_{p,\varphi}}{N_{p,\varphi}}\ \frac{A_p}{A_c}\ \left(1 + \frac{A_c \cdot z_{cp}^2}{I_c}\right) \wedge \text{Equil.}$$

$$\sigma_{p,\varphi} = \Delta\sigma_{p,c+s+r} = \frac{N_{p,\varphi}}{A_p} = \frac{n \cdot \phi_t \cdot \sigma_{cp,o} + E_p \varepsilon_s + \Delta\sigma_{p,r}}{1 + n\ \frac{A_p}{A_c}\ \left(1 + \frac{A_c\ z_{cp}^2}{I_c}\right)\ (1 + \chi \cdot \phi_t)}$$

Redistribution of stress in prestressed concrete member
(without $\Delta\ \sigma_{p,r}$)

$n = 6;\quad \phi_\infty = 2{,}5;\quad \sigma_{cp,o} = -10\ \text{N/mm}^2;\quad \varepsilon_{s\infty} = -30 \cdot 10^{-5}$

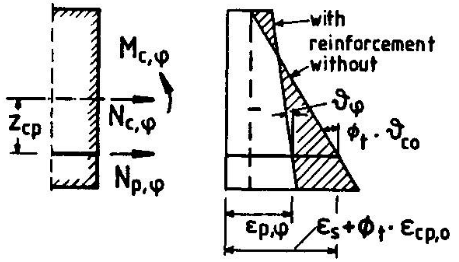
$$\sigma_{p,\varphi_\infty} = \frac{-6 \cdot 2{,}5 \cdot 10 - 2{,}0 \cdot 30}{1 + 0{,}1\ (1+2) \cdot 3} = -\frac{210}{1{,}9} = -110\ \text{N/mm}^2$$

denominator: $n \cdot \frac{A_p}{A_c} = 0{,}1;\quad \frac{A_c}{I_c}\ z_{cp}^2 = 2;\quad 1 + 0{,}8 \cdot \phi_\infty = 3$

Deformations:

$$\varepsilon_{p,\varphi} = \frac{N_{p,\varphi}}{E_p A_p} = \frac{\sigma_{p,\varphi}}{E_p} = \lambda(\phi_t \cdot \varepsilon_{cp,o} + \varepsilon_s)$$

$$0 < \lambda < 1 \quad \text{here } \lambda = \frac{1}{1 + 0{,}9} = 0{,}55$$



$$\varepsilon_{p,\varphi} = \frac{N_{p,\varphi}}{E_p \cdot A_p}$$

$$M_{c,\varphi} = -N_{p,\varphi} \cdot z_{cp}$$

$$N_{c,\varphi} = -N_{p,\varphi}$$

Rotation: $\vartheta_\varphi = \phi_t \cdot \vartheta_{c,o} + \frac{-N_{p,\varphi} \cdot z_{cp}}{E_c\ I_c} \cdot (1 + \chi \cdot \phi_t)$

Deflection: $f_\varphi = \phi_t \cdot f_{o,g+p} + f_{N_{p,\varphi}} \cdot (1 + 0{,}8 \cdot \phi_t)$

Fig. 4: Change of stresses and deformations in prestressed concrete members due to creep and shrinkage and relaxation (for abbreviation Index φ = c+s+r)

4.2.3.5.5. Loss of prestress

(9) Time dependent losses should be calculated from:

$$\Delta\sigma_{p,c+s+r} = \frac{n\cdot\phi(t,t_0)\cdot(\sigma_{cg}+\sigma_{cp})+\epsilon_s(t,t_0)\cdot E_s+\Delta\sigma_{pr}}{1+n\cdot\frac{A_p}{A_c}\cdot(1+\frac{A_c}{I_c}\cdot z_{cp}^2)\cdot[1+0,8\cdot\phi(t,t_0)]}$$

where :

