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Creep Effects on Structural Concrete

Effets différés dans les structures en béton

Auswirkungen des zeitabhängigen Materialverhaltens auf Betonverbundtragwerke

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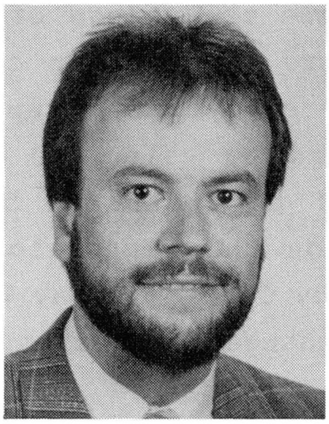

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SUMMARY

Starting with an analytical stress-strain-time relation of concrete, the numerical analysis of time-dependent effects on structural concrete using an equivalent-stiffness method and an iterative method are explained. If several abrupt loadings have to be superimposed these methods can be used with the help of an incremental constitutive law.

RÉSUMÉ

La relation algébrique contenant contrainte, déformation et temps intervient dans l'analyse numérique des effets différés du béton, et ceci en tant qu'élément de base; l'utilisation de la rigidité équivalente est expliquée, conjointement à la méthode itérative utilisée. Une relation constitutive est introduite afin de tenir compte de la superposition de plusieurs sauts de contrainte.

ZUSAMMENFASSUNG

Ausgehend von der algebraischen Spannungs-Dehnungs-Zeit Beziehung für Beton wird die numerische Berechnung der Auswirkungen des zeitabhängigen Betonverhaltens sowohl für die Methode der äquivalenten Steifigkeiten als auch für die iterative Methode erläutert. Anschliessend wird eine bei Anwendung dieser Methoden für die Superposition mehrerer Spannungssprünge vorteilhafte inkrementelle konstitutive Beziehung vorgestellt.

1. INTRODUCTION

The analysis of structural concrete at serviceability limit state regarding deflections, stress distribution and cracking due to creep and shrinkage is essential for a good performance. Nowadays two groups of methods are used for the numerical analysis of time-dependent effects on structural concrete. The first of them is the step-by-step method which is most often used for computer programs and allows to calculate the change of stress in short time steps. According to the nature of creep in concrete the change of stress has to be stored for every time step and every cross section or point of the structure to calculate the creep strain in further time steps. To avoid this storage of huge numbers of data the second method based on the algebraic stress-strain-time relation which was introduced by Trost [1] may be used. This relation allows the calculation of changes of stress as a result of creep and shrinkage in only one time step.

2. QUASI-ELASTIC METHODS OF CALCULATION OF TIME-DEPENDENT EFFECTS

Creep problems are generally solved by the incremental step-by-step analysis of structural concrete as a sequence of elasticity problems. Then the stress-strain relation within a time step is described by a linear function using the rectangle or the trapezoidal rule for time integration. These linear functions may be used to formulate an incremental elastic modulus according to the type of time integration (see [2]). The incremental method with short time steps based on the history integral is associated with the disadvantage that every preceding value of all stress components for each finite element must be stored. This may be reduced using differential-type formulations for the storage of stress history. But then computing time is still very long because only creep functions composed of e-functions can be used and short time steps are conditional.

The key idea to overcome these disadvantages and to obtain an efficient algorithm was to formulate an incremental constitutive law for long time steps in analogy to the stress-strain-time relation formulated by Trost [1]. Starting with

$$\varepsilon(t) = \frac{\sigma_c(t_0)}{E_c} [1 + \varphi(t,t_0)] + \frac{\sigma_c(t) - \sigma_c(t_0)}{E_c} [1 + \chi(t,t_0)\ \varphi(t,t_0)],$$

subtracting the elastic strain at loading age and replacing the change of strain $\varepsilon(t) - \varepsilon(t_0)$ by $\Delta\varepsilon(t,t_0)$ respectively the change of stress by $\Delta\sigma(t,t_0)$ results in

$$\Delta\varepsilon(t,t_0) = \frac{\sigma_c(t_0)}{E_c} \varphi(t,t_0) + \frac{\Delta\sigma_c(t,t_0)}{E_c} [1 + \chi(t,t_0)\ \varphi(t,t_0)] \qquad (1)$$

This equation describes the physically measurable change of strain in the time interval from t_0 to t as a sum of three fictitious strains: First the unrestrained creep strain caused by the abrupt stress change at t_0 which is furthermore called primary creep. Second the elastic strain and third the unrestrained creep strain caused by the steady change of stress in the time interval from t_0 to t. The third part as well as the steady stress change itself is caused by primary creep and is therefore called secondary creep.

