Modelling the transverse reinforcement in reinforced concrete structures

Modelling the Transverse Reinforcement in Reinforced Concrete Structures

Modélisation des armatures transversales dans les structures en béton armé

Modellierung von Querbewehrungen in Stahlbetontragwerken

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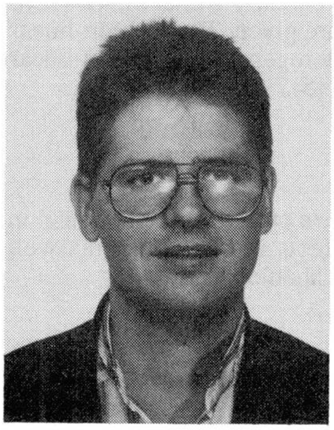

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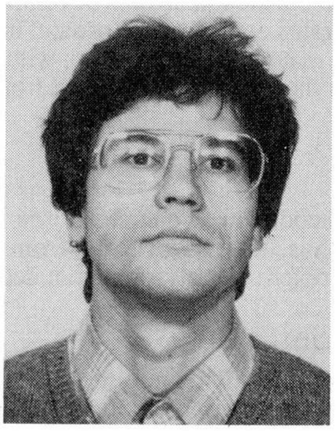

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SUMMARY

The authors present a simplified method to model the influence of transverse reinforcement on the behaviour of reinforced concrete elements. First, a numerical method to compute the three-dimensional stress and strain fields around heterogeneities like transverse reinforcement (stirrups in beams or hoops in columns) embedded in an elastic matrix is developed. Then, a non-linear homogeneous equivalent material is introduced, averaging the local heterogeneities. This unified method allows to treat both the shear reinforcement in beams and the confinement reinforcement in columns.

RÉSUMÉ

Les auteurs proposent une méthode simplifiée pour modéliser l'influence des armatures sur le comportement non linéaire d'éléments de béton armé. Dans un premier temps, une méthode numérique permettant d'obtenir les champs tridimensionnels de déformations et de contraintes localisés autour d'hétérogénéités de type armatures transversales (cadres dans les poutres ou frettes dans les colonnes) incluses dans une matrice élastique est développée. Ensuite, un matériau homogène équivalent rendant compte de façon moyenne des informations fines précédemment obtenues est construit. La méthode permet de traiter de façon unifiée le cas du frettage et celui du cisaillement dans les poutres.

ZUSAMMENFASSUNG

Eine vereinfachte Methode zur Modellierung des Einflusses von den Querbewehrungen auf das nichtlineare Verhalten von Stahlbeton-Konstruktionen wird hier vorgestellt. Zuerst wird eine numerische Methode zur Berechnung der 3D-Verformung- und Spannungsfelder zur Erfassung der Ungleichartigkeiten wie Querbewehrung (Umschnürungsbügel in Stützen oder Bügel in Träger) in einer elastischen Matrix vorgeschlagen. Darauf wird ein gleichartiger Äquivalentstoff mit verschmierten Ungleichartigkeiten formuliert. Die Probleme mit dem Umschnürungsbügel in Stützen und dem Bügel in Träger wird einheitlich behandelt.

1. INTRODUCTION

The behavior of reinforced concrete members including transverse reinforcement (shear reinforcement in beams or confinement reinforcement in columns) is not well understood until now and empirical design methods used in codes and specifications are very different around the world [1]. The concepts that underline current design practice are based partly on rational analysis, partly on test evidence, and partly on successful long-term experience with satisfactory structural performance. A theoretical treatment on the subject is needed.

The purpose of the paper is to model this behavior in the context of a general two-level approach [2] which consists in using for the structures global simplified methods issued from detailed local analysis:

- detailed analyses are first developed to compute the 3D stress and strain fields, warping of cross sections and other local informations in beams and columns taking into account the exact geometry of such elements (spatial distribution of reinforcement for instance).
- then global methods are built using global variables, such as generalized forces on the cross section, and taking into account the previous refined informations. These methods are simpler to use and quicker. The complex local behavior is integrated but not seen by the user who needs only a global response.

Following this general two-level approach, a numerical method to compute three-dimensional stress and strain fields localized around heterogeneities like transverse reinforcement (stirrups in beams or hoops in columns) embedded in an elastical matrix is developed. This method is based on Eshelby works [3, 4] where analytical stresses and strains around an ellipsoidal inclusion in an infinite body are given. Then, a non-linear homogeneous equivalent material is constructed, with an averaging of the local heterogeneities. The non-linear constitutive equations for the concrete are derived from continuous damage theory [5].

