Analyses based on the modified compression field theory

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Analyses basées sur la théorie modifiée du champ de contraintes en compression

Untersuchungen auf Grundlage der modifizierten Druckfeldtheorie

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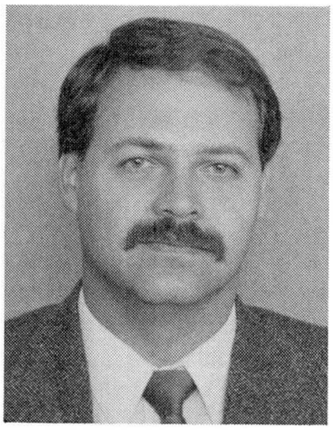

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SUMMARY

The Modified Compression Field Theory provides a unified, rational approach to the analysis of structural concrete elements under general in-plane stress conditions. Cracked reinforced concrete is treated as a nonlinear elastic orthotropic material based on a smeared, rotating crack assumption. Formulations satisfying equilibrium and compatibility conditions are developed, and new constitutive relations for the component materials are defined. The theory is incorporated into several analytical algorithms. Procedures have been developed for the analysis of membrane structures, beams, plane frames, plates and shells, and three-dimensional solids. Extensive corroborative testing has shown the theory to be able to model accurately the response of structural concrete.

RÉSUMÉ

La théorie du champ de contraintes en compression fournit une approche rationnelle et unifiée à l'analyse des éléments structuraux de béton, sous des conditions de contraintes planes. Le béton armé fissuré est traité comme un matériau orthotropique élastique non-linéaire, se basant sur un concept de fissuration uniforme. Des expressions satisfaisant les conditions d'équilibre ainsi que de comptabilité sont développées, et de nouvelles relations constitutives sont définies pour les matériaux. Des procédures sont développées pour l'analyse des membranes, poutres, cadres plans, plaques, coques et solides tri-dimensionnels. Une série d'essais démontrent que la théorie peut prédire correctement le comportement du béton.

ZUSAMMENFASSUNG

Die modifizierte Druckfeldtheorie stellt eine einheitliche und rationale Methode für die Untersuchung von Konstruktionsbeton-Elementen im ebenen Spannungsfeld dar. Gerissener Stahlbeton wird – basierend auf einem Rissmodell mit verschmierten und rotierenden Rissen – als nichtlineares, elastisches orthotropisches Material behandelt. Es werden Formulierungen, die Gleichgewichts- und Verträglichkeitsbedingungen genügen, entwickelt und neue konstitutive Beziehungen für die Teil-Materialien definiert. Die Theorie wird in mehrere analytische Algorythmen eingebaut. Für die Untersuchung von Membranwerken, Balken, ebenen Rahmen, Platten, Schalen und drei-dimensionalen Festkörpern werden Berechnungsmethoden entwickelt. Zahlreiche Versuche haben gezeigt, dass die Theorie das Verhalten von Konstruktionsbeton genau beschreiben kann.

1. INTRODUCTION

A unified, rational approach to the analysis and design of concrete structures does not currently exist. The difficulty stems from an inability to develop general material behaviour models for structural concrete that are simple, rational, consistent and accurate. This was clearly brought to light in the results of a competition conducted 10 years ago [1].

The international competition was organized to compare analytical methods for predicting the response of reinforced concrete elements subjected to general two-dimensional stress states. Entrants were asked to predict the strength and load-deformation response of four panels tested in a University of Toronto research program. While many of the predictions received were based on analyses conducted using complex nonlinear analysis procedures, a wide scatter in the predictions was evident. Clearly, the ability to accurately model behaviour, using finite element procedures or otherwise, was not very good due to a generally poor state-of-the-art in constitutive modelling.

Much work has been conducted recently at the University of Toronto in an effort to develop improved models. One outcome has been the formulation of the Modified Compression Field Theory.