$\Delta\sigma_{p,c+s+r}$	is the variation of stress in the tendons due to creep, shrinkage and relaxation at location x, at time t.
$\epsilon_s(t,t_0)$	is the estimated shrinkage strain, derived from the values in Table 3.4 for final shrinkage (see also 2.5.5 and Appendix 1).
E_s	is the modulus of elasticity for the prestressing steel, taken from 3.3.4.4.
E_{cm}	is the modulus of elasticity for the concrete (Table 3.2).
n	is E_s/E_{cm}.
$\Delta\sigma_{pr}$	is the variation of stress in the tendon at section x due to relaxation.
$\phi(t,t_0)$	is a creep coefficient, as defined in 2.5.5, equation (2.9) (see also Appendix 1).
σ_{cg}	is the stress in the concrete adjacent to the tendons, due to self-weight and any other permanent actions.
σ_{cp}	is the initial stress in the concrete adjacent to the tendons, due to prestress (see 4.2.3.5.3 P(3)).
A_p	is the area of all the prestressing tendons at the level being considered.
A_c	is the area of the concrete section.
I_c	is the second moment of area of the concete section.
z_{cp}	is the distance between the centre of gravity of the concrete section and the tendons.

Fig. 5: Excerpt of the Final Draft of EUROCODE 2

In structural concrete members we have two kinds of reinforcement: the prestressing steel (active reinforcement) and normal unprestressed steel (passive reinforcement) in some different layers.

For a precise calculation of the change of stresses in these reinforcement layers one can summarize the active and passive reinforcement $A_p + A_s = A_r$ in a resulting steel fibre r, but then the moment of inertia of all steel layers I_r has to be taken into account. With the compatibility condition for strain and for rotation the change of stresses in the different fibres of steel and in the whole concrete section are resulting out of the combination of longitudinal forces $N_{r,\varphi}$ or $N_{c,\varphi}$ and bending moments $M_{r,\varphi}$ or $M_{c,\varphi}$, related to the steel or concrete section. Naturally this leads to more complicated formulas as can be seen in the detailed investigations in the publication [6] and [9], but usually one can neglect the effects of $M_{r,\varphi}$ with the assumption $I_r \approx 0$. For uncracked sections under permanent loads and with $E_s \approx E_p$ the redistribution of steel stresses is then given in the same form as in eq.(11) with

$$\sigma_{r,\varphi} = \frac{N_{r,\varphi}}{A_r} = \lambda_r \cdot [n \cdot \phi_t \cdot \sigma_{cr,o} + E_p \cdot \epsilon_s + \Delta\sigma_{p,r}] \qquad (12)$$

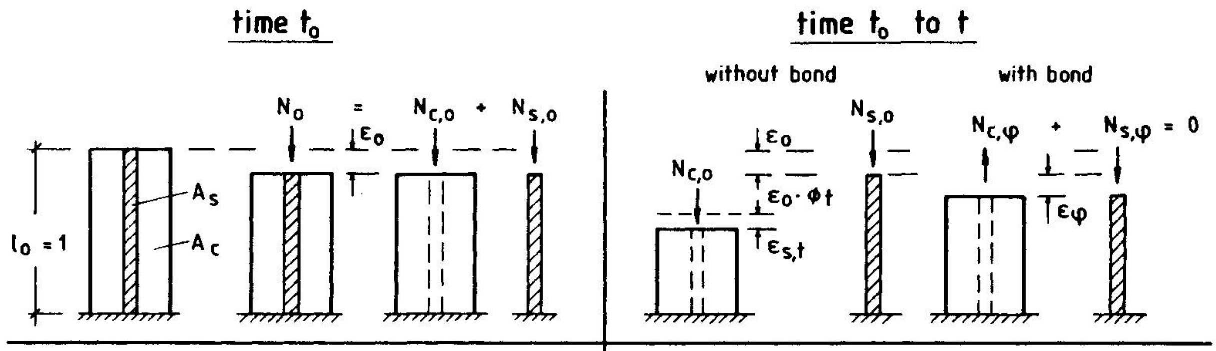
with the redistribution parameter in the resulting steel fibre r

$$1/\lambda_r = 1 + n \frac{A_s + A_p}{A_c} \left(1 + \frac{A_c}{I_c} \cdot z_{cr}^2\right)(1 + \chi \cdot \phi_t). \qquad (13)$$

