### **Dimensioning and detailing**

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Objekttyp: Article

Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band (Jahr): 62 (1991)

PDF erstellt am: 23.07.2024

Persistenter Link: https://doi.org/10.5169/seals-47668

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### **Dimensioning and Detailing**

### Dimensionnement et élaboration des détails constructifs

Bemessung und Konstruktion von Betontragwerken

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#### SUMMARY

Truss model approaches and related theories are presented in an attempt to provide a synthesis of recently-developed methods, permitting a consistent dimensioning and detailing of structural concrete members.

### RÉSUMÉ

Un aperçu est donné de quelques travaux récents basés sur des modèles de treillis afin d'arriver à une synthèse permettant un dimensionnement cohérent et une élaboration correcte des détails constructifs des structures en béton.

### ZUSAMMENFASSUNG

Fachwerkmodelle und verwandte Verfahren werden im Hinblick auf eine einheitliche Bemessung und Konstruktion von Betontragwerken im Sinne einer Synthese dargestellt.



#### 1. INTRODUCTION

The idea of using truss models for following the flow of internal forces in reinforced concrete structures, which emerged about one hundred years ago, was greatly advanced over the first two decades of this century [23]. Being based on careful observations of the behaviour of actual and test structures as well as a thorough understanding of basic principles, the clarity and the wide range of applicability of these concepts still deserve our admiration.

Truss models provide powerful tools for the structural dimensioning and detailing. Mörsch's classical 45-degree truss model concept has been adopted by most codes of practice as the basis of their shear and torsion design provisions. Unfortunately, the originally simple and transparent approach was obscured by many empirical modifications. As a result, rather than being simple, rational and general, like the methods for the dimensioning of sections subjected to flexure and axial load, shear and torsion design procedures became complex, empirical and restricted.

Despite the unfavourable code developments practising engineers have commonly applied truss models. However, as there was little if any official support, many engineers had some doubts about the justification of their methods. This has changed in recent years. The renewed interest in truss models has led to a universal acknowledgement of their potential but current code revisions have yet to avoid the danger of again introducing too many restrictions for their actual use.

This paper attempts to provide a synthesis of truss model approaches and related theories. The presentation starts with the familiar problem of beams subjected to flexure and shear. Next, geometric and static discontinuity regions are treated, followed by a chapter on the dimensioning of plates and shells for the effects of in-plane and transverse loading. The subsequent discussion of members subjected to combined actions is based on the treatment of plate elements subjected to in-plane forces. Finally, considerations of failure mechanisms involving discrete collapse cracks and slip lines are briefly reviewed and a set of conclusions regarding necessary further developments is given.



#### 2. SHEAR AND FLEXURE IN BEAMS

#### 2.1 Introductory example

To introduce a few basic concepts consider the uniformly loaded Ibeam represented in Figs. 1(a) and (b).

Fig. 1(a) shows the expected crack pattern due to the factored load of 200  $kNm^{-1}$ . It consists of vertical flexural cracks at midspan, inclined web shear cracks near the supports and inclined flexure-shear cracks in-between.

The crack pattern suggests that the beam acts like a truss with the flanges forming the chords, the stirrups acting as posts and the web concrete providing the compression diagonals. Three possible truss models for one half of the beam are shown in Figs. 1(c) through (e). In any case, the distributed load has been replaced by statically equivalent single loads and the effective depth of the web, dv, is equal to 1000 mm since the chords have been assumed to coincide with the flange centres. Resultant truss member forces (in kN) are indicated in the figures. It can be observed that choosing a flatter inclination of the truss diagonals leads to less stirrup reinforcement while more longitudinal reinforcement is required.

Fig. 1(f) shows a discontinuous stress field corresponding to the truss model of Fig. 1(d). Truss diagonals are indicated as lines AL, BJ and DH. While diagonals AL and DH correspond to fan-shaped stress fields in regions AKM and CEGI, diagonal BJ corresponds to a uniformly compressed band of web concrete, ACIK. Finally, truss posts BL and DJ correspond to uniformly distributed stirrups in regions ACKM and CEIK. Thus, truss diagonals and diagonal forces simply represent the lines of action and the magnitudes of the stress resultants in the associated fans or bands. Similarly, truss posts and post forces represent the position and magnitude of the resultant stirrup force within certain beam portions.