2. MODIFIED COMPRESSION FIELD THEORY

The Compression Field Theory (CFT) was first developed by Collins and Mitchell [2, 3], and co-workers, for the analysis of beams under combined torsion, shear and flexure. The theory provided a conceptual model for the behaviour of cracked reinforced concrete under two-dimensional stress states, following essentially a smeared, rotating crack idealization. Formulations satisfying conditions of equilibrium and compatibility in a continuum were based on average values of stress and strain in the component materials. It was assumed that the directions of the principal stresses coincided with the directions of the principal strains. Concrete in compression was modelled using the Hognestad parabolic curve; concrete in tension was assumed to carry no stress after cracking.

To develop more accurate constitutive relations for cracked reinforced concrete, a new testing facility was developed and utilized in an extensive experimental investigation. The 'Shear Rig' was capable of loading reinforced concrete panels under general conditions of uniform in-plane stress. In particular, it allowed for the first time anywhere, the testing of reinforced concrete under conditions of pure shear, as well as shear combined with biaxial normal stresses. [More recently, at the University of Toronto, the 'Shell Element Tester' facility was developed permitting the testing of larger elements under conditions involving both in-plane and out-of-plane loading]. In the initial series, 30 panels were tested in the years 1979-1981 [4].

Based on the results of these initial tests, the Modified Compression Field Theory (MCFT) was developed [4]. The refinements introduced by the MCFT related to: i) strain softening of concrete in compression, due to the action of transverse tensile strains; ii) tension stiffening effects in cracked concrete in tension, due to the continued presence of tensile stresses in concrete between cracks; iii) the transfer of stresses across cracks (ie. the need to consider local stress conditions at crack surfaces). These effects were embodied in the analytical model by a new set of constitutive relations.

3. EXPERIMENTAL CORROBORATION

To corroborate and refine the analytical models, and the finite element formulations developed subsequently, several test programs were undertaken involving membrane elements. Bhide and Collins [5] tested a series of thirty-one uniaxially reinforced concrete panels under various combinations of tension and shear. A series of ten panels, with centre perforations, were tested to study behaviour in situations involving stress disturbances due to structural discontinuities [6]. To examine predicted behaviour under conditions where a uniform system of cracks could not be assumed, panels containing a pre-cracked uniaxially reinforced shear plane were tested [7]. The behaviour of prestressed elements was investigated by Marti and Meyboom, through the testing of three large-scale shell elements. To investigate possible scale effects, panels of varying sizes were tested under similar load conditions using the Shear Rig, the Shell Element Tester and facilities elsewhere. The response of high strength concrete elements was investigated by test programs involving four shell elements and two panel elements. Finally, the influence of cyclic loading on the constitutive response of cracked structural concrete was briefly explored via three shell element tests [8]. Several other experimental programs are currently in progress.

Thus, since the development of the MCFT, several test programs have been undertaken involving a total of over one hundred membrane type specimens. The conditions investigated have encompassed a wide range of specimen construction details and loading conditions. In all cases, the MCFT was able to accurately predict behaviour in terms of crack patterns, deformations, reinforcement stresses, ultimate strengths and failure modes. Detailed comparisons of experimental versus theoretical response, for each of the test series, can be found in the references cited.

4. MEMBRANE STRUCTURES

The MCFT was incorporated into nonlinear finite element analysis algorithms, for membrane structures, using several alternative approaches. Adeghe [9] introduced the model's constitutive relations into the standard program ADINA. Stevens et al [8] developed program FIERCM, a nonlinear algorithm based on a tangent stiffness solution procedure and utilizing high-order quadratic strain elements. Cook and Mitchell [10] developed program FIELDS, also a nonlinear finite element algorithm based on a tangent stiffness scheme. Lastly, a secant-stiffness based algorithm was used by Vecchio [11, 12] in developing program TRIX.

In the secant-stiffness formulation, finite elements were developed such as to completely represent the formulations of the MCFT. By incorporating these elements into a iterative linear elastic procedure, nonlinear analysis capability was achieved. The resulting procedure demonstrated good numerical stability and good convergence characteristics. Extensions to the formulations were later developed [12] which permitted the consideration of prestrain effects in the component materials. Prestressing of the reinforcement, shrinkage or expansion of the concrete, or other types of strain offset effects could then be considered.

The accuracy of the finite element analyses were examined by predicting the response of the various membrane elements tested, as well as by modelling several 'benchmark' tests reported in the literature. In general, aspects of response pertaining to strength, stiffness, cracking patterns, reinforcement stresses, concrete distress regions, and failure modes were all predicted with good accuracy [6, 7, 8, 9, 10, 11, 12].

