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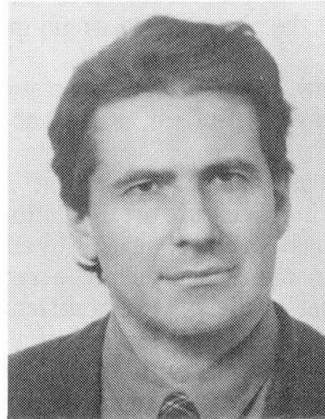
Influence of Tension Stiffening on Behaviour of Structures

Influence de la contribution du béton tendu sur le comportement des structures

Der Einfluss des Tension-Stiffening-Effekts auf das Verhalten von Tragwerken

Aldo CAUVIN
Prof. of Struct. Eng.
Univ. of Pavia
Pavia, Italy

Aldo Cauvin, born 1939, obtained his Civil Engineering Doctorate from Politecnico in Milan and is Full Professor since 1986. Author of several papers on nonlinear analysis of reinforced concrete structures and on buckling. Active for several years in the CEB Commission 2 and in the «Council on Tall Buildings» Committee 22. Currently devoted mainly to the problem of practical applications of nonlinear analysis in design.

SUMMARY

The problem of a simplified simulation of the so-called «Tension Stiffening» effect in the nonlinear analysis of reinforced concrete structures is treated. Its importance on the behaviour of Structural Concrete is emphasized, with reference to different kinds of loads and limit states. The uncertainties in the evaluation of the parameters which influence cracking of concrete and tension stiffening are discussed, with reference to the necessity to perform a safe design using simplified methods of nonlinear analysis.

RÉSUMÉ

L'article concerne la simulation par une méthode simple de l'effet de la contribution du béton tendu dans l'analyse non-linéaire des structures en béton armé fissurées. Son importance est mise en évidence, et ceci par rapport aux différentes situations de charge et d'appui. Les incertitudes dans l'évaluation des paramètres influençant la fissuration du béton et la contribution du béton tendu sont prises en compte par rapport à la nécessité de réaliser un projet qui respecte les conditions de sécurité, tout en utilisant des méthodes simplifiées d'analyse non-linéaire.

ZUSAMMENFASSUNG

Es wird das Problem der vereinfachten Erfassung des sogenannten «Tension stiffening» Effekts bei der Analyse von Stahlbetontragwerken behandelt. Die Wichtigkeit dieses Effekts für die Reaktion des Konstruktionsbetons wird unter Bezugnahme auf Belastungen und Grenzzustände hervorgehoben. Die Unsicherheit bei der Einschätzung der Parameter, die die Rissbildung des Betons beeinflussen, und das «tension stiffening» werden im Zusammenhang mit der Notwendigkeit betrachtet, ein sicheres Bemessungskonzept zu erarbeiten, das vereinfachte Methoden der nichtlinearen Analyse verwendet.

1. INTRODUCTION

As known concrete in tension between cracks has a stiffening effect on cracked reinforced concrete members.

As experience in nonlinear analysis of r.c. practical structures has demonstrated this phenomenon, known as "tension stiffening", may have a considerable influence on results both in terms of displacements and action effects.

On the other hand "tension stiffening" is not easy to simulate not only because it is in itself a complicated mechanism of interaction between the two materials, but also because it is necessarily based upon the cracking pattern which in turn depends on tensile strength of concrete (variable whose dispersion is very high), distribution and size of tensile reinforcement, and also the so called "size effect", according to the recent developments of fracture mechanics.

It is not true that disregarding or underevaluating tension stiffening will always lead to conservative results: certainly the opposite is true in some cases such as the evaluation of stresses due to thermal variations, where the disregarding of the phenomenon would inevitably produce grossly underevalued action effects.

In this discussion we intend to draw some relevant conclusion on the subject, basing on the accumulated experience, recent investigations in this specific field, and the objectives of this Colloquium[1][2] on which the author is partially in agreement.

As one of the basic objectives is to individuate a "trasparent" model for the design of structural concrete, it is necessary to choose, among the many approaches to the simulation of structural behaviour, the ones which best fit the need for both simplicity and adequate accuracy, meaning by adequate a degree of accuracy suitable for design. Therefore the possible approaches to nonlinear analysis in the cracked stage will be first examined and classified.

Secondly, as an example, a "trasparent" model of tension stiffening simulation will be briefly described, which seems to meet the objectives of simplicity and sufficient accuracy for design purposes.