Figs. 1(g) and (h) represent the variation of the chord forces according to both the truss model and the discontinuous stress field. It should be noticed that the two diagrams coincide at Sections CK and EI where the stirrup reinforcement is staggered. Furthermore, according to the discontinuous stress field, chord forces vary linearly along boundaries of stress bands, and parabolically along fan boundaries. In either case, the stirrups provide a uniform force but while the shear stresses acting on the chord are uniform for a stress band, they vary linearly along a fan boundary.

On principle, the bottom chord reinforcement may be curtailed such that the provided resistance matches the required resistance according to Fig. 1(g). It can be assumed that the provided resistance varies linearly over the development length of a bar,  $l_d$ .

Then, as shown in Figs. 1(i) and (j), two critical cases can be differentiated, where  $\Delta T$  denotes the resistance provided by the curtailed reinforcement.



Fig. 1 Uniformly loaded I-beam: (a) Loading and crack pattern; (b) Cross-section; (c) through (e) Truss models; (f) Discontinuous stress field; (g) Required factored resistance of bottom chord; (h) Factored top chord force; (i) Critical condition at bar cutoff; (j) Critical condition within development length. Note: Forces in kN and dimensions in mm. Figs. 1(d) and (g) demonstrate that a bottom chord force equal to 40 % of that required at midspan has to be anchored at the support. Assuming a bottom chord reinforcement consisting of eight bars as shown in Fig. 2(a), of which four are anchored behind the support using 90 degree bends as indicated in Fig. 2(b), the other bars may be curtailed.



<u>Fig. 2</u> Detailing of bottom chord reinforcement: (a) Cross-section; (b) Elevation at support.

Fig. 3(a) shows a truss model for the top flange. The compressive force of 1600 kN at midspan is split into two halves applied at the quarter points of the flange width and the shear transfer from the web into the flange is idealised with a series of 45-degree truss in Fig. 3(b). The resultant shear flow is models as shown illustrated by the diagram of Fig. 3(c) which represents one half of the derivative of the function given in Fig. 1(h). Since a 45degree truss model is used, this diagram also shows the necessary factored resistance of the transverse flange reinforcement and from Fig. 3(b) it is clear that this reinforcement has to be placed at a distance of 200 mm from the location where the flow. shear orignates.



<u>Fig. 3</u> Truss model for top flange: (a) Overview; (b) Detail; (c) Shear flow between web and either side of flange.

#### 2.2 Continuous beams and general loading

The truss model and discontinuous stress field approaches can easily be generalised for continuous beams and arbitrary loading.

For fixed loads acting on continuous beams the only difficulty consists in determining the support reactions. Once these are known the beam can be subdivided into segments bounded by sections of zero shear force and the different segments can be treated individually. The segments can be considered as free bodies with known forces and moments applied to them. The truss model approach allows to visualise internal force paths in each segment and it indicates how the reinforcement should be detailed such that the envisaged force flow may indeed develop.

Statically indeterminate support reactions are commonly determined by performing a linearly elastic analysis for the uncracked system. However, members usually crack and apart from applied loads there are typically restraints which may cause effects similar to those produced by the loads. Hence, while an elastic analysis provides a statically admissible equilibrium solution for given loads it should be realised that there will always be some redistribution of the internal forces and moments. This redistribution corresponds to a self-equilibriated (or residual) state of stress whose magnitude depends on the loading and restraining history.

Contrary to an analysis of an existing structure where the actual loading and restraining history may be reasonably approximated this is impossible in design. However, well proportioned and detailed structures are capable of adapting to a large variety of imposed actions in a ductile manner, resulting in considerable redistributions of the internal forces and moments. Hence, rather than trying to analyse these redistributions a designer will attempt to achieve a sufficiently ductile behaviour through good detailing, and determine concrete and reinforcement dimensions based on a simple elastic analysis taking some freely assumed redistribution of the internal forces and moments into account.