5. BEAM SECTIONS

The MCFT formulations were adapted to the analysis of structural concrete beams subjected to combined shear, flexure and axial loads by developing a layered section analysis procedure (program SMAL) [13]. In the layered procedure, the only sectional compatibility requirement enforced was that plane sections remain plane. Sectional equilibrium requirements included a balancing of the shear flows as well as of the member end actions. Beyond this, uniform stress conditions were assumed to exist in each concrete layer and each longitudinal rebar element. Conditions of compatibility and equilibrium in the concrete layers were enforced according to the formulations of the MCFT. Thus, given the sectional forces, the two-dimensional stress and strain conditions within each layer of the section could be computed. An iterative solution algorithm was employed.

The formulations were found to provide reasonably accurate predictions of the load-deformation response, ultimate load and failure mode of beam specimens. For example, the thirty-five T-beams tested at the University of Washington, under various conditions of shear and axial load, were modelled analytically and very good agreement was obtained. The ratio of experimental to predicted strength had a mean of 1.01 and a coefficient of variation of 15% [13].

6. PLANE FRAMES

A nonlinear frame analysis procedure (ie. program TEMPEST) was developed to model the response of reinforced concrete plane frames subjected to thermal and mechanical loads [14, 15]. The procedure was primarily based on performing rigorous sectional analyses of frame members at several points along their length, and then enforcing these sectional responses in the overall response of the frame. This was done by various means; for example, by defining effective sectional stiffness factors, unbalanced forces, fixed-end forces, or combinations of these. The sectional analyses were performing in the manner previously described for beam sections, but assuming uniform shear flow distributions through member cross-sections. Thus, the multi-layer section analyses incorporated into the frame procedure considered two-dimensional stress conditions in the manner of the MCFT.

Two large-scale frame models were fabricated and tested to corroborate the analysis program [15, 16]. The one-span, two-storey models had a centre-to-centre span of 3.5m and an overall height of 4.6m. The first specimen was subjected to a concentrated transverse load applied at the midspan of the first-storey beam. The second was subjected to constant axial column loads combined with monotonically increasing lateral load applied at the top storey level. The analysis procedure was found to give reasonably accurately predictions of the complex nonlinear response of the test frames. Shear-related effects contributed significantly to the deformations, and membrane action and geometric nonlinearity significantly affected the load capacities. These effects could only be captured in the theoretical analyses by considering two-dimensional stress conditions within the frame members.

7. PLATES AND SHELLS

Analytical procedures based on the MCFT were also developed for modelling the response of reinforced concrete plate and shell elements. Program SEP was initially developed by Kirschner and Collins [17] to predict the response of elements subjected to membrane forces, bending moments and torsional moments. A strain compatibility approach was coupled with a layered element technique. Appropriate assumptions were made regarding the distribution of strains across the thickness of the element. MCFT-derived constitutive relations were then used to calculate stresses within each of the layers. Program SEP was then further developed to account for the influence of out-of-plane shear [17]. This required a consideration of tri-axial stress conditions in each of the layers of the shell element, making the computational algorithm complex, time-consuming, and somewhat unstable. Thus, program SHELL474 was developed by Adebar and Collins [18]. Using the layered element approach, and program SEP as a subroutine, SHELL474 introduced a simplification whereby only the middle layer of the element was analyzed for the three-dimensional stress conditions.

The finite element analysis program APECS was developed to provide global nonlinear analysis capability [19]. The 42-degree-of-freedom quadrilateral shell element formulated allows the modelling of plate or shell structures containing arbitrary in-plane and out-of-plane reinforcement. Also employing a MCFT-based layered-element formulation, the analysis procedure is able to model response to general loading conditions, including a consideration of out-of-plane shear, material prestrains, membrane action, tension stiffening effects, and local stress conditions at crack locations.

Several series of specimens were tested in the 'Shell Element Tester' to corroborate the analytical formulations. Kirschner [17] and Polak [19] tested a total of ten shell elements under various conditions of membrane forces combined with flexure. Adebar [18] tested 9 shell elements and 27 beam elements involving conditions of membrane forces combined with out-of-plane shear. Other test programs are currently underway. In applying the analytical procedures to model the behaviour of the test specimens, good agreement was generally found.