Because the redistribution parameter λ_r contains the sum $A_s + A_p$ the forces according to the state of Eigenstresses must be calculated with

$$N_{p,\varphi} = \sigma_{r,\varphi} \cdot A_p;\ N_{s,\varphi} = \sigma_{r,\varphi} \cdot A_s;\ M_{c,\varphi} = -N_{r,\varphi} \cdot z_{cr} = N_{c,\varphi} \cdot z_{cr}. \qquad (14)$$

The change of internal forces and stresses in reinforced concrete columns due to creep and shrinkage can be computed by using the same procedure, as explained in fig. 6 for a concentrical load and a symmetrical reinforcement A_s. Because of the substantial amount of additional stress in reinforcement, which can add up to the two or threefold value of the initial stress, a careful design of stirrups is essential.



time t_o

$$N_o = N_{s,o} + N_{c,o}$$

$$\varepsilon_o = \varepsilon_{s,o} = \varepsilon_{c,o}$$

$$\varepsilon_o = \frac{N_{s,o}}{E_s A_s} = \frac{N_{c,o}}{E_c A_c}$$

$$N_{s,o} = n \frac{A_s}{A_{i,o}} N_o \; ; \quad N_{c,o} = \frac{A_c}{A_{i,o}} N_o$$

with $A_{i,o} = A_c + n \cdot A_s$ and $n = \dfrac{E_s}{E_c}$

time t_o to t

$$0 = N_{s,\varphi} + N_{c,\varphi} \qquad \text{Equilibrium}$$

$$\varepsilon_\varphi = \varepsilon_{s,\varphi} = \varepsilon_{c,\varphi} \qquad \text{Compatibility}$$

$$\varepsilon_\varphi = \frac{N_{s,\varphi}}{E_s A_s} = \frac{N_{c,o}}{E_c A_c} \phi_t + \varepsilon_{s,t} + \frac{N_{c,\varphi}}{E_c A_c}(1 + \chi \cdot \phi_t)$$

$$N_{s,\varphi} = -N_{c,\varphi} = \frac{n \frac{A_s}{A_c} \cdot \phi_t \cdot N_{c,o} + E_s A_s \cdot \varepsilon_{s,t}}{1 + n \frac{A_s}{A_c}(1 + \chi \cdot \phi_t)}$$

Fig. 6: Change of internal forces and stresses in reinforced concrete columns

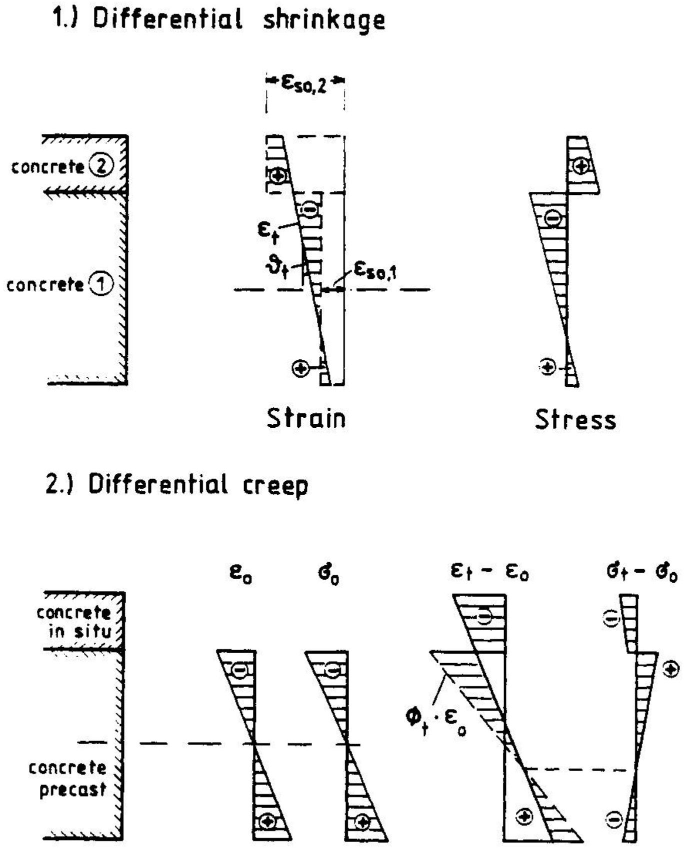


Fig. 7:

Redistributions of stresses in composite sections due to differential shrinkage and creep

In fig. 7 it is explained that the redistribution in composite sections due to differential shrinkage and creep can be calculated following the same principles.

