Imposed deformation and cracking

Autor(en): **Reinhardt, H.W.**

Objekttyp: **Article**

Zeitschrift: **IABSE reports = Rapports AIPC = IVBH Berichte**

Band (Jahr): **62 (1991)**

PDF erstellt am: **23.07.2024**

Persistenter Link: https://doi.org/10.5169/seals-47626

Nutzungsbedingungen

Die ETH-Bibliothek ist Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Inhalten der Zeitschriften. Die Rechte liegen in der Regel bei den Herausgebern.
Die auf der Plattform e-periodica veröffentlichten Dokumente stehen für nicht-kommerzielle Zwecke in Lehre und Forschung sowie für die private Nutzung frei zur Verfügung. Einzelne Dateien oder Ausdrucke aus diesem Angebot können zusammen mit diesen Nutzungsbedingungen und den korrekten Herkunftsbezeichnungen weitergegeben werden.
Das Veröffentlichen von Bildern in Print- und Online-Publikationen ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. Die systematische Speicherung von Teilen des elektronischen Angebots auf anderen Servern bedarf ebenfalls des schriftlichen Einverständnisses der Rechteinhaber.

Haftungsausschluss

Alle Angaben erfolgen ohne Gewähr für Vollständigkeit oder Richtigkeit. Es wird keine Haftung übernommen für Schäden durch die Verwendung von Informationen aus diesem Online-Angebot oder durch das Fehlen von Informationen. Dies gilt auch für Inhalte Dritter, die über dieses Angebot zugänglich sind.

Ein Dienst der *ETH-Bibliothek*
ETH Zürich, Rämistrasse 101, 8092 Zürich, Schweiz, www.library.ethz.ch

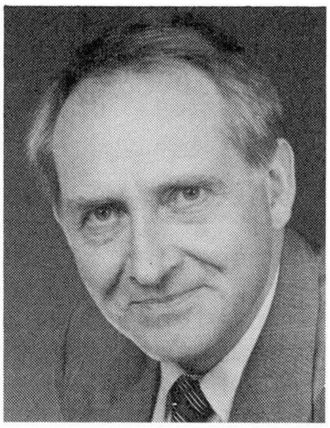
http://www.e-periodica.ch

Imposed Deformation and Cracking

Fissuration par déformation imposée

Rissbildung durch Zwang

H. W. REINHARDT
Prof. Dr.
Otto Graf Institute
Stuttgart, Germany



H. W. Reinhardt, born 1939, obtained his civil engineering degree at Stuttgart University. He performed research on concrete and concrete structures at Delft and Darmstadt. Since April 1990, he has been Professor for materials at Stuttgart University and the managing director of the Otto Graf Institute.

SUMMARY

Attention is focused on imposed deformation which may lead to cracking of concrete. Some typical situations are: stresses due to nonlinear temperature and shrinkage distribution, imposed deformation, and deformation with internal and external restraint. Some practical cases illustrate the relevance of these effects.

RÉSUMÉ

L'auteur traite les déformations imposées pouvant causer la fissuration du béton. On distingue des situations typiques: contraintes par distribution non-linéaire de la température et par retrait, déformation imposée, et déformation avec empêchement intérieur. Quelques cas pratiques illustrent l'importance de ces actions.

ZUSAMMENFASSUNG

Zentrischer Zwang mit möglicher Rissbildung wird behandelt. Dabei werden typische Fälle unterschieden: Spannungen infolge nichtlinearer Verteilung von Temperatur und Schwinden, aufgezwungene Verschiebung, innerer und äusserer Zwang. Praktische Beispiele verdeutlichen die Bedeutung dieser Belastungen.

1. MOTIVE AND SCOPE

There is an increasing number of structures with great demands on serviceability, especially on gas-tightness and liquid-tightness. We can think about structures for environmental protection such as catch basins, waste disposal sites, interim storage sites, treatment plants for contaminated water, about structures in chemical plants of refineries, but also about basements, pipes and ducts located in contaminated ground water. The common feature of these concrete structures is that they serve at least two purposes. First, they are structures which carry dead and life load, and second, they ought to be impermeable against gasses and fluids during a certain time. A main concern is cracking, i.e. occurence of cracks and crack width.