2. LOCAL ANALYSIS

A reinforcement bar embedded in a concrete matrix is a local perturbation (there is only a percent of steel in most R/C buildings). Thus, the use of tools from the "micromecanics of defects in solids" seems well adaptated to solve this problem. Equations are not detailed here, interested readers should refer to [4].

2.1 Ellipsoidal heterogeneity

The most popular author in this domain of the mechanics is Eshelby. He obtained in 1959, in an elegant way, the distribution of perturbations around an elipsoidal heterogeneity embedded in an uniformly loaded infinite elastic media [3] (Fig.1).
It can be expressed with:

$$\varepsilon(x) = S(x)\,[(K^*-K)S(x)+K]^{-1}\,(K-K^*)\,\varepsilon^o = H(x)\,\varepsilon^o \qquad (1)$$

where $\varepsilon(x)$ is the strain perturbation at point M with coordinates x, ε^0 the uniform strain at infinity, K^* and K the Hooke tensors of the heterogeneity and the matrix respectively and S is the Eshelby tensor which can be expressed from the two elliptic integrals:

$$\Psi(x) = \int_\Omega |x-x'|\,dx' \quad \text{et} \quad \Phi(x) = \int_\Omega \frac{dx'}{|x-x'|} \qquad (2)$$

These integrals are analytical for specials ellipsoids like sphere or infinite cylinders for instance.

2.1 Transverse reinforcement in concrete

In the case of transverse reinforcement studied here (Fig.2), the assumptions must be revised. These hypothesis are treated and discussed in [6] and will be the object of further publications. Only their list is given here.

2.2.1 Assumptions on geometry

- A stirrup or a hoop is not an ellipsoid: the tensor H of eq.1 must be computed with special technics (numerically).
- Reinforcement bars are not unique: interactions are neglected and perturbations are added.
- They are embedded in a finite body: the approximation of infinite media is still acceptable.

2.2.2 Assumption on loading

- In the case of a beam, the loading is not uniform: the beam is discretized in layers where the gradient of loading is supposed small.

2.2.3 Assumption on materials

- The behavior of concrete is not elastic: the continuous damage theory coupled with an homogeneisation procedure is used.

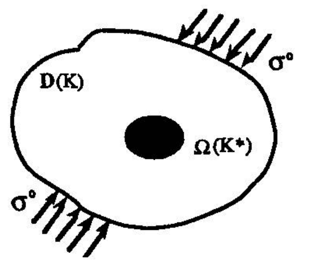


Fig.1 Ellipsoid in an infinite media

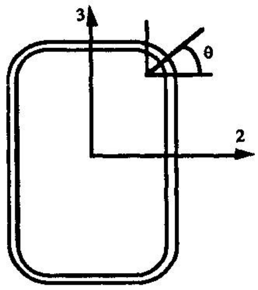


Fig.2 General shape for hoops and stirrups

3. GLOBAL ANALYSIS

3.1 Homogeneous equivalent material

The beam or the column is divided into elementary cells whose height depends of the number of layers choosen (depending on the sollicitation), whose width is the width of the beam and whose length depends on longitudinal distribution of transverse reinforcement (Fig.3,4).

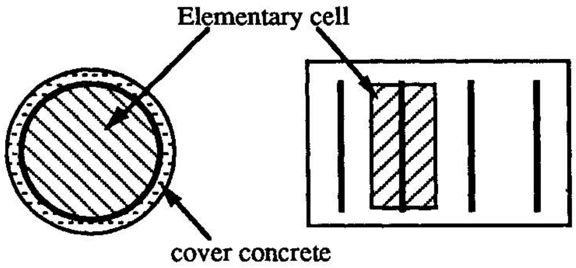


Fig.3 Hoop confined specimens

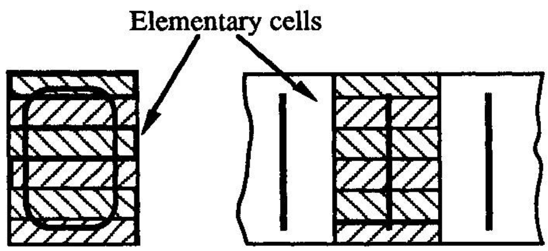


Fig.4 Beams subjected to shear

In relation with the choosen constitutive equations for concrete [5], the homogeneisation variable k is an average on the cell of the leading scalar variable of the damage model, which is calculated at each point within the cell from the positive principal strains $<\varepsilon_i>_+$:

$$\tilde{\varepsilon} = \sqrt{\sum <\varepsilon_i>_+^2} \tag{3}$$

$$k = \frac{1}{V} \int_{V_t} \frac{\tilde{\varepsilon}}{\tilde{\varepsilon}^o} \, dv \tag{4}$$

where V is the volume of the cell and $\tilde{\varepsilon}^o$ the value of $\tilde{\varepsilon}$ in the concrete alone (if there were no stirrup or hoop). Thus, during the monodimensionnal calculation described in the following, the variable $\tilde{\varepsilon}_c$ of the cell with transverse reinforcement influence will be calculated from $\tilde{\varepsilon}^o$ (case with no stirrup):

$$\tilde{\varepsilon}_c = k \ \tilde{\varepsilon}^o \tag{5}$$

The homogeneisation variable k varies with the geometry (% and distribution of steel), with the state of the materials (toughness or damage of the concrete matrix) and with the sollicitation of the cell (traction, compression or shear). k is less than one, so it is called the damage delaying indicator. On Fig. 5, one can see the variation of k with the damage variable:

$$D = \frac{E_c - E_{c0}}{E_{c0}} \tag{6}$$

where E_c and E_{c0} are the actual and initial Young modulus of concrete respectively. It can be seen that the more the concrete is damaged, the more the transverse reinforcement effects are great (k<1).

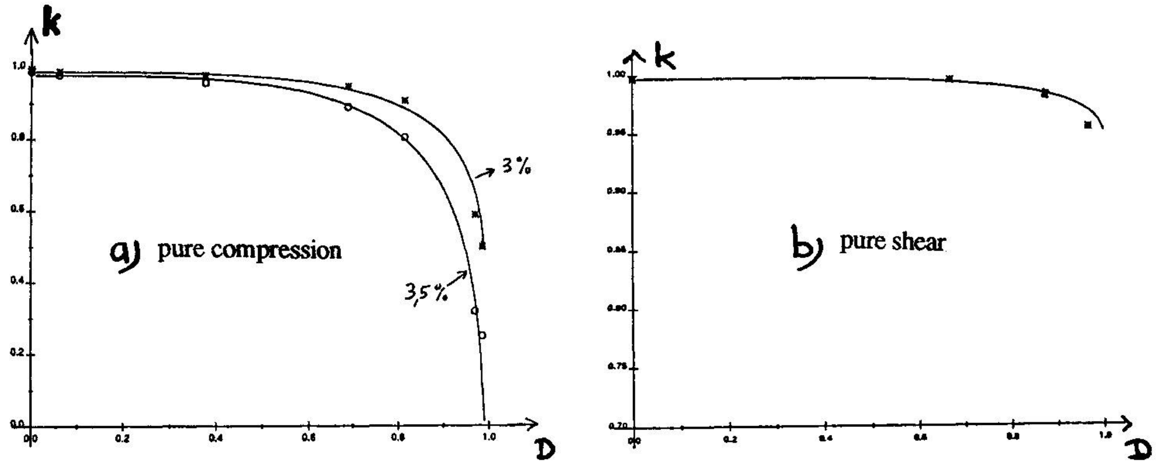


Fig.5 Homogeneisation variable k versus damage $D = \dfrac{E_c - E_{c0}}{E_{c0}}$

3.2 Global calculation

The heterogeneous elementary cell is replaced by an equivalent homogeneous cell with the same dimensions and whose behavior is affected by the presence of the steel. The transverse steel has no influence on the secant modulus of the cell which is the one of the matrix. But the degradation of the matrix is considerably delayed by the presence of transverse reinforcement.

3.2.1 Hoop confined specimens

The computation of a hoop confined specimen or column is first shown for simplicity:
- First stage: local computation (3D) and building of relations k(D) of Fig.5a.
- Second stage: global computation (1D):

i) The equivalent homogeneous cell is loaded in compression with ε^o , $\tilde{\varepsilon}^o$ is computed with eq.3
ii) The damage D_0 is calculated with the choosen damage law
iii) k is obtained from the curves Fig.5a (numerically stored at the first stage)
iv) The new state of the cell is computed with eq.5: $\tilde{\varepsilon}_c = k \ \tilde{\varepsilon}^o$
v) The damage of the cell D_c is calculated with the choosen damage law
vi) Return to step iii) until convergence (very fast in practice)
vii) Calculation of the stress $\sigma_c = E_{b0}\,(1 - D_c)\,\varepsilon^o$

The first and second stage are completely independant. The (long) three-dimensionnal local computation is done only once. The global monodimensionnal computation is very fast but takes into account the refined information of the first stage.