5. Deflections of structural concrete

It is well known that deflections of structural concrete members can be calculated by double integration of the local rotations with $\delta \triangleq f = \iint \vartheta \, dx \, dx$.

The entire rotation of concrete sections including time dependent effects is given in analogy to eq. (3) by

$$\vartheta_t = \frac{M_o}{E_c I_c}(1+\phi_t) + \frac{M_t - M_o}{E_c I_c}(1+\chi\phi_t) = \vartheta_o (1+\phi_t) + \frac{M_{c,\varphi}}{E_c I_c}(1+\chi\phi_t). \qquad (15)$$

This expression contains the elastic rotation at t_o and the total creep rotation, which is known, if the redistribution moment $M_{c,\varphi}$ of the concrete cross section due to creep and shrinkage according to eq. (14) is put in. Then the creep rotation is

$$\vartheta_\varphi = \vartheta_o \cdot \phi_t + \frac{-N_{r,\varphi} \cdot z_{cr}}{E_c I_c}(1+\chi\phi_t). \qquad (16)$$

The steel force due to creep and shrinkage $N_{r,\varphi}$ in this formula is known from eq. (12); compare also fig. 4.

The initial rotation ϑ_o and the initial concrete stress in the steel fibre $\sigma_{cr,o}$ in eq.(12) are dependent on the kind of bond between steel and concrete when permanent loads are implemented. This means for pre-tensioned beams or reinforced beams without prestressing

$$\vartheta_o = \frac{M_o}{E_c I_i}$$

$$\sigma_{cr,o} = \frac{N_o}{A_i} + \frac{M_o}{I_i} \cdot z_{cr}$$

and for post-tensioned beams

$$\vartheta_o \simeq \frac{M_o}{E_c I_c}$$

$$\sigma_{cr,o} \simeq \frac{N_o}{A_c} + \frac{M_o}{I_c} \cdot z_{cr}$$

under the premise that the section properties taking the area of ducts and reinforcing steel into consideration are nearly equal to the concrete section properties A_c and I_c (exact value see [6]). Here N_o and M_o contain all permanent loads including the internal forces due to prestressing as an artificial load.

Thus the rotation of post-tensioned cross sections (compare fig.4) may be written after dividing into the different actions as

$$\vartheta_\varphi = \phi_t \cdot \frac{M_o}{E_c I_c} - \lambda_r \cdot n \cdot \phi_t \cdot \frac{M_o}{E_c I_c} \cdot \frac{A_r z_{cr}^2}{I_c} \cdot (1+\chi\varphi_t)$$

$$- \lambda_r \left[n \cdot \phi_t \cdot \frac{N_o}{A_c} + E_p \epsilon_s + \Delta\sigma_{pr}\right] \frac{A_r z_{cr}}{E_c I_c} \cdot (1+\chi\varphi_t) \qquad (17)$$

with the redistribution parameter λ_r from eq. (13).

Starting with this equation the time dependent rotation ϑ_φ is subdivided into the share ϑ_φ^M (due to the initial bending moment M_o) and the share ϑ_φ^N (due to the centric longitudinal force N_o and shrinkage ϵ_s), which is only caused by an eccentrical reinforcement. This is useful, because the share due to M_o as bending moment of an indeterminate system causes no additional redistribution of the reaction forces and internal forces, which remain "Eigenstresses". Otherwise, the share due to N_o and ϵ_s can produce some correction in the reaction forces in an indeterminate system if the excentricity of the longitudinal reinforcement is taken into account (see [6,8]). But usually in a reinforced structure these refinements are totally neglected.

