Concrete columns

Concrete Columns

Colonnes en béton

Betonstützen

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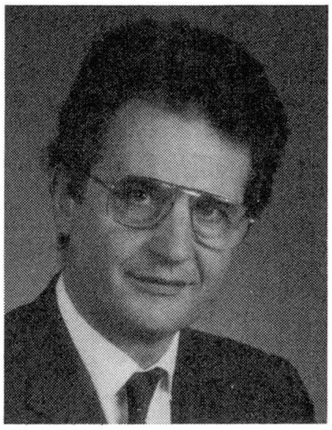

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SUMMARY

Compression forces and second order effects influence the behaviour of concrete columns and walls in terms of bearing and ductility capacity. An overview is given on the main aspects concerning phenomena, design, analysis and safety concepts.

RÉSUMÉ

Les efforts de compression et les effets du second ordre affectent le comportement de colonnes et murs en béton, en termes de résistance et de ductilité. Une vue d'ensemble est donnée sur les aspects principaux concernant les phénomènes, le projet, l'analyse et les concepts de sécurité.

ZUSAMMENFASSUNG

Druckkräfte und die Effekte aus der Theorie Zweiter Ordnung beeinflussen das Verhalten von Betonstützen und -wänden hinsichtlich ihrer Trag- und Verformungsfähigkeit. Es wird ein Überblick über die wichtigsten Gesichtspunkte bezüglich der auftretenden Phänomene, der Bemessung und Berechnung sowie der Sicherheitskonzepte gegeben.

1. INTRODUCTION

The main features of columns are that they sustain axial loads and they may be slender.

Therefrom, the capacity of such structures is affected also by additional load eccentricities produced by their deformation, and it is reduced, compared to the capacity of the cross-section alone. The phenomenon is called geometric non linearity, second order effects, stability of deformation, buckling.

Concrete has been viewed in its beginnings as giving bulky structures, not subjetc to slenderness problems. However, the increase of concrete strength, coupled with the needs of saving material reducing selfweight and gaining free space, have given rise to columns walls and piers that may show sensible second order effects. This because the stiffness does not grow up as much as the strength, unfortunately.

The above problems pertain to "long" columns, thus they are concerned namely with "B-regions" (according to the definition given in [18]) subject to bending and compression. Of course "D-regions" problems may appear in columns, namely at the ends, or in splice-joints of prefabricated units, or in "short" columns that might be acted by high shear and compression. However these are not so typical of columns, and will not be treated here as such.

The observations that follow, on various aspects involved in columns design, reflect the writer's views; thus, reference is made mostly on his previous papers, intended only as a partial justification of the statements.

2. COLUMNS CAPACITY

The parameters controlling the deformation of a column, given the loading and the geometry, are the shape of concrete stress-strain curve, its tensile strength, the stiffness of reinforcing steels and possibly the time. The first is governed by the initial E_c modulus, which may increase with the strength f_c but not proportionally; the same happens to the second (f_{ct}), which seems to have an upper limit and it is not fully reliable; the third (E_s) is independent of steel strength, up to yield stress f_y; long term strains roughly depend on E_c, too.

Moreover, the stiffness varies along the member following the local state of stress, thus depending partly on the second order effects themselves. This means that mechanical (material) nonlinearity and geometric nonlinearity interact in modifying the structural behavior.

The service limit states are normally not sensibly affected by second order effects, being the materials still in the linear range over the whole column.

Instead, under ultimate conditions, large deformations appear in critical regions, and the stiffness decreases also along the structure, due to lowering E_c in compression and cracking in tension. Considerable deformations and second order moments appear. The bearing capacity of the column is reduced. In fact, addition of bending moments is in any case unfavorable for the resistance of a concrete section.

Another relevant structural requirement is the ductility capacity, i.e. the ability of undergoing given "plastic" deformations with constant lateral force response, while dissipating energy. Also ductility also is influenced by the interacting nonlinearities. Normal forces reduce the elongation of tensile reinforcement available before concrete crushing, and second order effects are much increased by plastic rotations, even in columns non slender per se: this results in a reduction of moment redistribution capacity as well as in a rapid drop in the Force-Deflection curve of the column (fig. 1) and of the corresponding energy absorption [8,13].

Therefore, in design of seismic structures, ductility is preferably demanded to non compressed members, such as beams or bracing walls.