Two different ways of time dependent analysis of structural concrete may be adopted simply by transforming eq. (1). Furthermore the first of them is called equivalent-stiffness method. It requires the following conversion of eq. (1):

$$\Delta\sigma_c(t,t_0) = \frac{E_c}{1 + \chi(t,t_0)\ \varphi(t,t_0)} \left[\Delta\varepsilon(t,t_0) - \frac{\sigma_c(t_0)}{E_c} \varphi(t,t_0) \right]$$

This equation may be written as

$$\Delta\sigma_c(t,t_0) = E_{AAEM}\ \Delta\varepsilon(t,t_0) - E_{AAEM} \frac{\sigma_c(t_0)}{E_c} \varphi(t,t_0) \qquad (2)$$

with the age adjusted effective modulus (AAEM, see [2])

$$E_{AAEM} = \frac{E_c}{1 + \chi(t,t_0)\ \varphi(t,t_0)} \qquad (3)$$

The first part of eq. (2) corresponds entirely with Hooke's law and the second part contains the primary creep strain due to $\sigma_b(t_0)$ which can be treated like an imposed strain due to change of temperature. Thus the change of stress and strain in the time interval from t_0 to t may be calculated by the equivalent-stiffness method by considering primary creep as an imposed load and by modification of either the elastic modulus or more generally of the stiffness matrix of the structure in a way that secondary creep is enclosed. Then it is possible to calculate the change of stress and strain with conventional methods based on elastic theory.

The second method is called iterative method and may be explained with equation

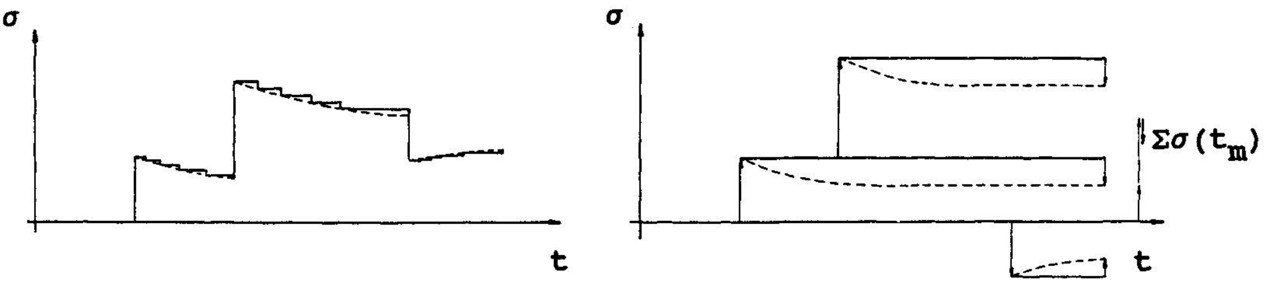
$$\Delta\sigma_c(t,t_0) = E_c\left[\Delta\varepsilon(t,t_0) - \frac{\sigma_c(t_0)}{E_c}\varphi(t,t_0) - \frac{\Delta\sigma_c(t,t_0)}{E_c}\chi(t,t_0)\varphi(t,t_0)\right] \quad (4)$$

which can easily be obtained from eq. (1). The first part of eq. (4) is identical with Hooke's law, the second part contains primary creep and the third part secondary creep.

The first iteration may be done setting $\Delta\sigma_c(t,t_0) = 0$ on the right side of eq. (4). Thus changes of stress and strain caused by primary creep as external load in the time step from t_0 to t can approximately be calculated with conventional methods based on elastic theory. Then the third part of eq. (4) can be estimated with $\Delta\sigma_c(t,t_0)$ taken from the results of the first iteration step. The second iteration step follows with an improved external load containing now the sum of primary and secondary creep. Thus it is possible to improve the change of stress and strain and the load including secondary creep iteratively. Zienkiewicz [3] described this method already but he didn't take $\chi(t,t_0)$ into consideration. Therefore he had to limit the method to short time intervals with $\chi(t,t_0) \approx 1$ [4] what is not generally necessary.