For variable loads, moment redistribution within certain limits is usually permitted for each individual loading case. Alternatively, moment and shear force envelopes can be adjusted by superimposing a single residual stress state for all loading cases [20]. The differences between the two approaches have not yet been fully investigated, and in conjunction with a discussion of conventional load and resistance factor design, would deserve the attention of researchers.

If deemed necessary, cracked member stiffnesses can be considered to get an estimate for the residual stress state. An interesting possibility to perform such an investigation is to use the truss model used for proportioning the member by assigning appropriate stiffness values to the individual truss members. However, it is

## Page missing

The cover elements are sujected to the membrane forces given in Fig. 13(d) and can be designed according to the principles outlined in Section 4.2 [21].



Fig. 13 Limit design of slab elements: (a) Sandwich model; (b) Pure shear in uncracked core; (c) Diagonal compression field in cracked core; (d) Forces acting on cover elements.

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Apart from shear transfer in the interior of slabs, shear transfer along slab edges should be considered. Twisting moments at slab edges correspond to a shear force of equal magnitude being transferred along the edge. The analogy to beams subjected to circulatory torsion suggests that slab edges should be similarly reinforced as the side faces of such beams. In particular, the inplane top and bottom reinforcements perpendicular to the edge must be connected through transverse reinforcements at the edge. The truss model of Fig. 14 illustrates this situation.



#### Fig. 14 Truss model for slab corner.

#### 5. MEMBERS SUBJECTED TO COMBINED ACTIONS

#### 5.1 Limit analysis and design

Starting from Fig. 15(a) and connecting several web elements along their stringers, one can build up arbitrary cross-sections which may be subjected to arbitrary combined actions, see Fig. 15(b) and (c). Conversely, arbitrary combined actions can be replaced by a system of axial and shear forces applied to the different stringers and wall elements. For example, side wall 1 in Fig. 15(c) has to resist a shear force of  $V_1 = T/(2a_2) - V_y/2$  where the shear is taken as positive if it acts in the direction of the circulatory shear flow  $S = T/(2a_1a_2)$  due to T. Assigning flexural moments and axial load to the stringers, stringer 1 has to take a force of  $T_1 = N/4 + M_y/(2a_2) - M_z/(2a_1)$ , etc. Alternatively, flexural moments and axial load can be carried by combined axial forces in the stringers and the different wall elements.

Considering a diagonal compression field in each wall element and selecting appropriate inclination angles  $\theta_i$ , the transverse reinforcements can be designed according to Eq. (1). The axial components of the diagonal compression forces,  $V_i \cot \theta_i$ , have to be resisted by the adjoining stringers. Thus, for example, stringer 1 of Fig. 15(c) has to resist T<sub>1</sub> +  $V_1 \cot \theta_1/2$  +  $V_4 \cot \theta_4/2$ .



<u>Fig. 15</u> Subdividing a cross-section into wall and stringer elements: (a) Single web; (b) Open cross-section; (c) Box section.

For a box section subjected to torsion, it is appropriate to assume that the shear flow, due to the torque, acts in the centre planes of the different side walls. A solid cross-section acts like a hollow one since its interior is subjected to triaxial tensile strains. Hence, neglecting the concrete tensile strength, only an exterior tube of diagonally compressed concrete is effective in resisting the torque [16].

Similar to the response to flexure a section subjected to a torsional moment tends to develop as large as possible lever arms of the internal forces resisting the torque. At ultimate, the concrete in the diagonally compressed tube will reach the crushing limit.

In dimensioning a member subjected to combined actions one may start with an estimate of the different wall thicknesses. This determines the lever arms of the internal forces and allows to compute these. Using Eq. (3) the adequacy of the initial estimate can be checked and the procedure can be repeated if necessary. Usually, one iteration is sufficient to get an acceptable solution.



Fig. 16 Connection of wall elements: (a) Force flow; (b) Detailing of transverse reinforcement.



In detailing the transverse reinforcement we have to consider the shear transfer between the interconnected wall elements. Fig. 16(a) illustrates the static conditions and Fig. 16(b) shows an appropriate detail. For ease of placing, hairpin-shaped bars can be spliced with straight bars in the walls.

