Design approaches for shear reinforcement in concrete beams

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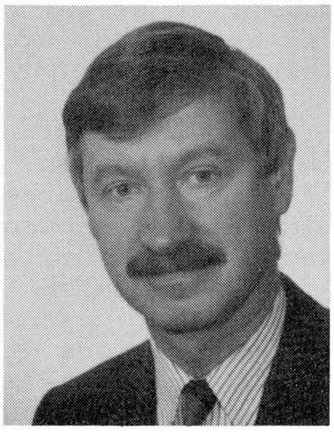
Design Approaches for Shear Reinforcement in Concrete Beams

Détermination de l'armature à l'effort tranchant

Ermittlung der Schubbewehrung von Betonbalken

Wolfgang MOOSECKER
Prof. Dr.
Fachhochschule Giessen
Giessen, Germany



W. Moosecker, born 1948, graduated from Techn. Univ. München and Univ. of Calif. Berkeley. He was a consulting engineer in Krumbach, Germany from 1982 to 1988.

SUMMARY
Most recent approaches to a rational design of shear reinforcement for reinforced concrete beams are formulated in terms of equilibrium of stress fields and compatibility of corresponding strain fields. The review points out common features and differences of these approaches.

RÉSUMÉ
Plusieurs nouvelles méthodes de détermination rationnelle de l'armature à l'effort tranchant des poutres en béton armé sont dérivées de l'équilibre des champs de contraintes et de la compatibilité du champ de déformations correspondant. Les points communs ou divergents de ces méthodes sont successivement traités.

ZUSAMMENFASSUNG
Die meisten neueren Ansätze zur rationalen Bemessung der Schubbewehrung von Stahlbetonträgern sind auf der Grundlage des Gleichgewichtes von Spannungsfeldern und der Verträglichkeit der zugehörigen Dehnungen abgeleitet. Der Überblick behandelt Gemeinsamkeiten und Unterschiede dieser Ansätze.

1. SCOPE

As pointed out in the Introductory Report by MACGREGOR, there is still no general agreement with respect to design of concrete beams for combined shear, bending moment and axial force. But, there is a clear tendency in recent research work on the subject towards physical models with complete description of equilibrium in the shear zone as a rational basis for concrete design.

The review is concerned with recent research work on the description of the equilibrium system of stresses and the compatibility of strains in the web of reinforced concrete beams containing shear reinforcement.

The review is limited to papers published since 1980. This limiting date was deliberately chosen, because the scientific discussion on design of shear reinforcement was decisively influenced by the COLLINS/MITCHELL paper of 1980 [1]. This influence is clearly demonstrated by the fact that all models treated in this review are formulated in terms of stress fields.

Most theories deal with beam regions in constant shear, but - since they are based on physical models - can be adjusted to situations of varying shear. It is assumed that shear failure is due to yielding of shear reinforcement and/or crushing of web concrete. Thus, bending or bond failure is not taken into account.

2. STRESS-FIELD CONCEPTS IN SHEAR DESIGN

In the webs of slender reinforced concrete beams with shear reinforcement, inclined cracks develop prior to shear failure. It is mostly assumed that these cracks are parallel and straight at approximately constant spacing. This simplification is particularly valid for T- or I-shaped beams.

If there are no bending moments in the concrete struts between inclined cracks, the inclined stresses in the concrete web form a continuous inclined compressive stress field σ_c, the angle α to the beam axis of which does not necessarily coincide with the angle α_{cr} of shear cracks. Furthermore, if the spacing of the shear reinforcement is sufficiently close, stresses in the shear reinforcement can be regarded as a continuous tensile stress field σ_t, the angle β to the horizontal of which is equal to the inclination of shear reinforcement.

The traditional truss model for shear design of concrete beams which dates back to the beginning of the century can also be interpreted as a stress field concept in which the inclination of the concrete compressive stress field is taken as 45°.

If the inclination of the compression stress field σ_c is taken as constant over the height of the beam and given the inclination β of the tension stress field σ_t, the stress intensities follow from equilibrium of vertical forces

$$\sigma_c = \frac{V}{b \cdot z} \cdot \frac{1}{\sin^2\alpha\,(\cot\alpha + \cot\beta)} \qquad (1)$$

$$\sigma_t = \frac{V}{b \cdot z} \cdot \frac{1}{\sin^2\beta\,(\cot\alpha + \cot\beta)} \qquad (2)$$

where V = applied shear force
b = web thickness
z = lever arm of bending stress resultants

Hence, there is no fundamental difference between truss analogy and stress field models if the inclination α of the compression stress field in the web is the same. The main problem is to determine this angle α in a rational manner.

3. LOWER BOUND PLASTIC SOLUTION [2],[3]

In theory of plasticity (also called limit analysis), it is assumed that materials exhibit unlimited plastic (i.e. irreversible) deformations when certain stress combinations (yield condition) are reached. Elastic deformations and workhardening effects are normally neglected.

Given that the function describing the yield condition is convex and that the plastic deformations are normal to the yield surface, upper and lower bounds for the failure load of any structure can be derived. The lower bound theorem of plasticity (limit analysis) states that a lower bound to the true failure load can be found from any stress field in equilibrium which does not violate the yield condition.