8. THREE-DIMENSIONAL SOLIDS

A nonlinear finite element program (SPARCS) was developed for the analysis of reinforced concrete solids [20]. The program was derived using the analytical models previously described for membrane elements, extrapolated to three dimensions. Thus, an iterative linear elastic formulation was used in which secant moduli were defined and progressively refined according to current local stress/strain states. The three-dimensional constitutive relations incorporated into the formulation were ones extrapolated from the two-dimensional models of the MCFT. An eight-noded regular hexahedral element was formulated accordingly.

To obtain an indication of the potential accuracy of the three-dimensional formulation, torsion beams tested by Onsongo [21] were modelled. The hollow, rectangular beams were subjected to varying conditions of torsion and flexure. They represented a stringent test of the analysis procedure because of the complex loading condition, and because the beams were generally over-reinforced and governed by failure of the concrete. The ultimate load and failure mode of the ten beams tested were predicted very well. The ratio of experimental to theoretical strength had a mean of 0.99 and a coefficient of variation of 6.1%. The load-deformation responses and local strain conditions were also modelled well.

9. DISCUSSION

Breen [22] has pointed to the need for "developing unified, consistent analysis and design approaches" which can be universally applied to "continuum of structural concrete", but has cautioned that "we must dispel the present preoccupation with complex analysis procedures". Further, he has suggested that "as nonlinear analysis packages develop, it is possible nonlinear finite element analysis may be useful" [in this regard]. Scordelis [23] has echoed these feelings, stating that "there is a need to develop a unified approach for the analysis and design of the entire spectrum of ... structural concrete systems which takes advantage of the latest analytical and experimental research, materials, computers, and practical design and construction experience". He adds that "it is desirable to have refined analytical models and methods of analysis which can trace the structural response ... under increasing loads through their elastic, cracking, inelastic, and ultimate ranges".

The MCFT is a conceptual and mathematical model for structural concrete that is consistent with the perceived needs stated by Breen and by Scordelis. As has been shown, the theory presents a unified, rational analysis approach that can be applied to structural concrete in many of its various forms and applications. The theory's formulations are simple, transparent and easy to implement. Further, emphasis is placed on accurately describing the constitutive behaviour of structural concrete, as oppose to developing complex and sophisticated mathematical solutions. The constitutive models incorporated have been corroborated extensively with experimental data. Thus, while MCFT analysis procedures may not always represent the optimal solution to routine analysis and design applications, they do provide for a consistent approach in performing an accurate 'trace of structural response' in situations where it is deemed necessary.

10. CONCLUSIONS

The Modified Compression Field Theory (MCFT) provides a simple, rational framework for modelling the behaviour of cracked reinforced concrete. The theory presents formulations for satisfying conditions of strain compatibility and stress equilibrium in a continuum and, most importantly, defines realistic constitutive relations for the concrete and reinforcement.

The accuracy and range of applicability of the MCFT has been extensively corroborated with additional experimental research since its formulation. The test programs undertaken, conducted on simple panel and shell elements under well controlled conditions, have covered a wide range of structural parameters and loading conditions. The theory, still in essentially its original form, has been found to provide fairly consistent and accurate results in all cases.

The simplicity of the MCFT formulations has allowed them to be easily adopted into various analytical algorithms. Procedures have been developed for the nonlinear analysis of membranes, beams, plane frames, plates and shells, and three-dimensional solids.

In applying the analysis procedures to the modelling of more complex structural systems, generally good correlation was found between predicted and observed responses. The theory was found to provide accurate modelling of crack patterns, deformations, reinforcement stresses, ultimate strengths, and failure modes.

REFERENCES

1. Collins, M.P., Vecchio, F.J., and Mehlhorn, G., "An International Competition to Predict the Response of Reinforced Concrete Panels", Canadian Journal of Civil Engineering, V. 12, No. 3, Sept 1985, pp 626-644.