Besides direct actions which are usually taken into account in designing there are indirect actions originating in imposed and restraint deformations. Causes can be due to shrinkage and swelling of concrete, thermal movements, chemical reactions, and differential settlement. This contribution deals with eigenstresses and actions due to imposed and restraint deformation. It will be shown that the boundary conditions have an essential influence on these actions. Practical examples will illustrate the findings.

2. TYPE OF ACTIONS

2.1 Non-linear temperature distribution

Heating and cooling of concrete elements due to air temperature variation and thermal radiation cause non-linear temperature distribution.

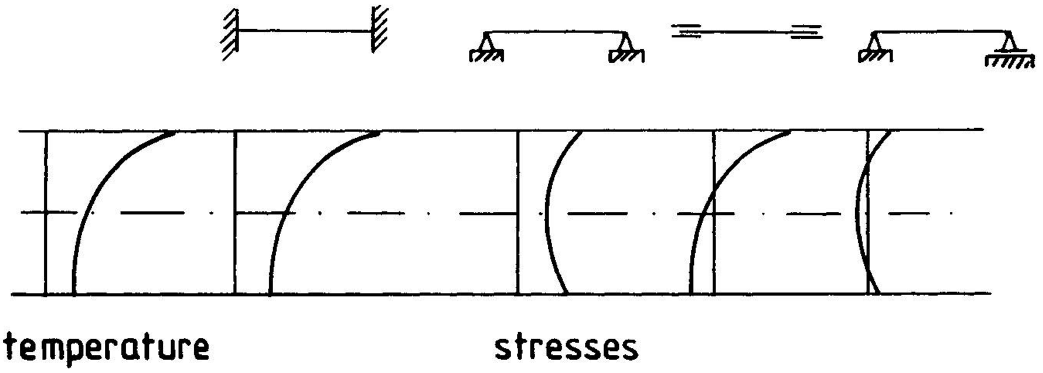


Fig. 1 Stresses due to non-linear temperature distribution with various boundary conditions.

Fig. 1 shows a typical example of a concrete slab on a foundation during nightly cooling. The surface temperature is lowest. Depending upon the boundary conditions the stress distribution is different. Even if the supports allow rotation and translation there are eigenstresses in the homogeneous cross-section, in this case tensile stresses at the surface.

In a composite cross-section with layers of material with different coefficients of thermal expansion there will be eigenstresses even if the temperature is raised uniformly. At the boundaries of the layers shear stresses develop.

Similarly to non-linear temperature distribution, drying and wetting of concrete cause eigenstresses due to shrinkage. However, the process is much slower. The governing parameter for temperature is the thermal diffusivity which is about $1*10^{-6} m^2\ s^{-1}$, and, for shrinkage, it is the diffusion coefficient which is about $2*10^{-10}\ m^2\ s^{-1}$. This means that shrinkage is about 5000 times slower than temperature movement.

2.2 Imposed deformation

If various concrete elements which are connected to each other undergo different thermal and shrinkage history there may mutually impose deformation. For instance, if an external column is heated up while the internal columns stay at the same temperature a deformation will be imposed on the slab resting on these columns. Those deformations are most relevant which cause tensile forces in an element or excentricity in a compressive part.

Common imposed deformations are due to differential settlement of hyperstatic structures.