Thirdly the importance of an adequate simulation of tension stiffening within the framework of a simplified method of nonlinear analysis is emphasized and an example is given.

At last the uncertainties in the assumption of basic input data are considered and final conclusions are drawn.

2. NONLINEAR ANALYSIS WITH REFERENCE TO CRACKING-SMEARED AND DISCRETE MODELS

As well known, the static-dynamic behaviour of reinforced concrete structures is markedly nonlinear in nature even in the elastic stage and difficult to model with accuracy, due to the micro-anisotropic, quasi brittle nature of concrete and the composite nature of the material. Problems arise mainly from uncertainties in definition of concrete constitutive law, complexities in interaction phenomena between the two materials, extensive cracking in tensile areas.

To describe this behaviour two different basic approaches are possible (fig. 1), using micro or macro-elements.

Only the first approach, coupled with a "discrete" representation of cracking, can (potentially at least) describe accurately the interaction between the two materials, as each element model a very small area of steel or concrete and the discontinuous nature of cracking is described.

This approach however can only be used in simulation of laboratory tests and for very simple structures or structural elements and is unsuited for design and analysis of real structures.

It is therefore necessary to formulate the stiffness of macro-elements of r.c. (and not concrete or steel only) which takes into account as well as possible the previously mentioned local phenomena.

In this formulation a "smeared" approach needs be adopted and therefore tension stiffening must be introduced in a suitable way.

Several approaches are possible[13]. In the following paragraph a simple and "trasparent" method is briefly described. More details can be found in[7],[8] and [12].

3. AN EXAMPLE OF "TRASPARENT" MODEL FOR "TENSION STIFFENING"

The problem of "tension stiffening" is concerned with the behaviour of steel bars embedded in concrete and subject to tensile forces. In a given crack, all the tension is trasmitted by the bars; between one crack and the following part of the tension is transferred to concrete, thus reducing the stresses in the steel.
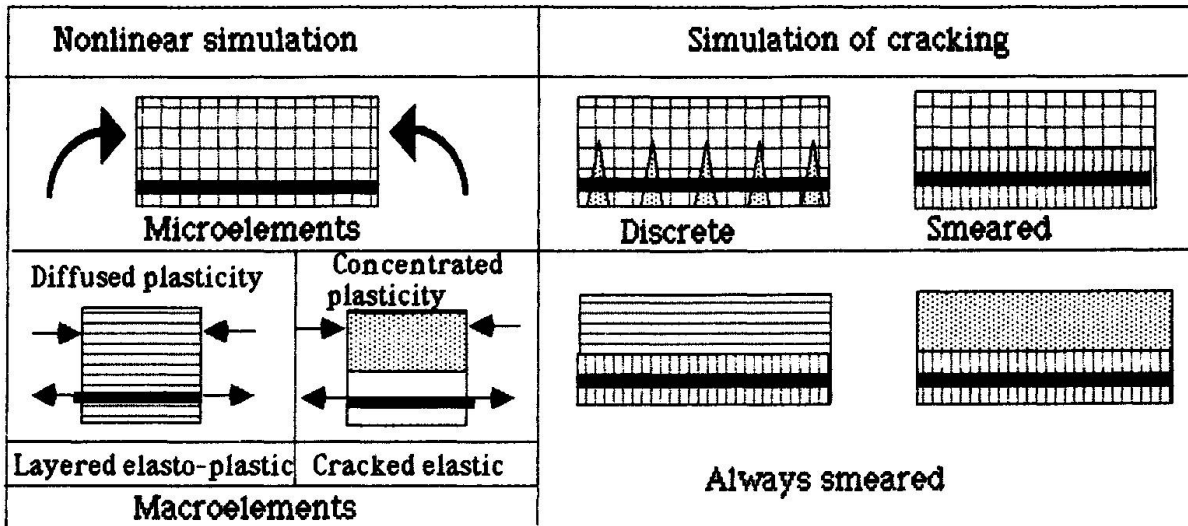
Nonlinear simulation	Simulation of cracking	
Microelements	**Discrete**	**Smeared**
Diffused plasticity ∣ **Concentrated plasticity**		
Layered elasto-plastic ∣ **Cracked elastic**	**Always smeared**	
Macroelements		

Fig. 1-Simulation of cracking

It is quite natural to think that things behave as if the tensile reacting part of the beam be constituted by the tensile reinforcement and an additional "virtual" reinforcement which represents the contribution of the concrete in tension between cracks. The method can be described as follows:

Each element belonging to the frame is divided in a given number of short macro-elements behavin elastically(as plastic behaviour is considered concentrated in "plastic hinges").For each element the moment of inertia of the cracked section is introduced when.within the element,the adopted tensile strength of concrete in tension is exceeded.In computing the moment of inertia of the cracked section the influence of"tension stiffening" is simulated by introducing a "virtual"additional steel area AS^* which can resist a constant force $K_1b(a-x)$, K_1 being the mean tensional stress between cracks which will be subsequently called "tension stiffening coefficient"(seefig.2)
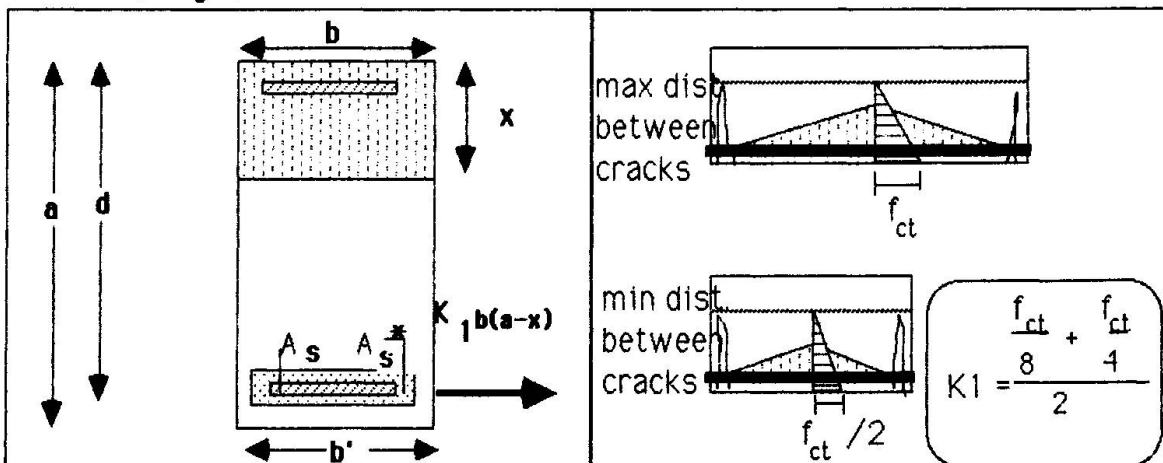

Fig.2-"Trasparent" simulation of "Tension Stiffening"

A_s^* decreases as loads increase,thus simulating the decreasing influence of" tension stiffening" with increasing loads(which canbe verifiedexperimentally).

The tension stiffening coefficient can be computed in function of the tensile resistance of concrete,adopting the simplified assumption of uniform distribution of adherence stresses along tensile reinforcement between one crack and the following.If this assumption is made tensile stresses in concrete vary linearly along the element axis.If again the assumption is made that tensile stresses also vary linarly along the section depth ,the mean value of these stresses is equal to $f_{ct}/4$ in the case two consecutive cracks form at the maximum possible distance(upperlimitsituation) and equal to $f_{ct}/8$ in the lower limit situation(minimum possible distance between

cracks). If again a mean value is assumed between these two extremes a value of the tension stiffening coefficient equal to $K1=3/16.f_{ct}$ is obtained.

4. IMPORTANCE OF "TENSION STIFFENING" ON STRUCTURAL BEHAVIOUR

One might wonder whether it is really important,to evaluate the structural behaviour of frames,to model accurately the influence of cracking and the related phenomenon of "tension stiffening",that is to evaluate the nonlinear behaviour in the elastic stage.
In fact experience tells us that these effects are essential to evaluate correctly,among others,the following phenomena:
-redistribution of moments due to cracking in beams,which can influence considerably the behaviour at Ultimate LimitState.
-elastic displacements at service load level
-Influence of cracking on second order effects in slender columns
-Thermal structural effects.
With reference to the latter phenomenon an example taken from [15] is briefly illustrated.
In the frame of fig.3 vertical distributed loads were applied to the beams first and then a thermal variation was applied to the column.
The frame was analyzed both linearly and non linearly and for different values of f_{ct} and corresponding values of K1(tension stiffening coefficient).The 6 cases which were considered are summarized in the table of fig.4.
The lag in formation of stabilized cracking(see following paragraph)was simulated in case n.5.
The load history is represented in fig.3:vertical loads were applied first to the beams in 10 steps so that cracking might take place due to external loads.An uniform thermal field of +40°C was then applied to the central column(again in 10 steps),so that moments of the same order as those produced by external loads(and of the same sign in the central section)were induced in the beams.
In figure 4 the moments due to thermal effect only for the various cases are represented graphically
As may be seen from this diagram the extreme cases(linear analysis of uncracked structure and nonlinear analysis disregarding tensile stress of concrete and tension stiffening)lead,on opposite side,to completely unreliable results. Taking into account these factors according to different evaluation of tensile strength of concrete leads to considerably different results. This consideration leads to the problem of tensile strength evaluation.

