Nonlinear behaviour of deep beams

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Comportement non-linéaire d'un élément porteur de type cloison

Nichtlineares Tragverhalten von Scheiben

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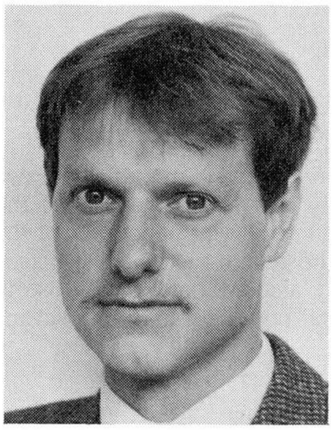

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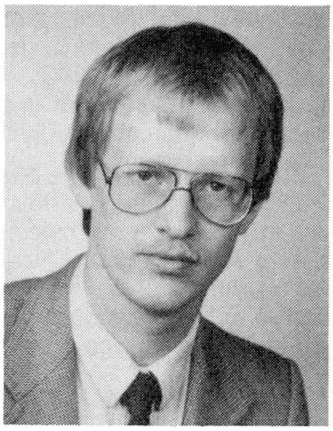

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SUMMARY

This paper deals with an analytical method, which is an extension of the method of Strut-and-Tie-Models. It enables the design engineer to calculate the nonlinear behaviour of deep beams in a simple way. It yields approximate values for the ultimate load, the support reactions of statical indeterminate systems, the important stresses and strains and the load deflection behaviour, respectively. Additionally, a simultaneous control of locally high stressed regions, e.g. near supports or loadings, is possible. The results of this analysis can be used also as an independent verification of the results of a nonlinear finite analysis.

RÉSUMÉ

On présente dans ce rapport une méthode dérivée de la modélisation par bielles (analogie du treillis) qui permet à l'ingénieur de calculer simplement le comportement non-linéaire d'éléments porteurs de type cloison. On obtient ainsi des valeurs approchées de la charge ultime, des réactions d'appui de systèmes hyperstatiques, des contraintes et des déformations déterminantes ainsi que des diagrammes force-déplacement. Le contrôle de régions particulièrement sollicitées, comme le voisinage des appuis ou le point d'application d'une force concentrée est de plus rendu possible. Les résultats obtenus par cette méthode peuvent être employés pour vérifier une analyse non-linéaire par éléments finis.

ZUSAMMENFASSUNG

In dieser Abhandlung wird ein Verfahren vorgestellt, das eine Erweiterung der Methode der Stabwerkmodelle darstellt und dem entwerfenden Ingenieur ermöglicht, das nichtlineare Tragverhalten seines Tragwerks vereinfacht zu untersuchen. Dabei können z.B. die tatsächlich erreichbare Bruchlast, Auflagerkräfte statisch unbestimmter Systeme, massgebende Spannungen, Dehnungen und Last-Verschiebungskurven ermittelt werden. Zusätzlich ist eine Kontrolle lokal hoch beanspruchter Knotenbereiche, z.B. im Auflager- oder Krafteinleitungsbereich, möglich. Das Verfahren kann z.B. auch als ingenieurmässiges unabhängiges Kontrollinstrument für eine nichtlineare FEA verwendet werden.

1. INTRODUCTION

For the analysis and dimensioning of deep beams of reinforced concrete, especially for statical indeterminate structures, the theory of elasticity still is the main basis. However, the real behaviour of these structures under increasing load is determined by the nonlinear behaviour of the materials. This has a major influence on the real ultimate load capacity and the load-deflection-response, respectively.

Up to now the calculation of such effects is only possible with the Nonlinear Finite Element Analysis (NLFEA) and therefore requires an enormous amount of computing. Moreover the results are very difficult to check. Additionally the sophisticated calculation of stresses and strains in every point of the structure is often of no interest (/1/).

This paper presents a practical tool, which enables the design engineer to analyse his concrete structure with respect to some interesting points, e.g. (s. also Fig.1):

- Magnitude of the real ultimate load after the mobilizing of all the bearing capacities within the structure;
- Determination of the structural elements which are probably responsible for the failure of the whole structure, e.g. node regions, parts of reinforcement etc.;
- Load-Deflection-Behaviour with increasing load;
- Distribution of the support reactions in statical indeterminate structures;
- Sensitivity of a structure to restraint;
- Response of the structure to variations in amount and arrangement of reinforcement (e.g. partially prestressed ties, redistribution of reinforcement between span and support-regions etc.).