With respect to web stresses of concrete beams, it follows from the lower bound theorem of plasticity that the inclination α of the web compression field can be freely chosen as long as yield (limit) conditions are not violated. These conditions are usually taken as the yield strength of shear reinforcement and the effective crushing strength f_c^* of concrete in uniaxial compression. The latter cannot be taken directly from normal specimen tests because of cracking and transverse strains in the web concrete.

A maximum of the lower bounds for the failure load of concrete webs in shear is determined, if the angle α of web compression is chosen in such a way that web crushing and yielding of shear reinforcement occurs simultaneously (web crushing criterion). It should be noted that for low amounts of shear reinforcement this assumption leads to inclinations α of web compression well below the crack inclinations observed in tests.

4. THE COMPRESSION FIELD APPROACH OF COLLINS/MITCHELL [1],[4],[5]

Provided that shear crack openings are "smeared" over the web of the beams, compatibility of average strains is governed by Mohr's circle. The compatibility relation between average strain ε_ℓ in the direction of the beam axis, ε_t perpendicular to this axis and the principal inclined compressive strain ε_d can be derived from Mohr's circle as

$$\tan^2 \alpha = \frac{\varepsilon_\ell + \varepsilon_d}{\varepsilon_t + \varepsilon_d} \qquad (3)$$

From eq.(3), the angle α can be determined, if the values of the average strains ε_ℓ, ε_t and ε_d are known.

With the assumption that the directions of the principal compressive strain ε_d and of the inclined web compression field σ_c coincide, this angle α can be used to determine the web stresses from the equilibrium equations (1) and (2). Using this assumption, there is no need to consider the question whether this angle α is equal to the direction α_{cr} of inclined cracks or not.

For stirrups perpendicular to the beam axis (β = 90), ε_t is equal to the average stirrup strain. It can be determined from the stress-strain relationship of stirrup steel, if the stiffening effect of concrete between cracks and the anchorage slip of stirrups are ignored.

The concrete strain in the direction of the inclined compression field, however, cannot be taken from uniaxial load tests, because the large transverse tensile strains exert a softening effect on the stress-strain relationship of web concrete. VECCHIO/COLLINS [4] propose the following relationship between principal compressive stress σ_c and principal compressive strain ε_d which depends also on the magnitude ε_1 of the principal tensile strain.

$$\sigma_c = \sigma_{c,max} \left[2\left(\frac{\varepsilon_d}{\varepsilon_c'}\right) - \left(\frac{\varepsilon_d}{\varepsilon_c'}\right)^2 \right] \qquad (4)$$

where $\sigma_{c,max} = \dfrac{f_c'}{0.8 - 0.34\,\varepsilon_1/\varepsilon_c'}$

$\varepsilon_c' = -0.002$

f_c' = concrete cylinder strength

The strength f_{du} of the web concrete in inclined compression is also influenced by the coexisting transverse strain. COLLINS/MITCHELL [1] propose the following relationship

$$f_{du} = \frac{5.5\, f_c'}{4 + \gamma_m/\varepsilon_d} \qquad (5)$$

where $\gamma_m = 2\varepsilon_d + \varepsilon_\ell + \varepsilon_t$

It must be noted that in the normal case of combined bending and shear the longitudinal strains in the concrete web vary over the beam height due to bending. The compatibility equation (3) in this case predicts an angle α which also varies over the beam height as a function of ε_ℓ. For normal design situations, it is recommended by the authors to consider the longitudinal strain at middepth of the beam and take the corresponding angle α as constant over the web height.

5. STRAIN COMPATIBILITY AND AGGREGARTE INTERLOCK [6],[7],[8],[9]

If it is assumed that the angle α of the inclined compression field in the web does not coincide with the angle α_{cr} of inclined cracks, forces must be transferred across the cracks by aggregate interlock. These forces depend on the displacements v and w of the crack faces tangential and normal to the crack direction.

The compatibility of strains can be considered independently for the strains in the concrete struts between cracks and for the average web strains (including "smeared" crack openings). The differences between both strain fields can be summed up to determine the crack displacements. For this, the crack spacing must be estimated from bond considerations.

The forces which are transferred across the inclined cracks by aggregate interlock depend on the crack displacements v and w. KUPFER and coworkers [6],[8] use relationships for aggregate interlock stresses determined by WALRAVEN, while DEI POLI et al. [7] consider equations derived by GAMBAROVA.

Taking into account aggregate interlock forces and strain compatibility, the angle α of the compression field can be determined by an iterative procedure.

Again, strain compatibility is dependent on longitudinal strains which normally vary across the web height. As a consequence, crack displacements, aggregate interlock stresses and the angle of the compressive stress field vary accordingly. To simplify calculations, it is again recommended to consider the strains at middepth and treat all related variables as constant over the web height.

REINECK/HARDJASAPUTRA [9] use a kinematic condition to determine the angle α of the inclined compression field. Following considerations of deformations of truss models, they assume that the resulting crack opening is always perpendicular to α. From this assumption, the angle α can be determined by an iterative procedure. Aggregate forces are taken into account by the model allthough their magnitude is not explicitely considered.

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