3. SUPERIMPOSING OF SEVERAL ABRUPT LOADINGS

The superposition of several abrupt loads is easily possible if any step-by-step method is used. In connection with the algebraic stress-strain relation of Trost [1] or equations (2) or (4) several abrupt loads have to be treated separately according figure 1b and summed up for time t in question. If stress or strain of any other time is needed the whole calculation has to be carried out again. These difficulties may be overcome by a combination of both methods using the incremental structure of the step-by-step method and the long time steps of Trost's method. Incremental structure means that the time axis is subdivided in intervals of any length and that the stress history is evaluated step after step from the beginning.



a) step-by-step method b) algebraic relation (Trost)

Fig.1 Superposition of several loads according to different methods

Limits of the time intervals should be set when abrupt loadings are implemented, when the cross section or the restraint conditions are modified and when values of stress or strain are needed. This means that the length of time intervals is optional - short or long.

4. INCREMENTAL METHOD WITH LONG TIME STEPS

An algebraic stress-strain-relation for the time interval from t_{m-1} to t_m which is needed for the incremental method was defined in [4]. It can be obtained from the integral equation (see [1]) by formulating the strain at t_m and subtracting the strain at t_{m-1} from it. Introducing the coefficient $\chi(t_m,t_{m-1})$ (exactly conform to the relaxation coefficient of Trost $\chi(t,t_0)$), the aging coefficient for the elastic modulus $k_E(t_m,t_{m-1})$ and the incremental aging coefficient for primary creep $\chi(t_m-t_{m-1},t_j)$ the remaining integrals can be transformed to algebraic expressions in analogy to Trost's algebraic relation.

The resulting constitutive law for uniaxial stress runs as follows

$$\Delta\varepsilon_c(t_m,t_{m-1}) = \Delta\varepsilon_{0,\varphi}(t_m,t_{m-1}) + \Delta\varepsilon_{0,S}(t_m,t_{m-1}) + \Delta\varepsilon_{0,T}(t_m,t_{m-1}) + \frac{\Delta\sigma_{c\varphi}(t_m,t_{m-1})}{E_c}\cdot\left[\frac{E_c\,k_E(t_m,t_{m-1})}{E_c(t_{m-1})} + \rho(t_m,t_{m-1})\,\varphi(t_m,t_{m-1})\right] \quad (5)$$

with the primary creep

$$\Delta\varepsilon_{0,\varphi}(t_m,t_{m-1}) = \sum_{j=1}^{m-1}\frac{\Delta\sigma_{cL}(t_j)}{E_c}\cdot[\varphi(t_m,t_j) - \varphi(t_{m-1},t_j)] + \sum_{j=1}^{m-2}\frac{\Delta\sigma_{c\varphi}(t_{j+1},t_j)}{E_c}\,\rho(t_m-t_{m-1},t_j)\,[\varphi(t_m,t_j) - \varphi(t_{m-1},t_j)] \quad (6)$$

and the unrestrained changes of shrinkage strain $\Delta\varepsilon_{0,S}(t_m,t_{m-1})$ and temperature strain $\Delta\varepsilon_{0,T}(t_m,t_{m-1})$ in the time interval from t_{m-1} to

t_m. Eq. (6) describes the unrestrained creep strain caused by changes of stress in the past, subdivided into abrupt changes $\Delta\sigma_{cL}(t_j)$ and steady changes $\Delta\sigma_{c\varphi}(t_{j+1},t_j)$. These steady changes took place during the time interval from t_j to t_{j+1} and do not exactly fit to the loading age t_j in eq. (6), but the incremental aging coefficient $\chi(t_m-t_{m-1},t_j)$ acts as a correction factor in this case. It's approximate value is 1,0 [4], differing from the well known average value of $\chi(t_m,t_{m-1}) = 0{,}8$ for the aging coefficient defined by Trost [1].

In consequence of the analogy between equations (5) and (1) the equivalent-stiffness and the iterative method may both be used in connection with the incremental constitutive law for the analysis of time-dependent effects in structural concrete. The first condition is that primary and secondary creep are strictly separated like in eq. (5) and treated properly as set forth in chapter 2.

5. CONCLUSIONS

Any algorithm or computer program based on elastic theory may be used for the analysis of time-dependent effects in structural concrete if quasi-elastic methods in connection with a suitable constitutive law are used. An incremental stress-strain-time relation like eq. (5) renders long time steps possible and reduces the necessity to store a lot of values in connection with short computing time. Some practical examples dealing with loss of prestress, stress redistribution in an inhomogenous cross section and bridges build in sections can be found in [4].

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