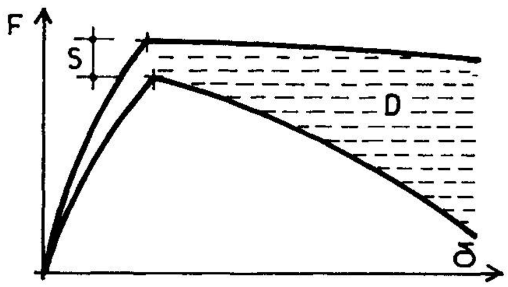


Fig. 1
Reduction due to 2nd order effects
S : Strength
D : Energy dissipation

3. PRESTRESSING

Prestressing may be a means of improving the bearing capacity of axially loaded slender columns [9]. In fact, it has an influence on the second order effects by modifying the stiffness of the cross-sections. When applied, prestressing is normally symmetric in the cross section, if bending has not a preferred sign. The influence in twoedged: on one hand, the increase of the average compressive stress tends to reduce the stiffness; on the other hand, it tends to improve it by delaying formation and propagation of cracks, as in the example in fig. 2. If the external axial force is very low, centric prestressing may even improve the cross-section strenght.

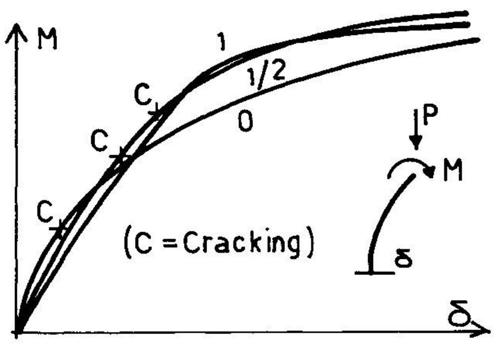


Fig. 2
Behavior of slender prestressed elements
1 : fully
1/2 : partially
0 : not

But, contrary to an external axial force, internal prestressing does not produce itself second order effects, as it follows the structure's displacements. Also the ductility reduction is less than that of an external axial force [13]. It should be well calibrated to be helpful (the more sensibly the more the column is slender). Partial prestressing is advisable and bonded tendons would also contribute to flexural strength.

Economy governs the "if and how much" to prestress. In the balance, the advantage may come of pretensioning precast columns for resisting transient loading conditions.

Unbonded external tendons are not fitted for the purpouse, as they do give second order effects, unless they are multilinked to the structure; the effects are theoretically less then those of a real external force, because eccentric though symmetric tendons strain favorably and oppose ends rotations.

4. RESTRAINTS

The effective restraint acted on by the supports of the columns must be known, both in statically determinate and indeterminate cases. In fact, second order effects render all structures statically indeterminate.

The actual stiffness of the restraint, which may vary according to the actual forces applied, determines in particular the "effective length" l_0 between virtual points of contraflexure. Even in the simpliest case of an isolated cantilever clamped at the base (as in fig. 1) account should be made in principle both for rotation of foundation, when evaluating l_0, and for second order moment, when verifying the foundation itself.

Often, columns are part of frames where the beams form the restraint. In principle, restraint parameters should be evaluated at a state of stress corresponding to the u.l.s. of the columns, which comes out automatically in a full nonlinear analysis of the frame, but requires some judjment if the column is taken out of its frame and analized as "isolated".

A major point is whether the frame's joints may or may not displace (sway vs. non sway frames). This distinction refers to the presence of a rigid structure bracing the whole frame, i.e. providing a fixed restraint to lateral displacements of all joints (fig. 3).

A frame, whose joints do not displace under the given loading condition, but is not itself positively braced, cannot be considered as non sway if second order effects are expected. Again, either a full nonlinear analysis is performed, including forces exciting lateral displacements, or criteria for evaluating the overall slenderness are needed.

The overall analysis of a sway frame is sufficient if the columns of every storey are regular, i.e. uniform in dimensions and loadings. Otherwise, it must be integrated by the check of the most slender columns in their isolated buckling mode.

Columns much more slender than the nearest become soft elements refusing increments of axial or lateral loadings from the structure, thus altering the force pattern in the beams (fig. 3). It is matter of proper design to avoid such situations. Slender sway frames would not be recommended for buildings anyway.

