Reinforced concrete plates under biaxial bending moments

Reinforced Concrete Plates under Biaxial Bending Moments

Dalles en béton armé soumises à des moments de flexion biaxiaux

Stahlbetonplatten unter zweiachsiger Biegung

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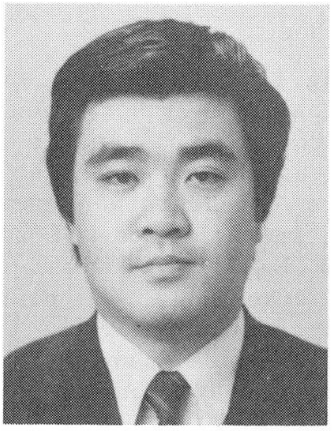

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SUMMARY

This paper proposes a method of analyzing the strength and deformation of reinforced concrete slabs for out-of-plane biaxial bending moments. In the analytical method, the tension zone and the compression zone were divided into two discrete layers, and compression field theory was employed in analyzing reinforced concrete slabs under in-plane forces. The analytical method was verified using a newly-developed apparatus for out-of-plane biaxial bending tests.

RÉSUMÉ

Cet article présente une méthode d'analyse de la résistance et de la déformation de dalles en béton armé soumises à des moments de flexion biaxiaux hors du plan. Dans la méthode analytique, la zone tendue de même que celle comprimée furent divisées en deux couches discrètes et la théorie du champ de compression fut appliquée dans l'analyse de dalles armées soumises à une force dans le plan. La méthode analytique développé fut vérifiée grâce aux tests effectués à l'aide d'un bâti spécialement développé sous effort de flexion biaxiale hors du plan.

ZUSAMMENFASSUNG

Es wird ein Berechnungsverfahren für das Trag- und Verformungsverhalten von Stahlbetonplatten unter zweiachsiger Biegung vorgeschlagen. Dabei werden die Zug- und die Druckzone als zweiachsig beanspruchte Scheiben nach der Druckfeldtheorie behandelt. Das Berechnungsverfahren wurde durch Versuche in einer neu entwickelten Prüfmaschine bestätigt.

1. INTRODUCTION

With the recent increase of reinforced concrete structures constructed underground, designers are being faced with many problems. Side walls of underground storage tanks for crude oil or LNG, for example, are subjected to out-of-plane biaxial bending or torsional moments. Design methods, however, for reinforced concrete plates (RC plates) under such moments yet remain to be established. In this paper, an analytical method for RC plates under out-of-plane biaxial bending moments is presented. This paper also describes newly developed apparatus for out-of-plane biaxial bending tests, by which the validity of the above analytical method is verified. This study assumes pure bending condition without the influence of out-of-plane shearing force and it considers the deviation of principal moment in the direction of reinforcing bars.

2. ANALYTICAL METHOD FOR OUT-OF-PLANE BIAXIAL BENDING MOMENTS

2.1 Analytical Method

As shown in Fig.1, this method analyzes RC plates, by applying compression field theory, under in-plane forces N_1 and N_2 assumed in the zone of tensile stress due to bending. The method calculates the strength and the deformation of RC plates taking into consideration compatibility conditions for deformation between the zone of tensile and compressive stresses due to bending. Given average strain ε_{ct} perpendicular to cracks in RC plates, the method can calculate the unit compressive force C'_1 due to bending from Bernoulli-Eulers' assumption, the stress-strain relationship of concrete and the conditions for equilibrium shown in Fig.2 and equation (1) ~ (3). Since the unit tensile force N_1 due to bending equals C_1 when strain reaches ε_{ct}, the location x of the neutral axis can be obtained by convergence calculation. Using the location x of the neutral axis, bending moment and curvature can be calculated and thus deformational behavior can be traced. Fig. 3 shows a flowchart of the above analytical procedure. The ultimate strength is judged when one of the following conditions was met:

i. Strain at the compressive edge of concrete after the yielding of x-axis reinforcing bars (reinforcing bars at a smaller deviation angle with the maximum principal moment M_1, see Fig. 4) reaches 0.35%.

ii. Y-axis reinforcing bars (reinforcing bars at a greater deviation angle with the maximum principal moment M_1) yields after the yielding of x-axis reinforcing bars.