Similar to the remark made in connection with Fig. 6(f) it can be observed that the tensile strength of the concrete has to be mobilised to activate the cover. In fact, under very high loads the cover may spall off and it would therefore be conservative to use spalled section properties. However, for ease of computation it is advantageous to use unspalled properties along with a cautious assessment of the effective concrete compressive strength  $f_c$ .

Interaction equations for the combined action of flexural and torsional moments and axial laods were derived [15,35,37], and collapse mechanisms were found which are compatible with the static conditions discussed above [24,25].

#### 5.2 Deformations

A variety of methods have been devised which permit to compute the deformations of thin-walled box girders subjected to combined actions in the cracked state [16]. Essentially they are based on the compression field approach and use either a space truss [34] or a finite element model [29].

Usually, flexural and axial load deformations dominate and, fortunately, are much easier to predict than the deformations due to shear and torsion. Thus, application of the referenced methods can be restricted to special cases with significant shear and torsional deformations.

#### 6. CONSIDERATION OF FAILURE MECHANISMS

The material presented in the previous chapters is essentially based on the static or lower-bound method of limit analysis. This chapter briefly reviews concepts of the kinematic or upper-bound method of limit analysis. This method has been widely used for slab design [13] but it has not received the same attention for other applications. However, although not normally stated, the familiar shear friction method [1,4] belongs to the category of upper-bound analyses. Furthermore, the analogy to slip line methods used in soil mechanics has been helpful in solving several problems in the area of concrete structures [27,28,20,25].

Using a lower-bound approach, one is forced to follow the flow of forces throughout a member. Hence, such approaches lend themselves



to dimensioning and detailing which must result in enough of the right material at the right place throughout the member. On the other hand, upper-bound methods are primarily suited for analysis, enabling quick checks of essential dimensions and details. Recent developments have favoured the application of static methods, while perhaps, the kinematic methods have been thrust too much into the background. This is unfortunate because there is much to be gained from such considerations.





Fig. 17 Failure mechanisms: (a) Discontinuity surface in corbel;
 (b) Collapse crack in beam; (c) Web crushing mechanism; (d)
 Web crushing and stirrup yielding in zone of homogeneous deformation; (e) Collapse crack mechanism for knee joint subjected to opening moment [41].

Frequently, we observe that failure of a member occurs at a distinct surface. For example, the corbel shown in Fig. 17(a) may fail along a discontinuity plane extending from the back side of the loading plate to the reentrant corner at the bottom connection of the corbel with the column. At failure, the portion of the corbel under the load moves down and away from the column. The work done by the externally applied load equals the dissipation in the concrete and the reinforcement crossing the discontinuity surface

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where all the deformation is concentrated. Considering a relative translation inclined at an angle a to the discontinuity the work as well as the dissipation can be expressed as functions of a and, on account of the upper-bound theorem, the optimum value of a resulting in the lowest upper-bound for the collapse load can be found.

Neglecting the concrete tensile strength and assuming a perfectly plastic behaviour in compression, the dissipation in the concrete per unit area of a discontinuity surface due to a unit relative displacement amounts to  $f_c(1 - \sin \alpha)/2$ . Thus, for  $\alpha = 90$  deg., i.e., a pure crack opening, there is no dissipation in the concrete. Müller [24] introduced the term "collapse crack" for such a situation. Fig. 17(b) shows an example of a collapse crack mechanism. The stirrups as well as the bottom chord reinforcement crossing the collapse crack are yielding while there is no dissipation in the concrete.

Figs. 17(c) and (d) illustrate web crushing mechanisms of a beam in which the stirrups yield, the web concrete crushes and the chord reinforcement remains elastic. These mechanisms correspond to the static conditions underlying Eq. (8) and the inclination of the discontinuity or slip line in Fig. 17(c) is twice that of the associated stress field [3,25]. The mechanism of Fig. 17(d) can be thought of as a series of slip lines similar to that in Fig. 17(c).

Fig. 17(e) shows a collapse crack mechanism for a knee joint. It appears that similar mechanisms could be developed for numerous other practical problems.