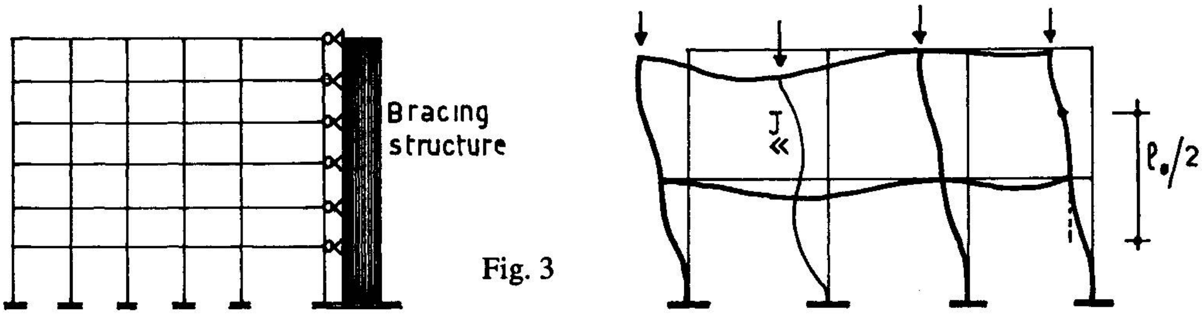


Fig. 3

5. SLENDERNESS LIMITS

The analysis of second order effects in a concrete structure implies more or less complex calculations, due to the interaction of geometric and mechanic nonlinearity. Even with the simplified methods and aids, it represents a computational cost. Theoretically, all members subject to external compressive forces show an increase of internal bending moments. Practically, the majority of compressed members may be excluded from that analysis.

5.1 Uniaxial bending

The parameter used in past codes, as for instance in CEB-FIP Model Code 78 [1], is the slenderness ratio λ in the plane of bending.

This ratio is merely geometric and derives from the elastic theory. However, even for the elastic cheks of steel structures, the lower bound of λ (combined with the well known coefficient ω) varies according to the different steel grades, thus revealing itself insufficient as such for being and absolute value.

In concrete structures, where the interaction of normal force N and bending M is more complex, and ω is not used, the slenderness ratio alone is a very poor parameter, not able to represent a significant boundary for the importance of second order effects. In fact, in former CEB Recommendations 1970, the adopted value was 50; because columns with $\lambda < 50$ may show important second order moments, subsequent MC 78 adopted the value 25, but the same drawback applies. Present draft of MC 90 [2] uses a different approach, as well as the future Eurocode 2.

A better parameter has been worked out [12] incorporating the specific normal force applied (ν), which is the other essential factor. The new parameter, formulated as $\lambda\sqrt{\nu}$, showed to detect whith much greater precision the bound where second order effects rise sensibly. It has been checked in numerical tests [12, 17] against the ratio μ of total to first order moment increasing by 10%; the useful limit appeared situated around the value $\lambda\sqrt{\nu} \cong 20$.

That parameter may be used also for defining an upper limit, above which the column would be too slender and is to be avoided : increase of moment $\cong$ 100%; $\lambda\sqrt{\nu} \cong 70$.

Of course the above values are averaging most cases, and do not give the exact solution, which needs the analysis. The use of $\lambda\sqrt{\nu}$ as indicative limits represents a substantial improvement with respect to λ, though not more complex (fig. 4).

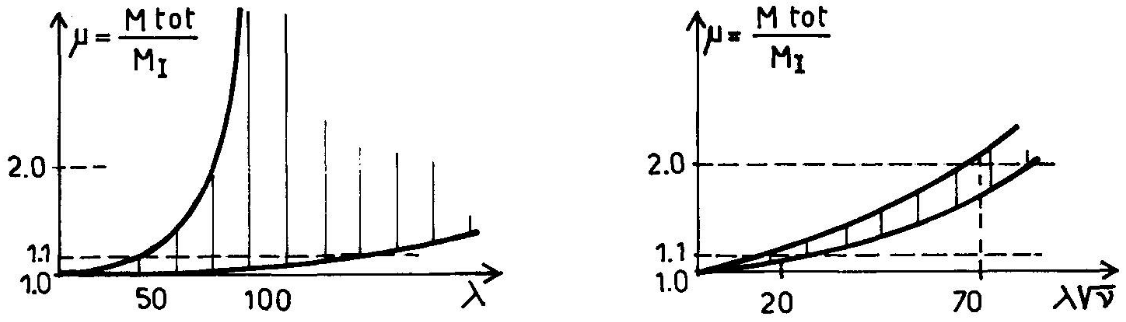


Fig. 4 - Compaction of second order limit estimation using $\lambda\sqrt{\nu}$ instead of λ

Limits of slenderness based on λ corrected with the ratio of load eccentricities at the ends of the column, $\lambda / (2 - e_{01} / e_{02}) = 25$, as adopted by some codes, may be misleading - especially when both eccentricities tend to zero - because they rely upon favorable moment distribution that may not influence really the bucking mode, and allow for λ's between 50 and 75 to be treated as non slender, independently of the normal force.