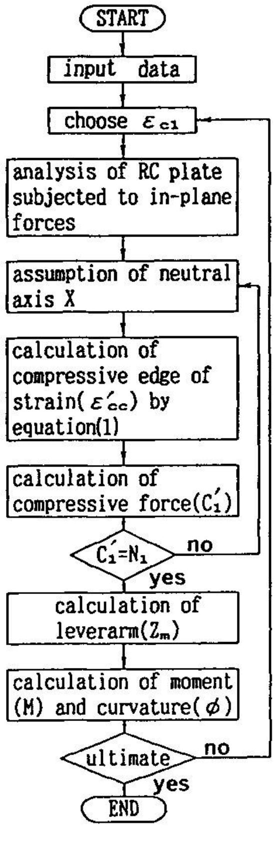

Fig.3 Flowchart for analytical method

$$\varepsilon'_{cc} = \frac{x}{d-x}\cdot\varepsilon_{ct} \quad (1) \qquad \sigma'_c = f'_c\,[2(\varepsilon'_c/\varepsilon_0) - (\varepsilon'_c/\varepsilon_0)^2] \quad (2) \qquad C'_1 = \int \sigma'_c\cdot dx \quad (3)$$

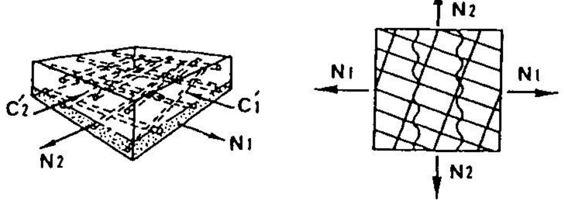

Fig.1 RC plate subjected to biaxial bending and RC plate subjected to in-plane forces assumed in tensile zone due to bending

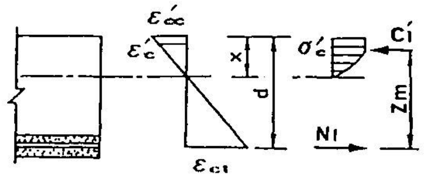

Fig.2 Assumption of strain profile and equilibrium of section

2.2 Analysis of RC Plates Under In-Plane Forces Based on the Compression Field Theory

2.2.1 Modeling of Cracks

A smeard crack model was employed, and RC plates with cracks were assumed to be continuous elements. Strain and stress dealt with in this paper, therefore, were assumed to be average strain and stress which were uniform within the element. It was assumed that the initial cracks occurred when tensile strain in the direction of the maximum principal stress reached 200μ. It was also assumed that even after the maximum principal stress reached the tensile strength, that stress was kept until tensile strain reached 200μ. Reinforcing bars in plates are usually arranged in two directions. It is likely, therefore, that reinforcing bars in the direction perpendicular to the principal moment which are virtually thought to be voids in concrete furthers the development of cracks. The cracking stress was hence determined according to equation (4) which takes this influence into consideration. Experiments conducted by the authors indicated that the direction of cracking, whether in one axis or two axes, remained throughout the cracking process. Consequently, it was assumed that the cracking direction was perpendicular to the maximum principal moment. Biaxial bending with a principal moment ratio of $Km=M_2/M_1=0.5$ or above causes cracking in two directions. To reflect this influence in the analysis, the authors decided to take account of the influence of bidirectional cracks in the evaluation of shear stiffness described later, while assuming that dominant cracks always occurred in a single direction.

$$f_t = 0.5\, f_c'^{\,2/3} \quad (kgf/cm^2) \qquad (4)$$

2.2.2 Stress-Strain Relationship

Stress was calculated using the average stress-strain relationships shown below. In the analysis, Vecchio-Collins Model which considers softening was used as the average stress-strain relationship in a direction parallel to concrete cracks [1], while Okamura-Maekawa Model which considers tension stiffening was used as the average stress-strain relationship in a direction perpendicular to the cracks [2]. For shear stiffness of RC plates after cracking, Aoyagi-Yamada Model was used [3]. It is to be noted, however, that since the average shear stiffness was being considered, the shear stiffness of uncracked portions and the Aoyagi-Yamada Model were serially connected to determine the average shear stiffness (equation (5) with $\beta = 1$).