2. Mitchell, D., and Collins, M.P., "Diagonal Compression Field Theory - A Rational Model for Structural Concrete in Pure Torsion", ACI Journal, V. 71, No. 8, Aug 1974, pp 396-408.

3. Collins, M.P., "Towards a Rational Theory for RC Members in Shear", Proc., ASCE, V. 104, ST4, Apr 1978, pp 649-666.

4. Vecchio, F.J., and Collins, M.P., "The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear", ACI Journal, V. 83, No. 2, 1986, pp 219-231.

5. Bhide, S.B., and Collins, M.P., "Influence of Axial Tension on the Shear Capacity of Reinforced Concrete Members", ACI Structural Journal, V. 86, No. 5, Sept - Oct 1989, pp 570 - 581.

6. Vecchio, F.J., and Chan, C.C.L., "Reinforced Concrete Membrane Elements with Perforations", ASCE Journal of Structural Engineering, V. 116, No. 9, Sept 1990, pp 2344-2360.

7. Vecchio, F.J., and Nieto, M., "Shear-Friction Tests on Reinforced Concrete Panels", ACI Structural Journal, (accepted for publication Sept 1990).

8. Stevens, N.J., Uzumeri, S.M., and Collins, M.P., "Analytical Modelling of Reinforced Concrete Subjected to Monotonic and Reversed Loadings", Publication No. 87-1, Department of Civil Engineering, University of Toronto, Jan 1987, 201 pp.

9. Adeghe, L.N., "A Finite Element Model for Studying Reinforced Concrete Detailing Problems", Ph.D. Thesis, University of Toronto, 1986, 264 pp.

10. Cook, W.D., and Mitchell, D., "Studies of Disturbed Regions near Discontinuities in Reinforced Concrete Members", ACI Structural Journal, V. 85, No. 2, Mar - Apr 1988, pp 206-216.

11. Vecchio, F.J., "Nonlinear Finite Element Analysis of Reinforced Concrete Membranes", ACI Structural Journal, V. 86, No. 1, Jan - Feb 1989, pp 26-35.

12. Vecchio, F.J., "Reinforced Concrete Membrane Element Formulations", ASCE Journal of Structural Engineering, V. 116, No. 3, March 1990, pp 730-750.

13. Vecchio, F.J., and Collins, M.P., "Predicting the Response of Reinforced Concrete Beams Subjected to Shear Using Modified Compression Field Theory", ACI Structural Journal, V. 85, No. 3, May - June 1988, pp 258-268.

14. Vecchio, F.J., "Nonlinear Analysis of Reinforced Concrete Frames Subjected to Thermal and Mechanical Loads", ACI Structural Journal, V. 84, No. 6, Nov - Dec 1987, pp 492-501.

15. Vecchio, F.J., and Emara, M.B., "Shear Deformations in Reinforced Concrete Frames", ACI Structural Journal, (submitted for publication July 1990).

16. Vecchio, F.J., and Balopoulou, S., "On the Nonlinear Behaviour of Reinforced Concrete Frames", Canadian Journal of Civil Engineering, October 1990.

17. Kirschner, U., and Collins, M.P., "Investigating the Behaviour of Reinforced Concrete Shell Elements", No. 86-09, Department of Civil Engineering, University of Toronto, Sept 1986, 83 p.

18. Adebar, P., "Shear Design of Concrete Offshore Structures", Ph.D. Thesis, University of Toronto, 1989, 197 p.

19. Polak, M.A., "Nonlinear Finite Element Analysis of Reinforced Concrete Shells", Ph.D. Thesis (in progress).

20. Vecchio, F.J., and Selby, R.G., "Towards Compression Field Analysis of Reinforced Concrete Solids", ASCE Journal of Structural Engineering, (accepted October 1990).

21. Onsongo, W., "The Diagonal Compression Field Theory for Reinforced Concrete Beams Subjected to Combined Torsion, Flexure and Axial Load", Ph.D. Thesis, Univ. of Toronto, 1978, 246 pp.

22. Breen, J.E., "Why Structural Concrete", Introductory Report, IABSE Colloquium, Stuttgart, 1991.

23. Scordelis, A., "Analysis of Structural Concrete Systems", IABSE Colloquium, Stuttgart, 1991.