The share of the bending moment leads from eq. (17) to

$$\vartheta_\varphi^M = \phi_t \cdot \vartheta_o \cdot \left[1-\lambda_r \cdot n \cdot \frac{A_r z_{cr}^2}{I_c} \cdot (1+\chi\phi_t)\right]$$

$$= \phi_t \cdot \vartheta_o \cdot \lambda_r \left[1+n \cdot \frac{A_r}{A_c} \cdot (1+\chi\phi_t)\right]. \qquad (18)$$

According to [6] and in analogy to [9,10] the deflection of post-tensioned structural concrete members in state I with one straight steel layer may be evaluated in relation to the inital elastic deflection $f_o = f_c$

$$f_{\varphi}^{M} = f_o \cdot \phi_t \cdot \lambda \; [1 + n \frac{A_r}{A_c} (1+\chi \cdot \phi_t)] = f_c \cdot \phi_t \cdot c \qquad (19)$$

with the reducing creep deformation coefficient $c < 1$ in the form

$$c = \frac{1 + n \cdot \frac{A_r}{A_c} (1 + \chi\phi_t)}{1 + n \cdot \frac{A_r}{A_c} \cdot (1 + \frac{A_c z_{cr}^2}{I_c}) \cdot (1 + \chi\phi_t)}, \text{ where } A_r = A_s + A_p. \qquad (20)$$

The reference deflections f_c are given in fig. 8c. This formula may also be used for post-tensioned curved tendons, if z_{cr} is taken from the middle of the span, as the variation of z_{cr} is nearly balanced by integration along the beam length. If $\chi = 0.8$ and $\phi_t = 2.5$ and $z_{cr} = 0.4 \cdot h$ are taken as regular values for long-term deflections, the coefficient c is only dependent on the factor $n \cdot A_r/A_c$ which is shown in fig.8a. The corresponding coefficient c for reinforced concrete (without prestressing) according to [9,10] is entered in fig. 8a as a broken line.

Deflections of state II for reinforced concrete may also be calculated with this coefficient c if the section properties of state II are taken into account. The result is also plotted in fig. 8a as a broken line. State II for prestressed members is much more complicated as the imposed longitudinal prestressing force has a strong influence on the shape of the compression zone. But pure state II is not a realistic premise for prestressed members under permanent loads, because as a rule state II is normally limited to short sections along the beam and the tension stiffening effect reduces local rotations even in these sections.

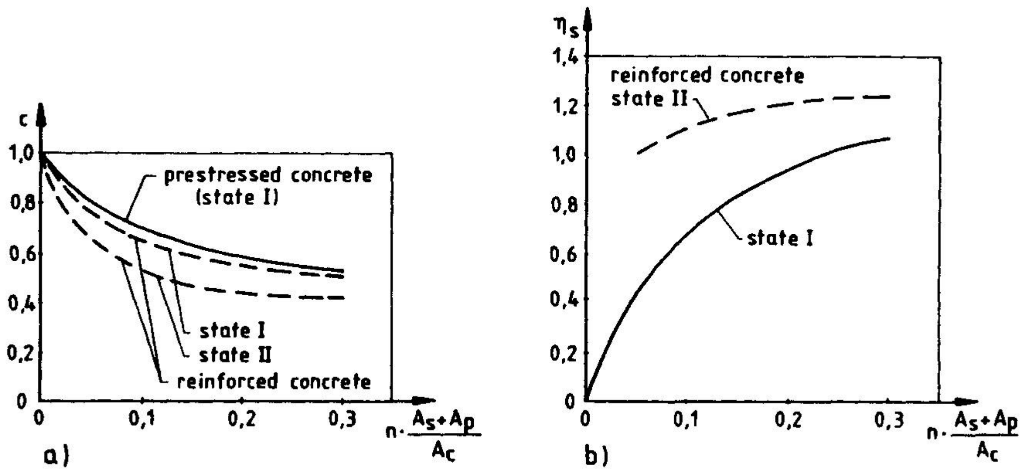
The evaluation of deflections - caused by eccentric reinforcement- due to longitudinal forces including prestressing force and shrinkage can be done in the same way solving eq. (17) in analogy to eq.(18). One has not to distinct between prestressed and reinforced members and describes the local rotations in state I with