2.2.3 Influence of Bidirectional Cracks

The analytical method described above is intented for concrete with unidirectional cracks. Experiments showed that cracks occurred in two directions for specimens under biaxial bending with a principal moment ratio of $Km=M_2/M_1=0.5$ or above. A modeling method, however, for bidirectional open cracks yet remains to be established. The authors had presumed that bidirectional cracks reduced the bond between reinforcing bars and concrete, thus decreasing tension stiffening and shear stiffness. From the results of trail calculation for test results, the influence of bidirectional cracks was clearly reflected in the shear stiffness model. When the ratio of the principal moments in two directions was $Km = M_2/M_1 = 1.0$ (cracks fully developed in both directions), the ability of shear transfer seemed to disappear almost. Hence the authors introduced the reduction coefficient β depending on the principal moment ratio Km and gave it as $\beta = 1-Km \leq 1.0$. Then shear stiffness Ga with bidirectional cracks was evaluated using equation (5).

$$Ga = \beta \cdot \frac{Gc \cdot Gcr}{(Gc+Gcr)} \qquad (5)$$

Note here that Gc which represents the shear stiffness of the uncracked portions is given as $Ec/2(1+\nu)$; where Ec is the elastic modulus of concrete, and ν is Poisson's ratio of concrete. Gcr is the shear stiffness of the cracked portions and is given as $36/\varepsilon_{ct}$ (Aoyagi-Yamada Model).

2.2.4 Analytical Expression of RC Plates Under In-Plane Forces

This analytical method is based on the compression field theory of Vecchio-Collins. However, this method is characterized by clear distinction between the directions of the maximum principal strain (α) and the maximum principal moment (θ). If it is assumed that Mohr's circle applies to the maximum principal strain ε_{c1}, the minimum principal strain ε_{c2}, strains ε_x, ε_y and ε_{xy} in the directions of reinforcing bars, strain ε_{ct} perpendicular to cracks, strain ε_{cv} parallel to cracks, and shear strain γ_{ctv}, compatibility conditions in equations (6) and (7) should be satisfied, where θ is a deviation angle between the direction of the maximum principal moment and the x-axis reinforcing bars and α is an angle between the direction of the principal strain of RC plates and the x-axis reinforcing bars.

$$\begin{aligned} \varepsilon_x &= \varepsilon_{c1}\cos^2\alpha + \varepsilon_{c2}\sin^2\alpha \\ \varepsilon_y &= \varepsilon_{c1}\sin^2\alpha + \varepsilon_{c2}\cos^2\alpha \\ \gamma_{xy} &= (\varepsilon_{c1} - \varepsilon_{c2})\sin 2\alpha \end{aligned} \qquad (6)$$

$$\begin{aligned} \varepsilon_{ct} &= \varepsilon_{c1}\cos^2(\alpha-\theta) + \varepsilon_{c2}\sin^2(\alpha-\theta) \\ \varepsilon_{cv} &= \varepsilon_{c1}\sin^2(\alpha-\theta) \quad \varepsilon_{c2}\cos^2(\alpha-\theta) \\ \gamma_{ctv} &= (\varepsilon_{c1} - \varepsilon_{c2})\sin 2(\alpha-\theta) \end{aligned} \qquad (7)$$

Loads on RC plates are resisted by the stresses of reinforcing bars and concrete, and they can be added. If the principal strain ε_{c1} is given, stress in concrete in the x and y-axis directions can be calculated by equation (8) based on strain's compatibility conditions, the stress-strain relationship and the equilibrium of forces of the free body shown in Fig. 4 and Fig. 5. Equation (9) is obtained from the equilibrium of external and internal forces. If ε_{c1} is established and ε_{c2} and α are assumed, all internal forces can be calculated. The convergence calculation is carried out until the resultant n_x, n_y and n_{xy} equal the principal moment ratio km and the deviation angle θ of the principal moment.