5.2 Biaxial bending

In the writer's opinion, when the slenderness lower limit in one principal direction is exceeded, biaxial check would be necessary, applying at least the minimum eccentricities provided by the codes.

In fact, the interaction of first order moments about two principal axes of a cross-section is low when the skewed eccentricity is close to one axis. But this may be not true for the second order moments, when slenderness ratios about both axes are much different: the reduced (accounting for second order effects) capacity about the stronger axis has two distinct values, if accounting for side buckling or not, as sketched in the reduced interaction diagram in fig. 7.

A criterion for avoiding biaxial check (when the loading condition lays in a principal plane) should be found, assessing a ratio λ_x / λ_y close to 1.

5.3 Frames

Slenderness limits for entire non braced frames have been sought. The problem is finding an equivalent λ for the storey, then it may be associated with an average specific normal force to enter in the criterion under § 5.1. If the frame is "regular" an average effective length of columns in lateral buckling mode may be easily estimated and assumed for the scope (fig. 3).

A simplified formula, very conservative, has been proposed [10] for a rough chek of the storey slenderness: $\lambda = \sqrt{(12\ K\ A / h)}$, where A is the sum of concrete areas of columns, h the storey height and K a conventionally calculated displacement.

6. ANALYSIS

6.1 General Methods

It is called "general method" any analytical procedure able to solving with sharp approximation the stress-strain state of the structure.

The great obstacle in these procedures is the computational work: the strain state must be determined all over the structure to work out the deformation, accounting for mechanical and geometrical nonlinearities. It is well knon that, by discretization, it is rather simpler to work out the internal forces over a section, given the strains, than vice-versa; while this is needed in the iterative procedures when searching for the deformations under the current tentative action effects along the structure.

To make this search easier, a criterion has been elaborated for building up the correct stiffness matrix of the most generic concrete structure, subject to axial force and biaxial bending, and responding to any given materials σ-ε relationships [5, 6, 15].

The criterion is based on the linearization, in every point of a cross-section, of the local stress-strain path during a given loading step; all the linearized path steps make up a section made of fictitious linear materials, each with a different modulus of elasticity; this can be treated, by a transformation into a common modulus, as a homogeneous elastic section, eventually with the theory of the ellipse of inertia, which figures the relationship between forces and deformations (fig. 5). Any phenomenon, like cracking, tension stiffening, bond, non elasticity, etc., may be incorporated for each material, provided it can be fit into a stress-strain curve.

The above relationships must be found anew at every iteration, as the stresses change and with them the fictitious elastic section. However, they are the most effective in driving the iterations toward convergency.