Fig. 18 Failure mechanisms for isostatic beam [6].

Finally, Figs. 18 and 19 illustrate a number of possible collapse crack and web crushing mechanisms for isostatic and continuous girders. Note that flexural failure mechanisms such as Mechanisms IV and V in Fig. 18 are special collapse crack mechanisms. Furthermore, mechanism II in Fig. 19 addresses the problem of insufficient extensions of the longitudinal reinforcement in the vicinity of points of contraflexure, while mechanisms IV and V address the problem of insufficient extensions of the top reinforcement in spans adjoining the most heavily loaded span.



Fig. 19 Failure mechanisms for continuous girders [20].

#### 7. CONCLUSIONS

Truss model approaches and related theories have been presented in an attempt to provide a synthesis of recently developed methods permitting a consistent dimensioning and detailing of structural concrete members. It has been demonstrated that all the different methods supplement each other, having one common base.

Truss or strut and tie model procedures inherently relate to dimensioning and detailing. Having selected initial member sizes application of such procedures is characterised by a trial and adjustment process leading to final dimensions and details. Provision of an adequate minimum reinforcement, development of an appropriate equilibrium model to follow the flow of the internal forces, and consequent dimensioning and detailing are the essential features in this process.

Major areas deserving further work include:

- (i) There is a need for better understanding of bond and development of reinforcement. Associated principles should be brought in line with strut and tie model concepts. This would lead to improved guidance on the effective concrete strength of nodal zones.
- (ii) Models for prestressed members are not yet entirely consistent.
- (iii) More guidance on the selection of minimum reinforcement is required. Possible restraints, deformations under service conditions, and the contribution of such reinforcement to the ultimate strength as well as its effect on the need for compatibility considerations during the development of strut and tie models should be taken into account.
- (iv) Failure mechanism considerations should be further developed to obtain reliable and easily applicable methods for standard design situations.
- (v) A compromise is needed regarding simplified design procedures permitted by codes.
- (vi) Appropriate information and education of the profession are necessary to make the transition to the envisaged unified design approach for structural concrete possible.

#### ACKNOWLEDGEMENTS

Discussions with Dr. David Rogowsky contributed much to clarifying the views expressed in this paper. Regina Nöthiger and Viktor Sigrist assisted in preparing the manuscript. Their contribution is also warmly acknowledged.





### REFERENCES

- [1] ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-89) and Commentary - ACI 318R-89", American Concrete Institute, Detroit, 1989, 353 pp.
- [2] Baumann, T., "Zur Frage der Netzbewehrung von Flächentragwerken (On the Problem of Net Reinforcement of Surface Structures)", Bauingenieur, Vol. 47, No. 10, Oct. 1972, pp. 367-377.
- [3] Braestrup, M.W., "Plastic Analysis of Shear in Reinforced Concrete", Magazine of Concrete Research, Vol. 26, No. 89, Dec. 1974, pp. 221-228.
- [4] Canadian Standards Association, "Design of Concrete Structures for Buildings (CAN3-A23.3-M84)", Rexdale, Ontario, 1984, 281 pp.
- [5] Comité Euro-International du Béton, "CEB-FIP Model Code 1990", First Draft, Bulletins d'Information, No. 195 and No. 196, Lausanne, March 1990.
- [6] Cerruti, L.M., and Marti, P., "Staggered Shear Design of Concrete Beams: Large Scale Tests", Canadian Journal of Civil Engineering, Vol. 14, No. 2, April 1987, pp. 257-268.
- [7] Collins, M.P., "Towards a Rational Theory for Reinforced Concrete Members in Shear", Proceedings, ASCE, Vol. 104, No. ST4, April 1978, pp. 649-666.
- [8] Collins, M.P., and Mitchell, D., "Shear and Torsion Design of Prestressed and Non-Prestressed Concrete Beams", Journal of the Prestressed Concrete Institute, Vol. 25, No. 5, Sept.-Oct. 1980, pp. 32-100. Also, Discussion, Vol. 26, No. 6, Nov.-Dec. 1981, pp. 96-118.
- [9] Collins, M.P., and Mitchell, D., "A Rational Approach to Shear Design - The 1984 Canadian Code Provisions", Journal of the American Concrete Institute, Vol. 83, No. 6, Nov.-Dec. 1986, pp. 925-933.
- [10] Collins, M.P., and Mitchell, D., "Prestressed Concrete Basics", Canadian Prestressed Concrete Institute, Ottawa, 1987, 614 pp.
- [11] Cook, W.D., and Mitchell, D., "Studies of Disturbed Regions near Discontinuities in Reinforced Concrete Members", ACI Structural Journal, Vol. 85, No. 2, March-April 1988, pp. 206-216.
- [12] IABSE, "Colloquium on Plasticity in Reinforced Concrete", Copenhagen 1979, International Association for Bridge and Structural Engineering, Introductory Report, Vol. 28, 1978, 172 pp; and Final Report, Vol. 29, 1979, 360 pp.
- [13] Johansen, K.W., "Yield-Line Theory", Cement and Concrete Association, London, 1962, 181 pp.
- [14] Kupfer, H., "Erweiterung der Mörsch'schen Fachwerkanalogie mit Hilfe des Prinzips vom Minimum der Formänderungsarbeit (Generalization of Mörsch's Truss Analogy Using the Principle of Minimum Strain Energy)", Comité Euro-International du Béton, Bulletin d'Information, No. 40, Paris, Jan. 1964, pp. 44-57.
- [15] Lampert, P., and Thürlimann, B., "Ultimate Strength and Design of Reinforced Concrete Beams in Torsion and Bending", International Association for Bridges and Structural Engineering, Publications, Vol. 31-I, 1971, pp. 107-131.