The reaction of a structural member to an imposed deformation δ can be illustrated by Fig. 2. If a displacement δ_0 is imposed the member will crack

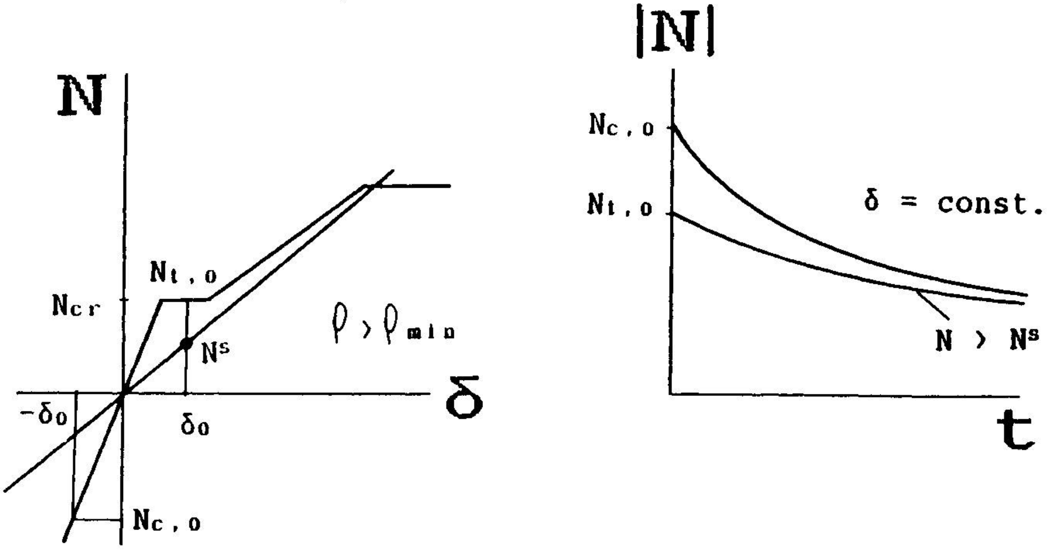


Fig. 2 a) Normal force vs. imposed deformation
b) Normal force vs. time with constant deformation

at N_{cr} (tension) or will react with $N_{c,o}$ (compression). The forces will decrease due to relaxation of the concrete. In case of tension the force cannot become lower than N^s. If the reinforcement ratio is smaller than $\delta_{min} = f_t/f_{sy}$ the steel will yield after cracking remaining at a constant force $N_y = A_s\ f_{sy}$. This is shown by Fig. 3

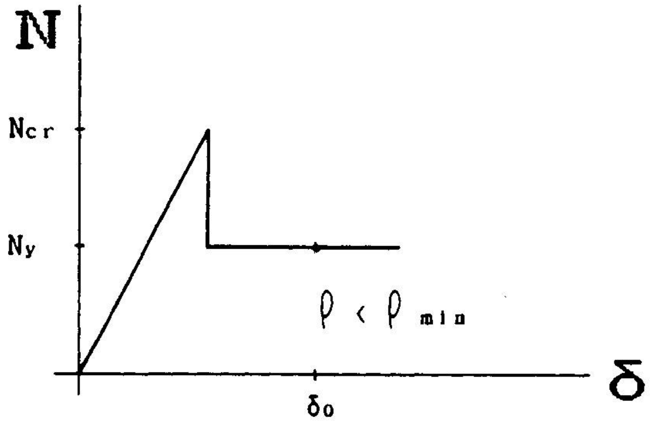


Fig. 3 Normal force vs. imposed deformation for $\delta < \delta$min.

To judge the effect of imposed deformations realistically the correct stiffness of the member has to be chosen. An analysis with initial elastic stiffness may lead to a great overestimation of reaction forces. Crack width and crack spacing can be obtained in the same way as for imposed loads [1].

2.3 Restraint deformation

If deformation is caused by temperature, humidity, chemical reactions, i.e effects which cause length changes of the concrete, and if this deformation cannot occur freely stresses will develop in the cross-section. There are two different cases: the movement will be restraint externally by supports or internally by reinforcement. The two cases lead to quite different stresses and crack widths. This will be elaborated in more detail in the following chapter.