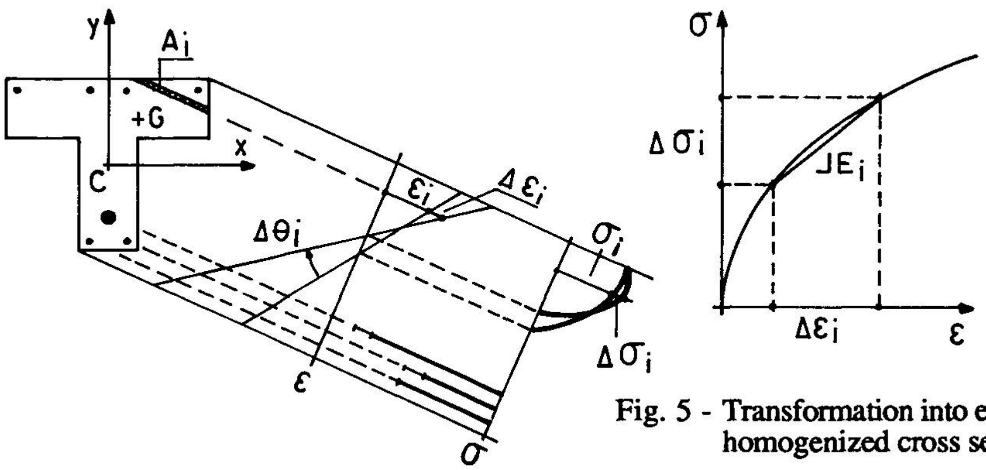


Fig. 5 - Transformation into elastic homogenized cross section

6.2 Approximated Methods

6.2.1 Uniaxial bending

For a simple cantilever column, loaded only at the top, the integration of deformations is solved approximately in the "model column" method [4] by assigning the structure a shape function (sinusoidal), whose amplitude is determined by imposing the compatibility, of the curvature of the axis line (1 / R) and of the deformed cross section (θ), only in the critical section at the base.

That assumption permits to derive easily the second order moment M_{II} in the critical section:
$M_{II} = N \cdot \delta = N \cdot \Theta \; l_0^2 / \pi^2 \cong 0.4 \, (l_0 / 2)^2 \cdot \; \Theta \cdot N$
Thus, M_{II} is represented by a straight line on the Moment-Curvature (M-Θ) diagram of the critical

section (for the given N). M being the total moment, a column can be acted only by a first order moment M_I given by the difference M- M_{II} (fig. 6).

Therefrom, it is easy to work out the so-called "reduced interaction diagram", by plotting the M_{Imax} for all values of N. This diagram becomes referred to the structure, not only to the section, M_{Imax} being a means of representing the applied forces net of 2nd order effects. Such diagrams are also prepared in nondimensional form as design aids; naturally, they may be built-up using exact methods.

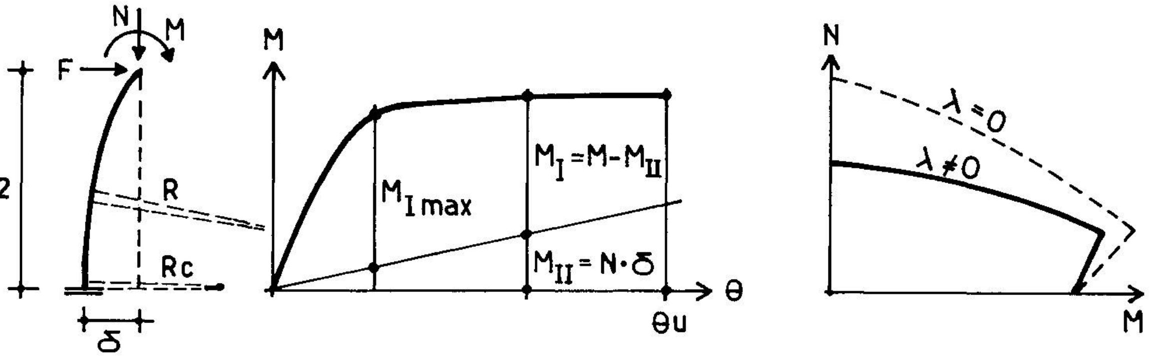
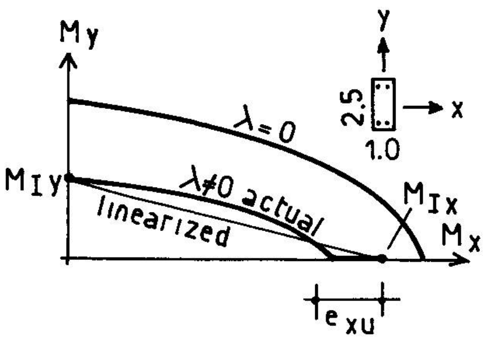


Fig. 6 - Model Column and Reduced Interaction Diagram (R.I.D.)

6.2.2 Biaxial bending

The model column may be used for biaxial actions, too. But the approximation is worst, ad it disregards that the deflection is not plane (fig. 8) and varies in direction when loads increase; in fact, internal forces vary along the structure and, with them, the angle of skew.

Nevertheless, reduced interaction diagrams may be worked out too, by more refinite means. Being the action triaxial (N, M_x, M_y) it is worth to plot them in M_x, M_y planes, for various N (fig. 7).