- [16] Marti, P., "Strength and Deformations of Reinforced Concrete Members Under Torsion and Combined Actions", Comité Euro-International du Béton, Bulletin d'Information, No. 146, Jan. 1982, pp. 97-138.
- [17] Marti, P., "Basic Tools of Reinforced Concrete Beam Design", Journal of the American Concrete Institute, Vol. 82, No. 1, Jan.-Feb. 1985, pp. 46-56. Also, Discussion, Vol. 82, No. 6, Nov.-Dec. 1985, pp.933-935.
- [18] Marti, P., "Truss Models in Detailing", Concrete International: Design and Construction, Vol. 7, No. 12, Dec. 1985, pp. 66-73. Also, Discussion, Vol. 8, No. 10, Oct. 1986, pp. 66-68.
- [19] Marti, P., "Staggered Shear Design of Simply Supported Concrete Beams", Journal of the American Concrete Institute, Vol. 83, No. 1, Jan.-Feb. 1986, pp. 36-42.
- [20] Marti, P., "Staggered Shear Design of Concrete Bridge Girders", Proceedings, International Conference on Short and Medium Span Bridges, Ottawa, Aug. 1986, Vol. 1, pp. 139-149.
- [21] Marti, P., "Design of Concrete Slabs for Transverse Shear", ACI Structural Journal, Vol. 87, No. 2, March-April 1990, pp. 180-190.
- [22] Mitchell, D., and Collins, M.P., "Diagonal Compression Field Theory - A Rational Model for Structural Concrete in Pure Torsion", Journal of the American Concrete Institute, Vol. 71, No. 8, Aug. 1974, pp. 396-408.
- [23] Mörsch, E., "Der Eisenbetonbau Seine Theorie und Anwendung (Reinforced Concrete Construction - Theory and Application)", 5th Edition, Vol. 1, Part 2, K. Wittwer, Stuttgart, 1922.
- 5th Edition, Vol. 1, Part 2, K. Wittwer, Stuttgart, 1922.
  [24] Müller, P., "Failure Mechanisms for Reinforced Concrete Beams in Torsion and Bending", International Association for Bridge and Structural Engineering, Publications, Vol. 36-II, 1976, pp. 147-163.
- pp. 147-163. [25] Müller, P., "Plastische Berechnung von Stahlbetonscheiben und -balken (Plastic Analysis of Reinforced Concrete Walls and Beams)", Institute of Structural Engineering, ETH Zürich, Report No. 83, 1978, 160 pp.
- Report No. 83, 1978, 160 pp. [26] Nielsen, M.P., "On the Strength of Reinforced Concrete Discs", Civil Engineering and Building Construction Series, No. 70, Acta Polytechnica Scandinavica, Copenhagen, 1971, 261 pp.
- [27] Nielsen, M.P., "Limit Analysis and Concrete Plasticity", Prentice-Hall, 1984, 420 pp.
- [28] Nielsen, M.P., Braestrup, M.W., Jensen, B.C., and Bach, F., "Concrete Plasticity", Special publication, Dansk Selskab for Bygningsstatik, Copenhagen, 1978, 129 pp.
- [29] Potucek, W., "Die Beanspruchung der Stege von Stahlbetonplattenbalken durch Querkraft und Biegung (Stresses in Webs of Reinforced Concrete T-beams Subjected to Flexure and Shear)", Zement und Beton, Vol. 22, No. 3, 1977, pp. 88-98.
- [30] Rogowsky, D.M., Mac Gregor, J.G., and Ong, S.Y., "Tests of Reinforced Concrete Deep Beams", Journal of the American Concrete Institute, Vol. 83, No. 4, July-August 1986, pp. 614-623.
- [31] Schlaich, J., and Weischede, D., "Detailing Reinforced Concrete Structures", Canadian Structural Concrete Conference 1981, Proceedings, Department of Civil Engineering, University of Toronto, Toronto, 1981, pp. 171-198.