3.2.2 Beams subjected to flexure and shear

The procedure is about the same but a little more complicated because the cells are loaded with an axial strain ε and a shear strain γ. The corresponding stresses are deduced from:

$\sigma = E_{c0}\,(1\text{-}D_c)\,\varepsilon$ and $\tau = G_{c0}\,(1\text{-}D_c)\,\gamma$ ()

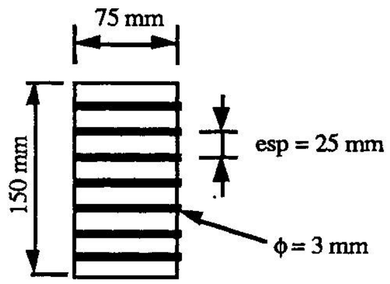
where E_{c0} and G_{c0} are the Young and shear modulus of the sound concrete and D_c the damage of the cell.
The distribution of the strains along the height of a section of the beam are obtained following the iterative method described in [7]: ε is proportional to the rotation ω of the section (supposed to remain plane) and γ is obtained by equilibrium conditions on a layer.

The homogeneisation variable k of a cell subjected to σ and τ is obtained by combination of the curve for pure compression (Fig.5a) and the curve for pure shear (Fig.5b) computed during the first stage.
Interested readers should refer to [6, 7].

4. RESULTS AND COMPARISON TO TEST DATA

4.1 Hoop confined specimens

The method was applied to the hoop confined cylinders described in Fig.6. The parameters of the damage law were identified from the stress-strain curves of the plain concrete. The stress-strain curves of the confined concrete (core concrete) were then calculated by the proposed method. This was done for two different concretes, one with low qualities (f_c = 26 MPa, Fig.7a) and the other with better qualities (f_c = 52 MPa, Fig.7b).
One can see that numericals results are in good agreements with experimental data [8]: the presence of hoops increases just a little the strength of the concrete (pic stress) but a lot its ductility. The better the concrete, the more prononced these effects.



ρ_t = 1,6 %

Fig.6 Hoop confined cylinder

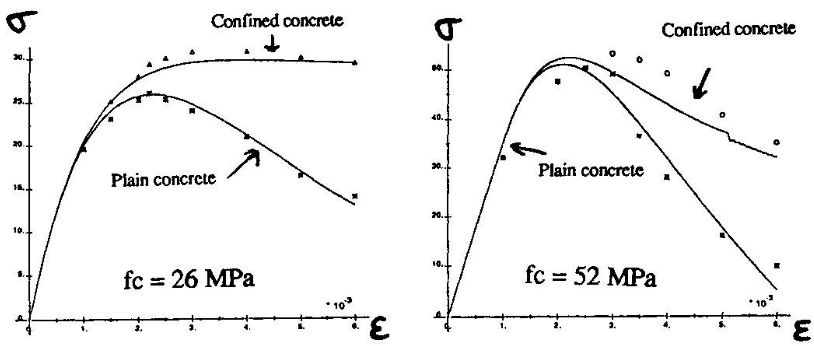
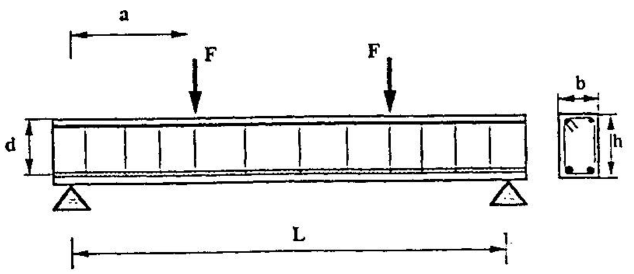


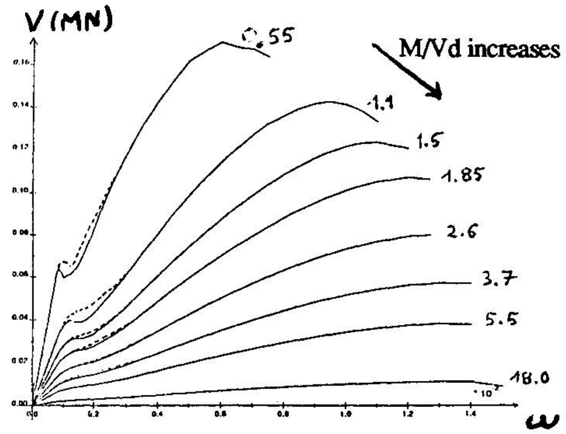
Fig.7 Stress-strain curves — numeric xo exp.