3. UNIFORM SHRINKAGE IN A REINFORCED MEMBER

3.1 Internal restraint

It is assumed that the concrete shrinks and that this movement is restraint by embedded bars. Since the concrete member is simply supported there will be no external forces. Steel strain and concrete strains are equal, $\epsilon_s = \epsilon_c$. The forces are equal with opposite sign, $N_c = -N_s$. The concrete strain consists of the initial shrinkage strain ϵ_0 and the elastic strain due to composite action:

$$\epsilon_c = \frac{N_c}{A_c\ E_c} + \epsilon_0 \qquad (1)$$

The steel strain is given by

$$\epsilon_s = \frac{N_s}{E_s\ A_s} \tag{2}$$

With $\rho = A_s/A_c$, $n = E_s/E_c$, $\epsilon_s = \epsilon_c$ it yields

$$\epsilon_s = \frac{E_c\ A_c}{E_s A_s + E_c A_c} = \frac{1}{1 + n\rho}\ \epsilon_0 \tag{3}$$

The elastic stresses are

$$\sigma_s = \frac{E_s\ \epsilon_0}{1 + n\rho} \quad \text{and} \quad \sigma_c = -\ \rho\ \sigma_s \tag{4}$$

As soon as the tensile strength of concrete is reached cracks will develop. This is true for $\sigma_c = f_{ct}$ and the appropriate strain ϵ_0^{CR} becomes

$$\epsilon_0^{CR} = \frac{1 + n\rho}{\rho}\ \frac{f_{ct}}{E_s} = \frac{1 + n\rho}{n\rho}\ \frac{f_{ct}}{E_c} \tag{5}$$

In the vicinity of a crack, bond stresses between steel and concrete are activated. Assuming a constant bond stress τ_0 (for a more accurate treatment see [1]) the stress is linearly distributed between two cracks with the maximum $\sigma_{c,max} = f_{ct}$. The elastic steel deformation is then

$$\Delta l_s = \frac{1}{2}\ \frac{1 + n\rho}{\rho}\ \frac{f_{ct}}{E_s}\ l_{CR} \tag{6}$$

and the mean steel strain

$$\bar{\epsilon}_s = \frac{\Delta l_s}{l_{CR}} = \frac{1}{2}\ \frac{1 + n\rho}{\rho}\ \frac{f_{ct}}{E_s} \tag{7}$$

with l_{CR} = crack spacing.

The mean strain of concrete is at this moment

$$\bar{\epsilon}_c = \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s} - \frac{1}{2} \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s}$$

$$= \frac{1}{2} \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s} \qquad (8)$$

Mean steel and concrete strain are the same, the crack width is zero.

With increasing ϵ_0, concrete will slip on the steel since τ_0 = const. Concrete strain increases according to

$$\epsilon_c = \epsilon_0 - \frac{1}{2} \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s} \qquad (9)$$

while steel strain remains the same. The mean crack width becomes

$$\bar{w} = l_{CR} \left(\bar{\epsilon}_c - \bar{\epsilon}_s\right) = l_{CR} \left(\epsilon_0 - \frac{1 + n\rho}{\rho} \frac{f_{ct}}{E_s}\right) \qquad (10)$$

Crack spacing is given by

$$l_{CR} = \frac{1}{2} \frac{f_{ct}}{\tau_0} \frac{d_s}{\rho} \qquad (11)$$

with d_s = bar diameter. Three stages can be distinguished if concrete shrinkage increases continuously (see Fig. 4). These are:

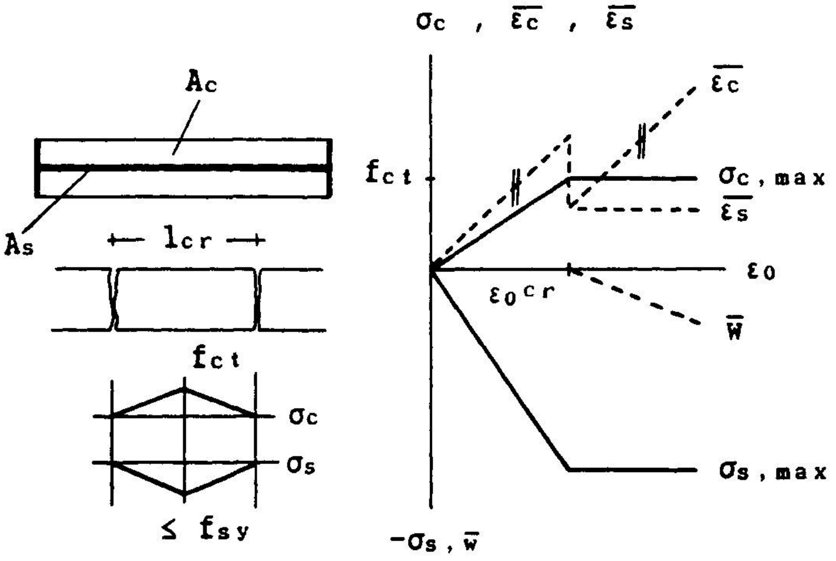


Fig. 4 Stress, strain, and crack width vs. initial strain ϵ_0

1) No cracks, shortening of the composite member.
2) At cracking, the member length increases again; crack width is zero.
3) The length of the bar remains constant and the crack width increases.

It should be noted that $\varrho \geq f_{ct}/f_{sy}$ because otherwise the steel would yield.

3.2 External (and internal) restraint

If the composite member is fixed at the ends and shrinkage occurs then

$$\sigma_c = -\epsilon_0 E_c \quad \text{and} \quad \sigma_s = 0 \qquad (12)$$

Cracks develop at $\sigma_c = f_{ct}$. Then, concrete gets shorter and forces the steel to become shorter. Since the ends are fixed a tensile force will develop in the steel. Steel stress and strain are

$$\sigma_{s,max} = -\frac{1+n\varrho}{\varrho} f_{ct} \quad \text{and} \quad \bar{\epsilon}_s = -\frac{1}{2}\frac{1+n\varrho}{\varrho}\frac{f_{ct}}{E_s} \qquad (13)$$

The elastic elongation $\bar{\epsilon}_s = \frac{\sigma_s^{CR}}{E_s}$ has to cancel the shortening which makes

$$\sigma_s^{CR} = \frac{1}{2}\frac{1+n\varrho}{\varrho} f_{ct} \qquad (14)$$

which is half as much as in the simply supported case. However, the sign of the stress changes along the bar. The crack width can be calculated from the initial strain minus elastic elongation of concrete:

$$\bar{w} = l_{CR}\left(\epsilon_0 - \frac{1+n\varrho}{2}\frac{f_{ct}}{E_c}\right) \qquad (15)$$

Comparing eqs. (10) and (15) tells that crack width is larger in case of external restraint than it is at internal restraint. Crack spacing is the same. The minimum reinforcement ratio at external restraint is half of the one at internal restraint. The schematics of this situation are given by Fig. 5.

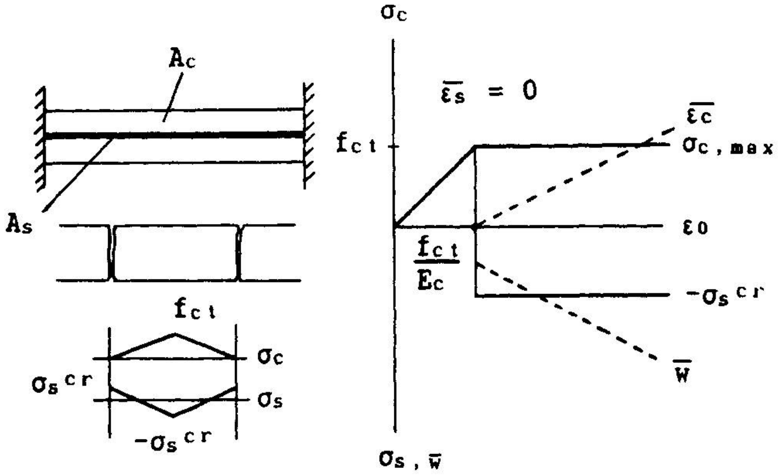


Fig. 5 Stress, strain and crack width vs. initial strain ϵ_0

3.3 General remark

The derivations made in the two preceeding chapters are intentionally based on crude material modelling in order to make the main point clear, the importance of the boundary condition. The results can be improved by assuming time dependent properties of concrete [2], realistic bond behaviour, and scatter of material properties.