The presented method is an extension of the Strut-and-Tie-Model (STM) analysis and therefore draws attention to those elements, which mainly influence the load bearing capacity and the behaviour of the structure. The assumptions for the numerical calculation, e.g. for the strength and nonlinear behaviour of the materials, are always present because of the small number of elements. So the response of the structure remains transparent and the results are easy to control. Therefore this tool may also become a very useful educational aid (/3/). Because of the permanent check of the highly stressed local regions this tool performs both at global as well as at local level, which is strictly demanded in /4/.

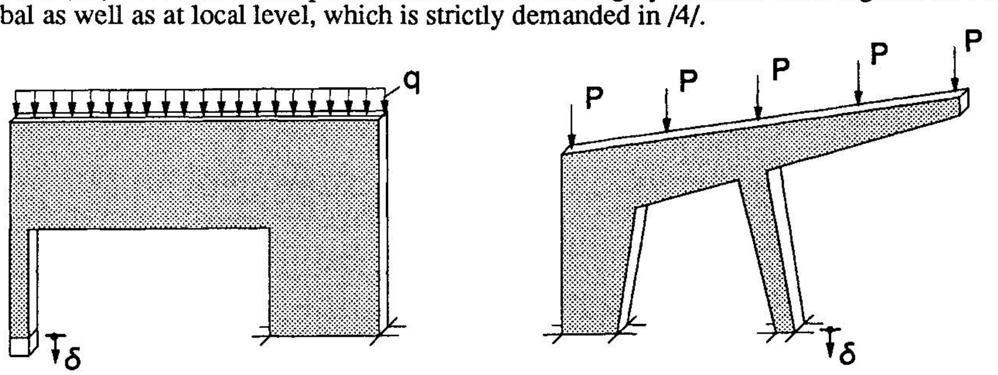


Fig.1 Two typical deep beams, for which a study of the load carrying behaviour due to load (q and P) and settlement (δ) may lead to a better estimation of the real structural safety.

2. FORMULATION OF THE METHOD

The **Flow of Forces** in the structure can be simply modeled either by hand ("Load Path Method", /2/) or by a more refined analysis of the structure with a Linear Finite Element Analysis (/5/). The result is a STM with struts, reinforced ties and nodes, s. Fig 2(a).

The **Strut-and-Tie-Net** separates the predominantly one-dimensional stress fields of the struts and ties from the two-dimensional stress fields of the nodes. With the fixed netpoints and the assumption of some effective widths of the struts, s. fig 2(b), the geometry of the Strut-and-Tie-Net can be found as a result of the current geometry of the STM, s. fig 2(c). Now the effective widths of the struts and ties and consequently their stresses, which govern the capacity and behaviour of these elements, can be computed.

The bearing capacity of the individual **Ties** is determined by the amount and strength of the reinforcement. The nonlinear behaviour can be calculated either according to the well-known regulations in codes (e.g. MC 90, EC 2 etc.) or according to appropriate publications. In this paper a simple formulation is used, which is an extension of the tension stiffening relations given in /7/. With its help the number of cracks, the crack-width and the average deformation of the ties can be calculated. Additionally an early state of cracking can be assumed. The different kinds of reinforcing-steel (e.g. in a partially prestressed ties) can be taken into account by their stress-strain-curves including the strain hardening, s. fig. 3(a).

The capacity of the **Struts** is governed by the stresses at the borderline between the node and strut regions.

For simplification the stresses are calculated in the so called "Transition-point", s. fig. 4. The description of the nonlinear behaviour follows a modified rule in the MC 90 and the DIN 1056, respectively. With the help of 3 factors, which determine the actual strength (α_f), the tangent modulus at the origin (α_E) and the strain reached at ultimate stress (α_ε) a wide range of nonlinear response of the struts can be covered, s. fig. 3(b). The values of these factors depend e.g. on the geometrical form of the struts (fan- or bottle-shaped) and their structural design (longitudinal or transversal reinforcement).