4.2 Beams subjected to flexure and shear

With the proposed method, it is also possible to study beams subjected to shear (Fig.8). In this isostatic case, the sections of the beam differ by the ratio M/Vd where M is the bending moment in the section, V the shear effort and d the effective depth of the beam. The greatest M/Vd ratio is the shear span to depth ratio of the beam a/d (in the section under the applied load), which is a well known influencing parameter in such a problem.



b = 150 mm
h = 300 mm
d = 270 mm
Stirrup: ϕ 6 mm
Longi. bars: top ϕ 10 mm
bottom ϕ 22 mm



--- with stirrups
—— without stirrup

Fig.8 Beam subjected to shear

Fig.9 Shear effort V versus rotationof the sections ω

On Fig.9 one can see the simultaneous evolution of the different sections of the beam: the shear effort V is directly related to the applied effort F (V = -F) and loading the beam increases V on the diagramme 9. The sections which have a small M/Vd ratio have two pics. The first pic is due to crushing of concrete under shear in the middle of the section and the second pic correspond to plastification of longitudinals bars or failing of the concrete in compression. The sections with a large M/Vd ratio have only the second pic, the influence of shear being lower.

The ultimate state of the beam is reached when one of the sections reaches a pic. If a/d is large, the beam fails in bending in the sections located in its middle (vertical cracks, Fig.10a). For beams with a small a/d ratio, sections with large M/Vd ratio do not exist and the beam fails in shearing of a section located between the support and the point of application of the load (horizontal cracks, Fig.10b).
The presence of transverse reinforcement has a significant influence only on the first pic in the sections where the M/Vd ratio is small (Fig.10). Thus, the stirrups modify the failure mode of the short beams.

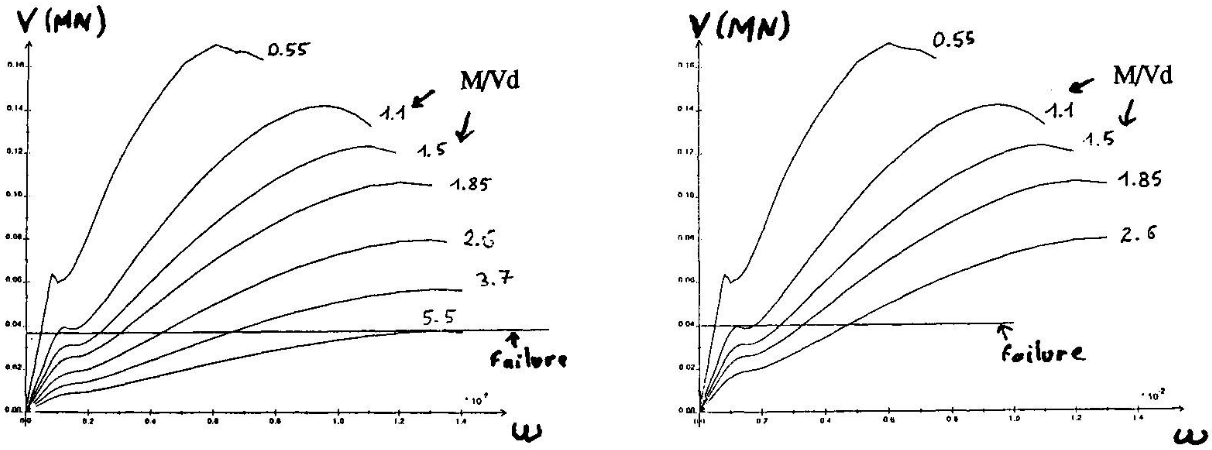


Fig.10 a) Beam with a/d = 5.5 $\frac{Mu}{Mfl} = 1.$ b) Beam with a/d = 2.6 $\frac{Mu}{Mfl} = 0.51$

5. CONCLUSION

Usually, the shear reinforcement in beams and the confinement reinforcement in columns are treated using very different concepts. In the present paper, a unified approach is proposed and the two problems are treated in the same way. This simplified method allows to treat these problems rapidly but with a good accuracy The first results are in good agreement with experimental observations.

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