4. PRACTICAL CASES

4.1 Car-park

The roof of an underground parking facility consists of a reinforced concrete slab supported by beams. On the roof, there is a water drainage but no isolation. Fig. 6 shows the situation after a year of service. Cracks have developed in the beam due to differential shrinkage. The slab does not shrink, it may even swell whereas the beam does shrink due to heating and ventilation of the parking deck. The crack width is the largest at places with no longitudinal reinforcement (w = 0.3 to 0.5 mm) and smallest near the longitudinal bars (w = 0.1 to 0.2 mm).

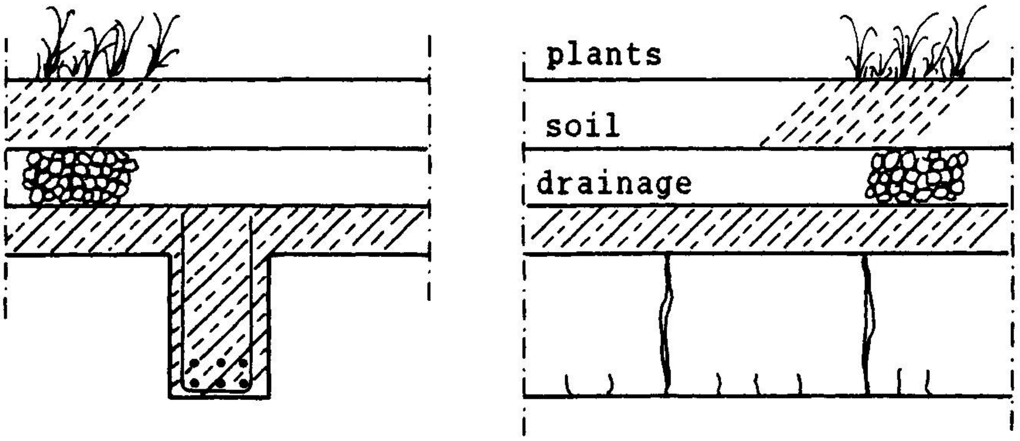


Fig. 6 Cracks in the beam under the roof slab

4.2 Baking furnace

Anodes for the electrolytic production of aluminium are manufactured in a baking furnace at about 1050°C. Although the interior of the furnace is strongly insulated the concrete structure is warmed up to about 200°C at the inner face. Temperature distribution gives rise to curvature of the walls (see Fig. 7). The horizontal displacement was large enough to touch the columns of the superstructure. The walls cracked and the columns were loaded by an almost constant force, i.e. imposed deformation on the walls and imposed force on the columns.