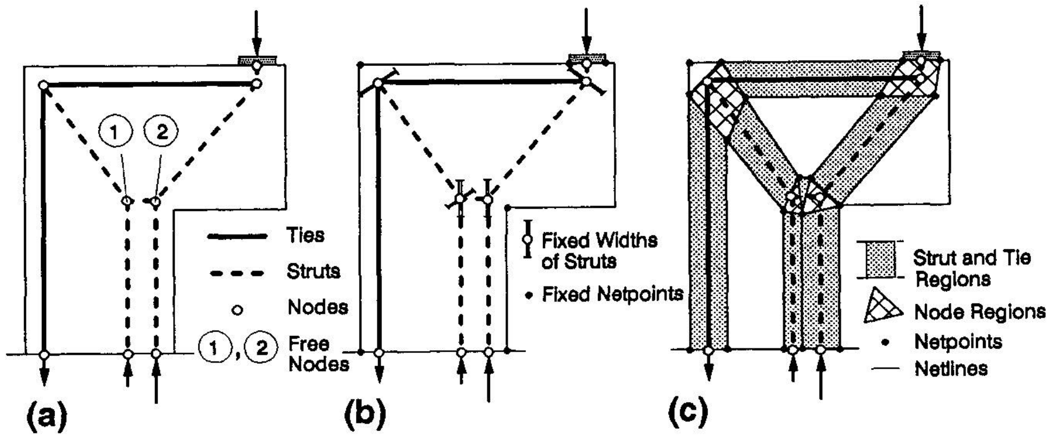


Fig.2 Evaluation of the Strut-and-Tie-Net demonstrated for a cantilever wall:
(a) The structure with its borderline, external forces and the STM
(b) Determination of the fixed netpoints and some widths of struts
(c) Complete Strut-and-Tie-Net with the effective idealized fields of struts, ties and nodes

The bearing capacity of the **Nodes** is also determined by the stresses in the "Transition Point", s. fig. 4. It depends on the kind of node (e.g. pure compression- or bond-node) and its detailing (e.g. kind of anchorage, amount of transverse reinforcement etc.) and is described by a strength factor (α_n), too.

The **time dependent behaviour** is determined by the creep coefficient, which can be adopted from code instructions. For this method the behaviour of struts and ties is considered differently: the struts have a creep strain, whereas the cracked ties have a time dependent decrease of the bond-stresses.

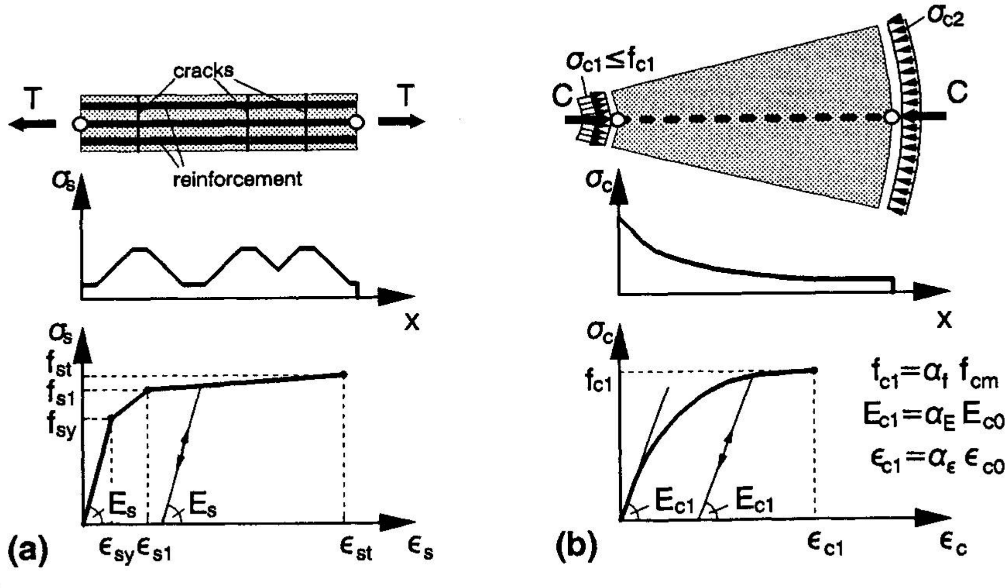


Fig.3 Calculation of the nonlinear behaviour of Ties (a) and Struts (b)

The positions of the **Free Nodes** are determined by applying an energy-criterion. This yields the best simulation of the real load bearing behaviour regarding to the accuracy of the chosen STM. The applied energy-criterion of the extreme value of the overall potential is computed by the internal deformation energy and the potential of external forces and support settlements. The compatibility of this equilibrium state is thus automatically guaranteed.