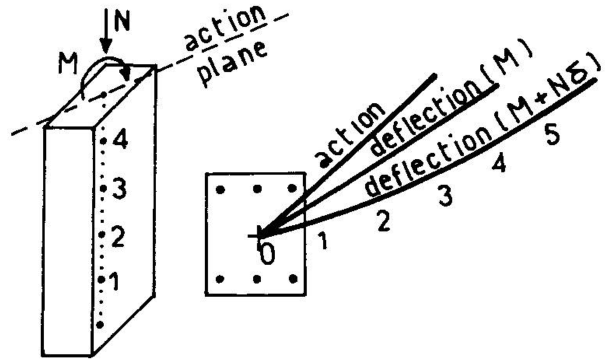


Fig. 7 - R.I.D. of biaxial bending

Fig. 8 - Non-plane deflection in biaxial bending

Due to the complexity of biaxial verification with general methods, a rough simplified criterion of check for the most general case was also proposed [7].

Given a column, with any possible kind of section, restraint, loading, it is analized in uniaxial bending on both directions x, y with an appropriate method, accounting for second order effects. The critical multipliers α^*_x and α^*_y, applied to all design loads acting in principal planes x and y respectively, are found. Finally, an interaction diagram is drawn with a straight line between both found values, in the plane of the multipliers α_x, α_y.

The diagram is referred directly to the actions (through the multipliers α) and not to the action effects on a section, because in a general case critical sections may be more than one. The check is satisfied if the point ($\alpha_x = 1$, $\alpha_y = 1$), representing the design load combination considered, is contained in the safe diagram (fig. 9).

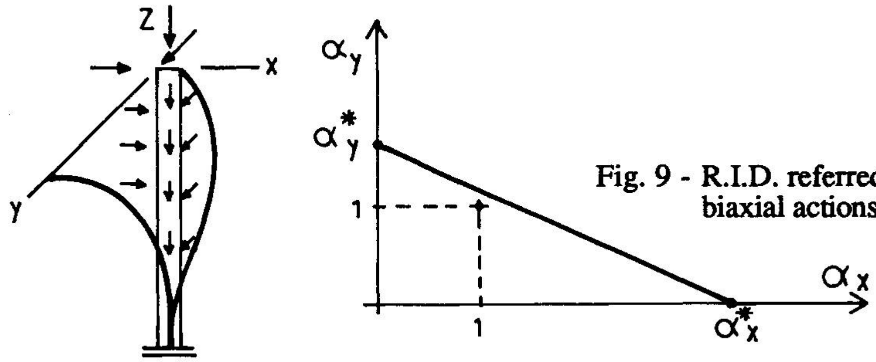


Fig. 9 - R.I.D. referred to generic biaxial actions and structures

The criterion is generally conservative, but in a small regions close to the axis of the greater α* in cases of much different slenderness about both planes, as it was mentioned. However, this must be covered by the minimum eccentricity (fig. 7).

6.2.3 Frames

A traditional approximated method for overall check of sway frames is the so-called P-δ method. It consists in an iterative analysis of the frame, acted by lateral forces at the storey levels, "equivalent" to the second order effect ($F \cdot h = P \cdot \delta$), F being the lateral resultant, P the vertical one, h the height and δ the deflection, all referred to the storey.

At each iteration the equivalence is checked with the obtained δ, until convergency is reached. The difficulty, determining the degree of approximation, is assigning correct stiffness to the elements, consistent with the u.l.s.

6.2.4 Walls

Walls are bi-dimensional plane structural elements, subject to complex in-plane and out-plane forces. Concrete walls are usually slender out of their plane. They might be analized accurately be means of appropriate non linear methods. However, in building practical design they are treated with approximated criteria, reducing the problem to that of an equivalent one-dimensional element, i.e., a column.

As well as for columns restrained at the ends in a frame, nomograms are given for estimating the reduction of the effective buckling length, as function of the restraints on the four edges and of their respective distance. Codes give guidance for estimating conventional and additional eccentricities. The checks are then performed accordingly, on the critical vertical strips of wall acted by a normal force equivalent to the strip cross-section normal stress resultant, deriving from the overall analysis of the structure.

A particular case is represented by non reinforced concrete walls. For them, the slenderness limits cannot be based on the criterion of 10% increase of the moment due to the second order effect, because the moment capacity depends almost totally on the normal force, and the eccentricity becomes the critical parameter.