5. UNCERTAINTIES IN THE SIMULATION OF CRACKED BEHAVIOUR AND TENSION STIFFENING
ASSUMPTION OF A SUITABLE VALUE FOR TENSILE STRENGTH

Every simulation of the cracking formation process must necessarily be based on an assumed value of concrete tensile strength. On the other hand it is well known that this parameter can only be determined with much uncertainty given the considerable dispersion of test results. Besides other factors influence crack formation and crack propagation[14](size and quantity of reinforcement,cover,size effect,moment gradient)
Therefore we cannot ,in evaluating structural behaviour in the cracked stage,reach the same degree of accuracy that we can obtain in the calculation of yielding moments in critical sections,which are in most cases mainly influenced by steel strength fsy,and to a lesser degree by concrete compressive strength fck.
As we cannot hope,even using sophisticated methods,to obtain very accurate results,the choice of the tensile strength fct must be based on safety probabilistic considerations,which are in turn influenced by the kind of results we need from structural analysis.
Also it is not convenient to adopt a very sophisticated procedure for tension stiffening simulation
If the purpose of analysis is to calculate deflections,the choice on the safe side is to overevaluate them , therefore to underevaluate fct and ,as a consequence,tension stiffening. The most logical choice according to CEB Model Code philosophy [5][6]is to adopt a characteristic value with a 5% probability of not being exceeded (fctk.05)
If the purpose is to evaluate redistribution of moments due to cracking ,it is not easy (and probably not even possible)to establish which value of fct would yield the safest distribution of action effects. In this case it is most reasonable to look for the "most probable" result and therefore choose a mean value of resistance(fctm)
If at last we need to evaluate thermal effects,the choice on the safe side is of course to overevaluate them and therefore to overevaluate fct(and tension stiffening).
A value of fct with a 95% probability of not being exceeded can be adopted(fct0.95).
This may however not be enough.
The method of simulating stiffness reduction due to cracking which was described in the previous paragraph(as well as most other methods of this kind)are based on the assumption that,as soon that,in a given element,the

MOMENTS DUE TO THERMAL EFFECT

M [KN.M]

Case	fck	fsy	fct	K1	Comments
1	35	440	0	0	Tensile strength and tension stiffening disregarded
2	35	440	22	4.12	Tensile strength of concrete assumed=fctk0.05
3	35	440	31	5.81	Tensile strength of concrete assumed=fctm
4	35	440	40	7.50	Tensile strength of concrete assumed=fctk0.95
5	35	440	40	7.50	Lag in formation of stabilized cracking taken into account
6	35	440	Linear case		

Sez.A-A 500x500 mm.

Sez.B-B 800x500 mm.