For the **numerical calculation** the loading, settlements and creep coefficients can be increased step by step, whereby the best position of the free nodes is computed in every loading step. If some struts or ties start to fail near the ultimate load, the increments are reduced to 1/5 of the original value. This ensures a sufficient redistribution of the inner forces. The results of the computation are directly presented in diagrams, which allow a fast and engineering analysis of the structure (e.g. load deflection curves, important strains and stresses, crack-widths, support reactions etc.).

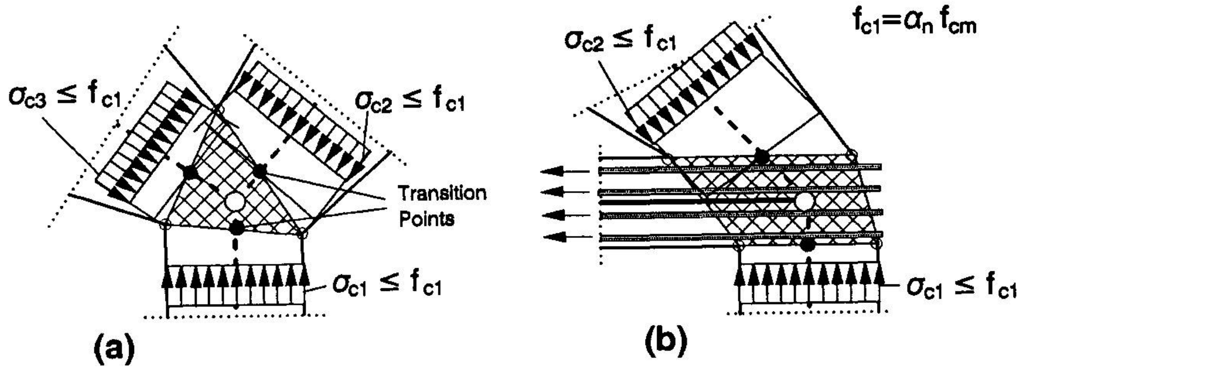
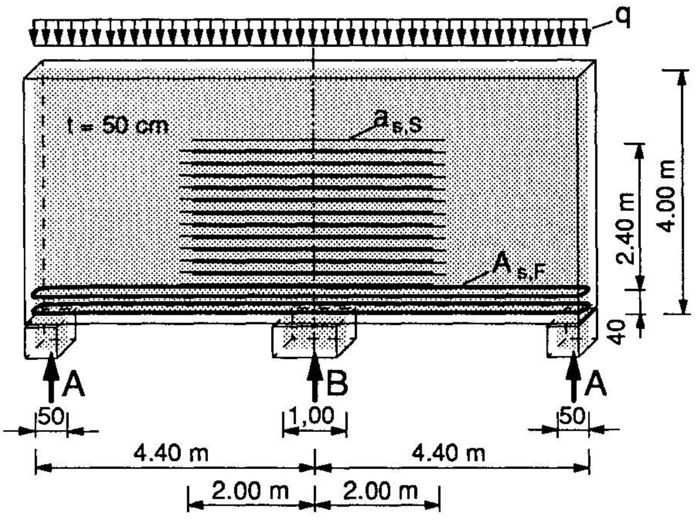


Fig. 4 Two examples for node regions and their governing stresses
(a) Pure compression node (CCC)
(b) Node with deviation of struts due to the anchorage of a tie (CTC)

3. ANALYSIS OF AN EXAMPLE

In fig. 5 a deep beam with 2 spans is shown with its geometry, loading and two cases of reinforcement. In case 1 the reinforcement is chosen according to a linear-elastic analysis. In case 2 the amount of the span-reinforcement is substantially increased, while simultaneously the reinforcement at support is reduced as against case 1. The load carrying behaviour for both cases now will be examined with the above presented method.



Concrete: f_{cm} = 36.0 MN/m^2
f_{ctm} = 3.20 MN/m^2
E_{co} = 37,000 MN/m^2
ε_{co} = 2.2 %.