$$\vartheta_{\varphi}^{N} = \frac{\phi_t \cdot \frac{N_o}{A_c E_c} + \epsilon_s + \frac{\Delta\sigma_{p,r}}{E_p}}{h} \left[\frac{h}{z_{cr}} \cdot \lambda_r \cdot n \frac{A_r}{A_c} \cdot \frac{A_c z_{cr}^2}{I_c} (1 + \chi\phi_t)\right] \quad (21)$$

The deflections can be directly calculated in analogy to eq. (19)

$$f_{\varphi}^{N} = - \alpha_s \cdot l^2 \cdot \frac{\phi_t \cdot \frac{N_o}{A_c E_c} + \epsilon_s + \frac{\Delta\sigma_{p,r}}{E_p}}{h} \cdot \eta_s \quad (22)$$

with α_s according to fig. 8c as coefficient for curved tendons and restraint conditions.



a: Creep deformation coefficient c acc. eq.(20)
b: Shrinkage deformation coefficient η_s acc. eq.(23)

	$f_c = \alpha \frac{max. M \cdot l^2}{E_c I_c}$	M x	M x	M x	M x
α	x	$\frac{1}{3}$	$\frac{1}{12}$	$\frac{1}{20,12}$	$\frac{1}{24}$
		$\frac{1}{4}$	$\frac{1}{9,6}$	$\frac{1}{23,08}$	$\frac{1}{16}$
		$\frac{1}{5}$	$\frac{1}{9,84}$	$\frac{1}{27,95}$	$\frac{1}{16,39}$
α_s	formula (22)	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{32}$	0
		$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{14}$	$\frac{1}{18}$

c: Coefficient α and α_s

Fig. 8: Coefficients for a rational calculation of deformations in structural concrete members

The shrinkage deformation coefficient η_s for shrinkage ϵ_s and including longitudinal force N_o is given with the dimensionless value

$$\eta_s = \frac{h}{z_{cr}} \cdot \frac{n \cdot \frac{A_r}{A_c} \cdot \frac{A_c z_{cr}^2}{I_c} (1 + \chi\phi_t)}{1 + n \cdot \frac{A_r}{A_c} \cdot (1 + \frac{A_c z_{cr}^2}{I_c}) \cdot (1 + \chi\phi_t)} \tag{23}$$

and can be taken from fig. 8b for mean values $\phi = 2.5$; $\chi = 0.8$ and $z_{cr} = 0.4 \cdot h$ in dependence of the parameter nA_r/A_c.

In consequence there are two ways to calculate the deflections in structural concrete: First the exact method analyzing the integral

$$\delta(t) - \delta_o = \delta_\varphi \mathrel{\hat{=}} f_\varphi = \iint (\vartheta_\varphi^M + \vartheta_\varphi^N) \; dx \; dx$$

with the local rotations according to eqs. (18) and (21) and second the simplified but less accurate solutions using eqs. (19) and (22) under reading off the required coefficients given in fig. 8.

Finally, it should be noted that the values of the coefficients c and η_s for the long-term deflections are strongly reduced if a compression reinforcement with the ratio A_s'/A_s is taken into account (compare [9,10]), while this is insignificant for the elastic or initial deformation. Due to the restraining effect of the existing bonded reinforcement the creep deflection caused by bending is only a fraction of the ϕ-fold initial one and the differences between state I and II are reduced with time, otherwise the deformation due to axial forces and shrinkage is only produced by the eccentricity of the resulting reinforcement in the structural member.

6. Summarizing commentary

With the described methods one can assess the influence of time-dependent material properties, especially the effects of limit values of the final creep and shrinkage coefficients, considering the known uncertainties in the design process.

For the whole range of the structural concrete, one could give the following recommendation: Only as much prestressing as necessary in order to achieve a favourable behaviour in the service state, but not less reinforcement as reasonable in order to assure the durability and the reliability concerning crack width control.

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