$$\begin{aligned} \sigma_{cx} &= \sigma_{ct}\cos^2\theta + \sigma_{cv}\sin^2\theta - \tau_{ctv}\sin 2\theta \\ \sigma_{cy} &= \sigma_{ct}\sin^2\theta + \sigma_{cv}\cos^2\theta + \tau_{ctv}\sin 2\theta \\ \tau_{cxy} &= (\sigma_{ct} - \sigma_{cv})\sin\theta\cos\theta + \tau_{ctv}\cos 2\theta \end{aligned} \qquad (8)$$

$$\begin{aligned} n_x &= p_x\sigma_{sx} + \sigma_{cx} \\ n_y &= p_y\sigma_{sy} + \sigma_{cy} \\ n_{xy} &= \tau_{cxy} \end{aligned} \qquad (9)$$

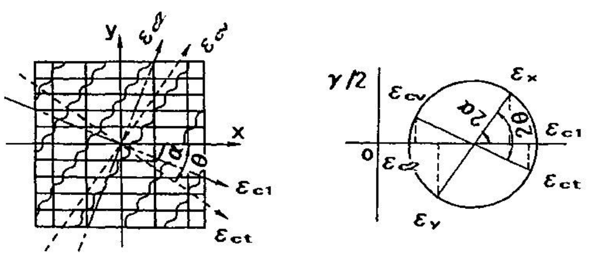

Fig.4 Average strains condition for RC plate element

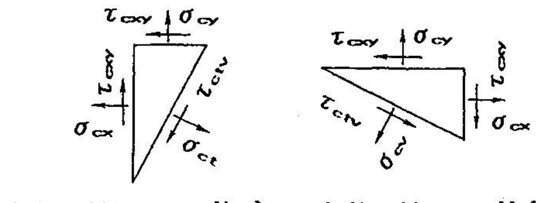

Fig.5 Average stresses acting on free-body in RC plate

3. COMPATISON OF TEST DATA AND CALCULATED VALUES

3.1 Biaxial Bending Test

An analytical model of this type requires verification using test data. However, reliable data on biaxial bending tests had not been available because of difficulty. Therefore an apparatus for biaxial bending tests of RC plates, was newly designed. It did not have the interaction of bending moments between each directions shown in Fig. 7. Using cruciform specimens shown in Fig.6, reliable test data on biaxial bending could be obtained. Table 1 summarizes the type of specimens, the compressive strength of concrete, cracking moment Mcr and the maximum moment M_{1u}.

3.2 Comparison of Test Data and Calculated Values

3.2.1 Comparison on Deformation

When the deviation on angle θ between the directions of principal moment and reinforcing bars under uniaxial bending moment increases, the RC plates show a marked tendency toward decreased flexural rigidity. Comparison of test data and calculated values for the moment-curvature relationship is shown in Fig.8. For RC plates under biaxial bending moments, comparisons on the moment-curvature relationship and the moment-reinforcing bars' average strain relationship are shown in Fig. 9 and Fig. 10, respectively. This analysis can trace deformation until the ultimate state of RC plates because it considers the compatibility conditions of deformation between the tensile and compressive stresses zone due to bending. In this analysis, the reduction coefficient $\beta = (1-K_m)$ is also introduced. As shown in figures, both curvature and average strain calculated accurately represent the actual behavior of deformation from the beginning of cracks through the ultimate state. The figure also shows calculated values which do not consider β. The influence of not considering β is clearly reflected in the results of the evaluation of the reinforcing bars' strain. This implies that if β is not taken into consideration, deformation could be underevaluated when the reinforcement ratio in the y-axis direction is lower than in the x-axis direction.