Reinforcement:
f_{sy} = 500 MN/m^2
f_{st} = 550 MN/m^2
E_s = 210,000 MN/m^2

Case 1: Reinforcement due to linear-elastic analysis
$A_{s,F}$ = 8 ϕ 20 = 25.12 cm^2
$a_{s,S}$ = ϕ 12/20 cm = 11.3 cm^2/m

Case 2: Increased span reinforcement
$A_{s,F}$ = 12 ϕ 25 = 58.92 cm^2
$a_{s,S}$ = ϕ 10/25 cm = 6.28 cm^2/m

Loading: q = 1.0 MN/m (service load)

Fig.5 Geometry, reinforcement and loading of the example

The structure is modeled with a very simple STM, s. fig. 6(a), which was found by the "Load Path Method". The geometry of this STM determines the initial position of the free nodes for both cases. In order to leave open the relations of the support reactions (A and B) and also the internal lever arm in the span (distance between tie 1 and strut 13) the free nodes 7 to 10 can vary in any direction. To allow a free distribution of the tie forces 7 and 17, the nodes 5 and 6 can vary in their x-coordinates. Regarding the symmetry and the internal coupling of nodes there are altogether 3 degrees of freedom for the whole system. The ultimate stresses at the supports are set to f_{c1} = **1.20** f_{cm} (α_n=**1.20**) because of the very proper node design with loops, while all other factors (α_f, α_E and α_ε) are simply set to **1.0**. To avoid a contribution of the concrete tensile strength over the middle support, the Ties 7 and 17 are assumed to be precracked with one crack, whereby the tie 17 is assumed to be reinforced like tie 7. The load increment is Δq = 0.50 MN/m.

In the first load step (q = **0.50 MN/m**) the structure behaves in a linear-elastic mode. The position of the free nodes, calculated by the energy-criterion, lead to the geometry of the STM shown in fig. 6(b). The internal lever arm and the distribution of stresses over the middle support agree very well with a linear-elastic FEA.

With increasing load the geometry of the STM changes, which is caused by a redistribution of the inner forces. The internal lever arm increases in both cases, until the geometries for the STMs at the ultimate loads are obtained, see fig. 6(c). In case 1 the ultimate load is q_u = **3.60 MN/m** and the failure of the structure is initiated by a simultaneous failure of the reinforcement in the span and at support. In case 2 the ultimate load is q_u = **4.60 MN/m** and the failure of the structure is caused by a failure of the nodes at the supports, which have nearly all the same pressures at the ultimate load.

The relation of **B/A**, see fig 6(d), shows a distinct increase for the wall in case 1. This is due to a redistribution of forces from the span to the support region because of the relatively weak stiffness of the tie in the span. In case 2 the ratio B/A remains nearly constant, it even deminishes nearby the ultimate load.