Α

- [32] Schlaich, J., and Schäfer, K., "Konstruieren in Stahlbetonbau (Detailing in Reinforced Concrete Design)", Betonkalender 1984, Part 2, W. Ernst, Berlin, 1984, pp. 787-1005.
- [33] Schlaich, J., Schäfer, K., and Jennewein, M., "Toward a Consistent Design of Structural Concrete", Journal of the Prestressed Concrete Institute, Vol. 32, No. 3, May-June 1987, pp. 74-150.
- [34] Thürlimann, B., and Lüchinger, P., "Steifigkeit von gerissenen Stahlbetonbalken unter Torsion und Biegung (Stiffness of Cracked Reinforced Concrete Beams Subjected to Torsion and Flexure)", Beton und Stahlbetonbau, Vol. 68, No. 6, June 1973, pp. 146-152.
- [35] Thürlimann, B., Grob, J., and Lüchinger, P., "Torsion, Biegung und Schub in Stahlbetonträgern (Torsion, Flexure and Shear in Reinforced Concrete Girders)", Institute of Structural Engineering, ETH Zürich, 1975, 170 pp.
- [36] Thürlimann, B., "Shear Strength of Reinforced and Prestressed Concrete Beams - CEB Approach", Concrete Design: U.S. and European Practices, SP-59, American Concrete Institute, Detroit, 1979, pp. 93-115.
- [37] Thürlimann, B., "Torsional Strength of Reinforced and Prestressed Concrete Beams - CEB Approach", Concrete Design: U.S. and European Practices, SP-59, American Concrete Institute, Detroit, 1979, pp. 117-143.
- [38] Thürlimann, B., Marti, P., Pralong, J., Ritz, P., and Zimmerli, B., "Anwendung der Plastizitätstheorie auf Stahlbeton (Application of the Theory of Plasticity to Reinforced Concrete)", Institute of Structural Engineering, ETH Zürich, 1983, 252 pp.
- [39] Vecchio, F.J., and Collins, M.P., "The Response of Reinforced Concrete to In-Plane Shear and Normal Stresses", Publication No. 82-03, Department of Civil Engineering, University of Toronto, March 1982, 332 pp.
- [40] Vecchio, F.J., and Collins, M.P., "The Modified Compression Field Theory for Reinforced Concrete Elements Subjected to Shear", Journal of the American Concrete Institute, Vol. 83, No. 2, March-April 1986, pp. 219-231.
- [41] Müller, P., "Consistent Connection Design", Session on 'Innovations in Shear Design', 1990 ACI Annual Convention, Toronto, March 20, 1990.

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