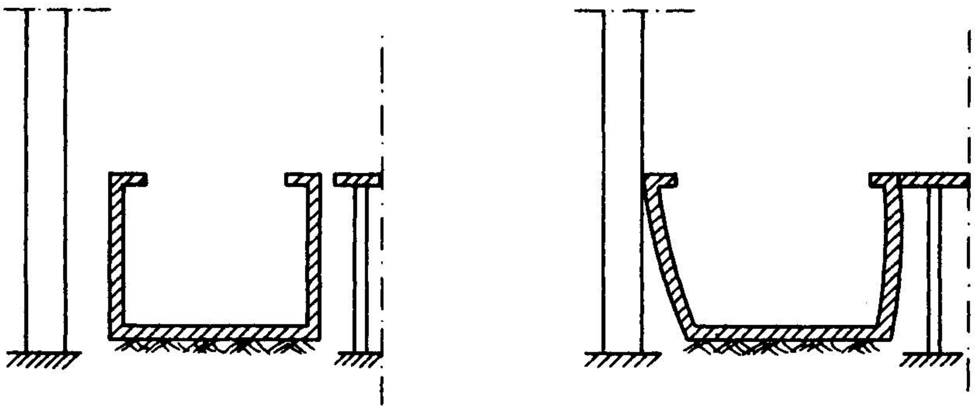


Fig. 7 Baking furnace with superstructure

4.3 Sedimentation-basin

The sedimentation basin of a process-water treatment plant cracked due to temperature differences between the hot water (38°C) and the cold air and structure. Fig. 8 shows the basin with vertical cracks at the

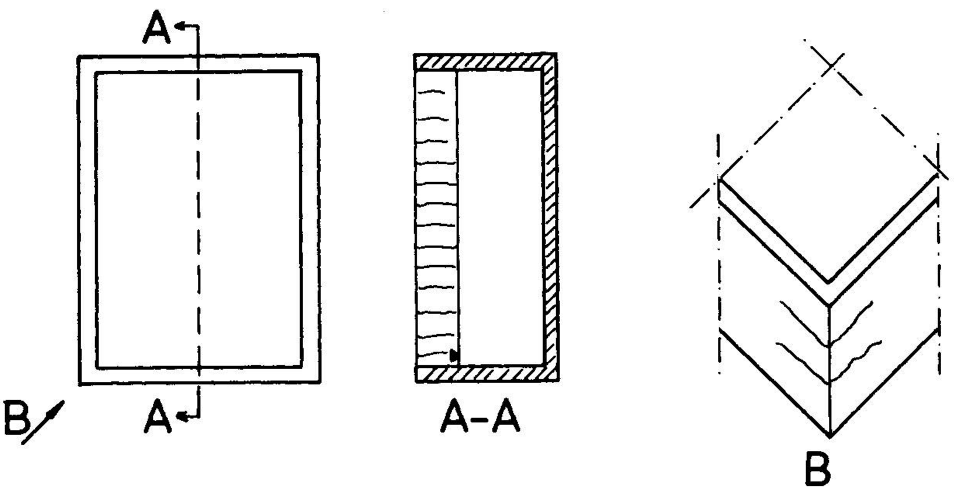


Fig. 8 Sedimentation-basin with cracks indicated

crown of the walls and horizontal cracks at the edges. The basin has been designed and constructed very carefully without any cracks due to early thermal movement. However, during operation cracks appeared the cause of which are rather obvious. There are thermal stresses in wall direction because the lower part of the structure warms up while the top part does not. In the middle of the basin the wall can bend and rotate freely whereas the edges restrain the curvature. Here the inner edge expands due to heating and the outer edge is stressed in such a way that cracks occur. It will be interesting to observe the cracks and to see whether they propagate into the concrete due to inelastic cyclic loading [3].

5. CONCLUSION AND OUTLOOK

Imposed deformations are usually treated as secondary effects which may be neglected in design of concrete structures. However, these effects should receive due attention in all cases where tightness against gasses and fluids play an important role.

It has been shown how cracks develop and that crack width is greatly influenced by boundary conditions. Models should be developed which enable the structural engineer to judge the behaviour of a structure under restraint deformations, similarly to what has been developed for imposed loads.

6. REFERENCES

[1] BRUGGELING, A.S.G.; Theorie en praktÿk van het gewapend beton. Uitgave PBF, 's-Hertogenbosch 1985.

[2] TROST, H.; Creep relaxation and shrinkage of structural concrete. Introductory Report to this colloquium.

[3] HORDIJK, D.A.; REINHARDT, H.W.; Growth of discrete cracks in concrete under fatigue loading. "Toughening Mechanisms in Quasi-Brittle Materials", ed. S.P.Shah, Evanston 1990, pp 553-568.