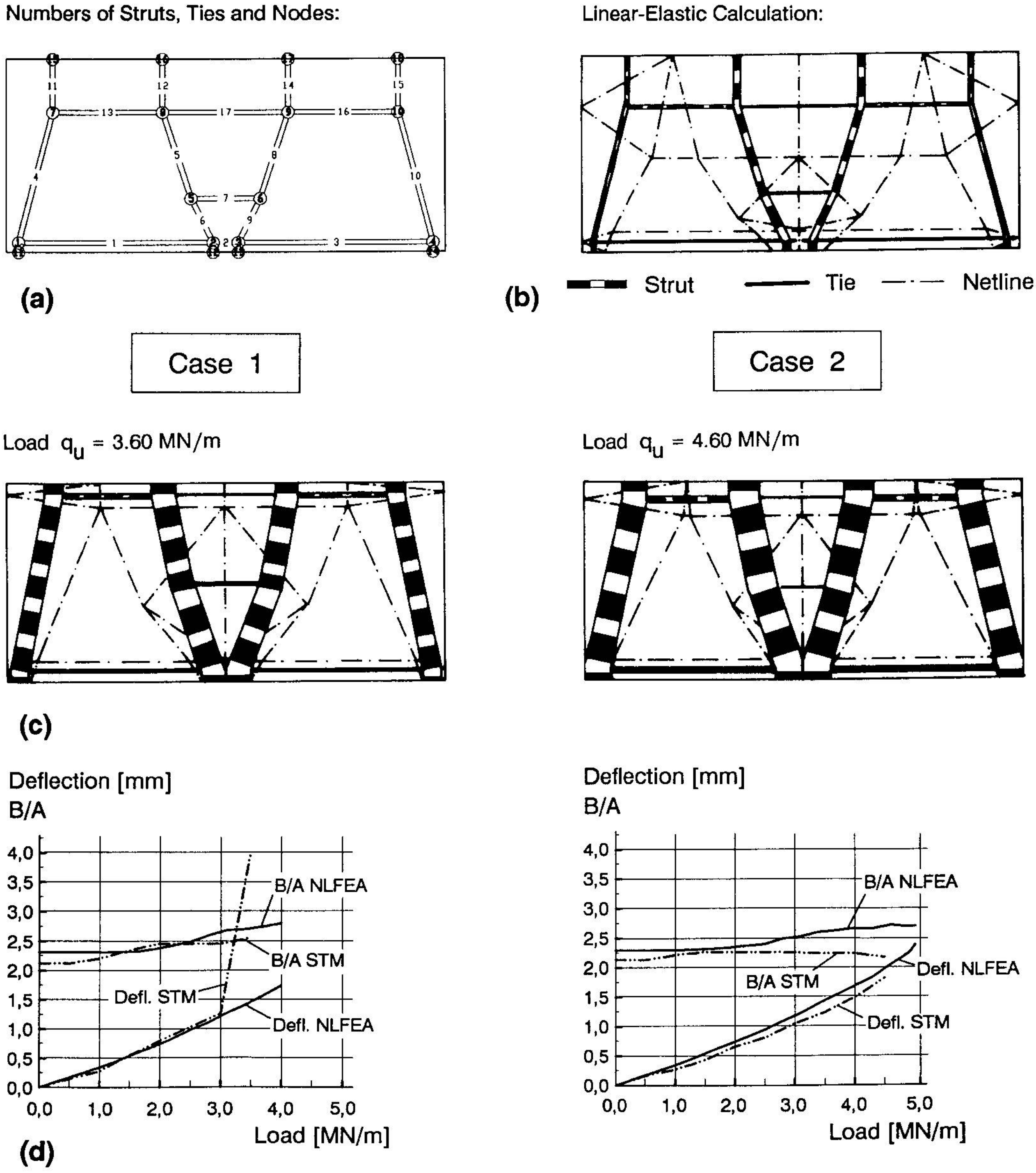


Fig. 6 Response of the structure

The **load-deflection-curves** in fig. 6(d) show a quite linear increase of the deflections because of the overall stiffness of the structure. Only in case 1 the deflection rapidly increases shortly before the ultimate load is reached. This is due to the yielding of the span- and support-reinforcement.

Additionally a NLFEA was carried out using the program SBETA described in /8/. The aim was to proof the reliability of the demonstrated STM method. Some results are shown in the diagrams (s. fig. 6(d)) in comparison to the calculations with the STMs. The ultimate loads of the NLFEA (q_u = **4.10 MN/m** for case 1 and q_u = **5.20 MN/m** for case 2) are slightly higher than the values of the STMs. In case 1 the increase of **B/A** occurs at higher loads in the NLFEA than in the STMs. This is due to the uncracked state, which is conserved up to higher loads in the NLFEA. In case 2 the relation B/A is higher in the NLFEA, which is also caused by uncracked areas especially at the top region over the middle support. The **load-deflection-curves** are in good agreement, apart from the case 1, where a difference occures near the ultimate load, because of a greater stiffness of the wall in the NLFEA. A more refined STM would of course improve the simulation of this deep beam. More details about the non-linear calculations with STM and the NLFEA including further examples can be found in /6/.

4. CONCLUSIONS

It could be demonstrated that the overall load carrying behaviour is simulated quite well with the presented tool. The discussed method especially provides the following advantages:

- The flow of forces remains transparent throughout the whole nonlinear analysis because of the small number of load bearing elements.
- The capacity of locally high stressed regions can be adapted separately according to their structural design.
- The whole numerical analysis runs on a PC with small equipment and takes only a very short time.
- This tool enables the engineer to analyse the behaviour of his structure under varying boundary conditions (e.g. with or without concrete tensile strength, with or without creep effects etc.).
- An adaptation of this model to future code regulations is easily possible.

When applying this method, the following points should be paid attention to:

- The chosen STM must be able to follow all the possible internal redistributions of the forces.
- The method cannot cover all structural effects, especially in the uncracked state, because of the simple modelling.

This practical tool promotes a very good understanding of the whole structure and the interaction of its components. This leads to a better design and to a more reliable estimation of the safety of the analysed structure.

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