Starting from the basic work [3], graphs and coefficients are given for the reduction of capacity of wall strips, accounting for the multiple parameters involved, including safety elements. Some reliance is made both on tension stiffening and on tensile strength of concrete, justified by the redundancy of the bi-dimensional behavior.

Short (horizontally) walls, unrestrained along vertical edges, become "non reinforced columns". Such seldom concrete elements should not rely upon tensile strength and possibly not be affected by second order effects, thanks to geometry or to loadings.

7. SAFETY

7.1 Code provisions

According to Level I approach, characteristic values of the relevant variables involved in a limit state are to be given partial safety factors, to obtain the "design values": generally, actions are amplified, strengths reduced, possibly the model uncertainty is covered by special factors; the corrisponding "state" of the structure must be within the given limit. The u.l.s. of slender structures is affected by the deformation, thus the relevant variables affecting it should be modified, too, for the verification. This is done by the following conventional means:

i an initial unintentional inclination is introduced, or an equivalent eccentricity;
ii the deformabilities of materials are factored;

furthermore:

iii creep deformations due to sustained loadings are added;
iiii possible deformation of restraints is accounted for.

Some observations are deserved.

The first item (i) accounts for unavoidable geometrical imperfections, that in slender structures may raise or amplify the deflections. Standard specified inclinations are quite severe, between 1/150 and 1/200, thus some reduction are allowed, depending on site controls; it could even be neglected, when many connected columns act in parallel.

Item (ii) implies γ_c factor being applied not only to the strength, but to the whole $\sigma-\varepsilon$ curve, reducing the initial modulus E_c, too. In fact, it is the same model used for checking the cross-section resistance; but here it assumes the particular meaning of increasing the "design deformation" of the structure for u.l.s. However, the initial part of the curve being relevant, it should be assumed more realistic than the conventional parabola attached at a fixed strain value to the rectangle, in order to better match E_c.

The question is under discussion whether the same γ_c should factor f_c and E_c: in fact, by factoring deformability and loads, the deformation is factored twice. Also the stiffening effect of concrete in tension is to be judged: whereas, for small size isostatic columns, it should be considered naught (consistently with the local effect governing the whole behavior), for large structures like towers, or for highly redundant frames, it would be really too conservative.

The initial deformability of steel is commonly not factored; yielding point is automatically lowered by factoring the strength, thus influencing the ultimate deformability.

Prestressing factors are not mentioned by codes in this context. In fact, it seems reasonable to factor steel curves as for ordinary steel (i.e. only beyond yielding point) and to apply the nominal tensioning strain unfactored for the analysis.

Item (iii), creep deflection, is generally calculated under the unintentional inclination and the quasi-permanent load combination (how factored is also matter of discussion). For sake of simplicity, the creep deflection is calculated separately, then added to the initial one (i); finally the check is performed with all design loads considered as short time.

Item (iiii) covers an obvious extension of the criteria to the connected bodies involved in the deformation. The deformabilities should be referred to the action effects intervening at the u.l.s. of the column.

7.2 Safety analyses

A set of checks of MC 78 provisions, applied to 60 different columns, were performed by a Level II method [16], in order to assess the consistency of the rules.

Some indicative conclusions were drawn, so summarized:

- partial factor γ_F on variable axial forces should be increased, compared with that on dead loads;
- the special γ_F on permanent loads, only for calculation of creep deflection (assumed = 1.1), should be increased;
- the minimum reinforcement, assumed $A_{smin} = 0.8\% \ A_c$, should be increased in slender members (as function of concrete grade or of design axial force).

8. DETAILING

Few remarks may be set on detailing of columns themselves (apart their D-regions):

- longitudinal reinforcement: minimum value, expressed as % of concrete area as in MC 78 (see § 7.2) should be increased for slender columns as function of design axial force, and in general, for columns with higher concrete grades, to provide bending capacities proportional to the implicit higher axial force capacities;
- confinement: overall confinement of axially loaded slender columns does not increase their bearing capacity, as u.l.s.. is governed by bending; placed in the end sections, it increases the ductility capacity.

Walls generally carry lesser average normal stress than columns and may need a lesser minimum vertical reinforcement. Instead, significant horizontal reinforcement is needed, to follow the plate effect. In "non reinforced" walls, minor reinforcements (bars or meshes) are placed for taking over local tensile stresses; care should be taken to prevent the vertical bars from buckling and spalling concrete.

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