Adopted mesh

G+Q=50 KN/m

$D_T = 40\,°c$
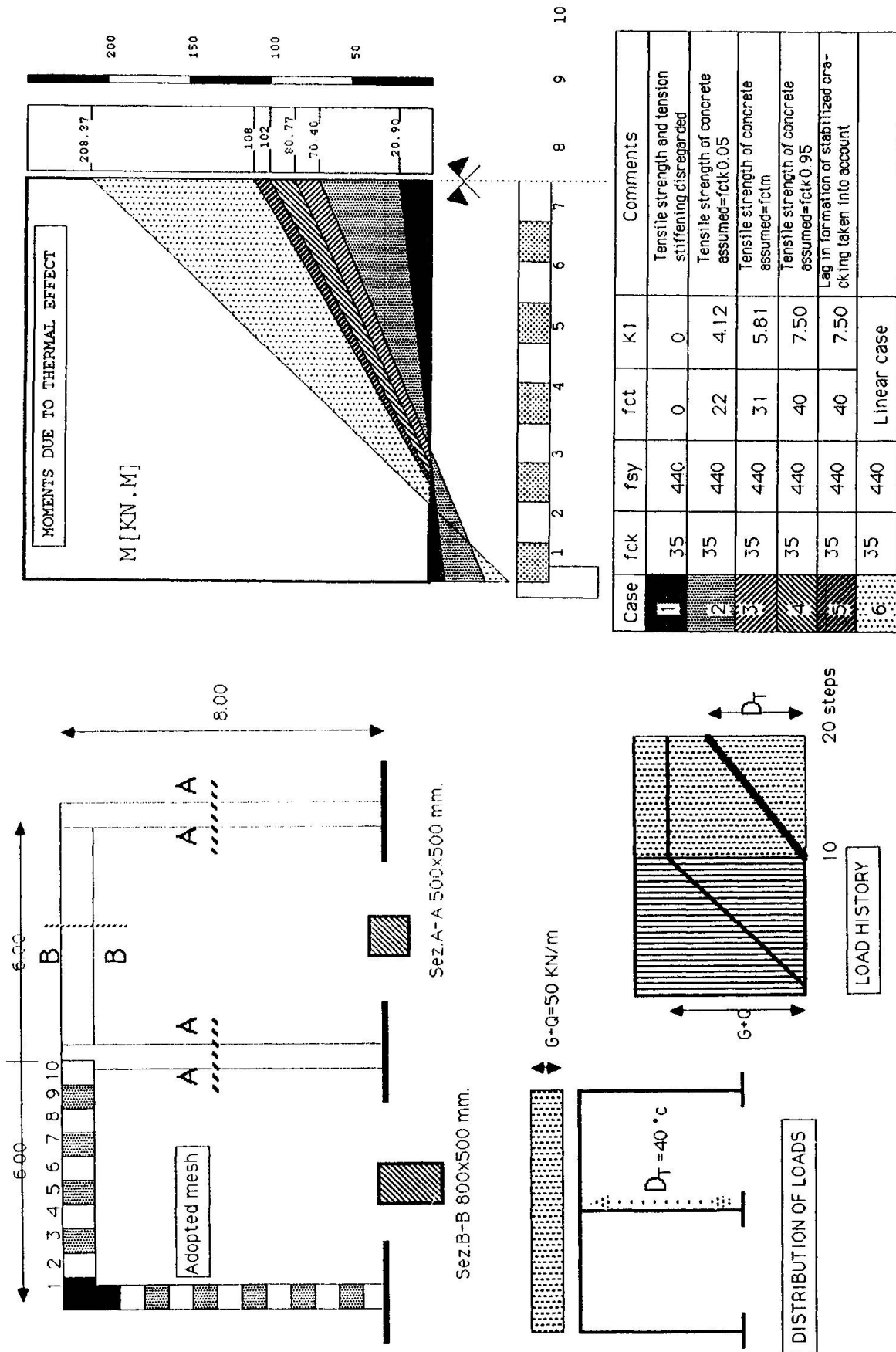
DISTRIBUTION OF LOADS

LOAD HISTORY

20 steps

Fig. 3/4 - Example: structural effect of thermal variation

tensile strength of concrete is reached in the most stressed fiber,fully opened "stabilized" cracks form in the element and its stiffness can be reduced accordingly.

However this assumption does not correspond to results of experimental tests In fact there is a "lag" between the reaching of the limit value of fct in a zone with constant momentand the formation of stabilized cracking in the same zone. An approximate method for simulating this lag is given in[15].

It may be concluded that a situation of fully open and stabilized cracks takes place gradually in the element and is fully developed for values of maximum tensile stress in concrete that can be much higher then the tensile strength fct.

This fact may be explained intuitively:in fact when fct is reached in the most stressed fiber a crack must begin;however an increase in stress in the adjacent fibers is required for the extension of this crack toward the neutral axis;this can happen only by furtherly increasing external actions also because tensile reinforcement is an obstacle to this propagation.

6.CONCLUSIONS:HOW "TENSION STIFFENING" SHOULD BE TREATED ACCORDING TO THE PHILOSOPHY OF THIS SYMPOSIUM

From the above considerations the following conclusions can be drawn:

-The influence of cracking and "tension stiffening" on structural behaviour must be taken into account in many practically important cases.

-Given its intrinsic complexity,the phenomenon must be simulated using simplified methods,which,to be accepted and correctly used by the engineer,must be sufficiently "trasparent",as the one which has been,as an example,presented.

-Given the uncertainties related to the prevision of the stress level at which a stabilized crack patterns takse places,the adopted input data and,in particular,the concrete tensile strength should be chosen so that results on the safe side be obtained.This means that values to be adopted must depend on the kind of load to be applied and the kind of situation to be studied.

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