3.2.2 Comparison on Ultimate Strength

Ultimate strength was estimated on the basis of the definition of the ultimate state described above. Comparison of the estimated values and the test values was shown in Table 1. Note that this comparison included data on not only RC plates under biaxial bending moments, but also those under uniaxial

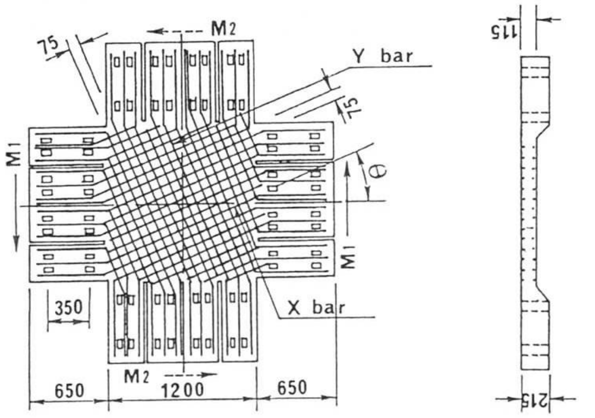

Fig.6 Shape and dimensions of typical specimen

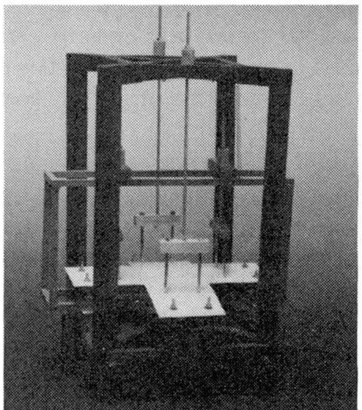

Fig.7 An apparatus for biaxial bending tests

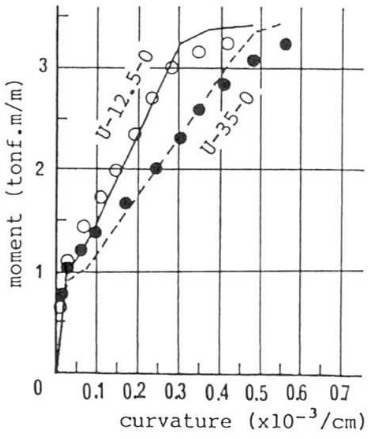

Fig.8 Comparison of test data and calculated values on moment -curvature relationship (uniaxial bending)

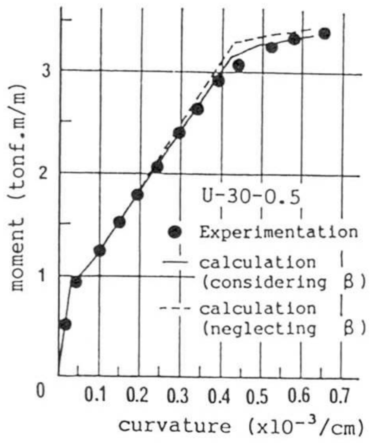

Fig.9 Comparison of test data and calculated values on moment -curvature relationship (biaxial bending)

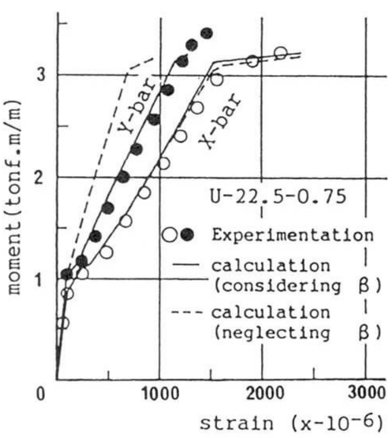

Fig.10 Comparison of test data and calculated values on moment -reinforcing bars' average strain relationship (biaxial bending)

bending moment. The analysis using ten data showed that the ratio of the test values to the calculated values averaged 1.03, giving the coefficient of variation of 4.8%. Although the average value was slightly high, the ultimate strength of RC plates under biaxial bending moment could be estimated with reasonable accuracy.

Specimen			Test Results			Calculation	M_{1utes}
Mark	θ	K_m (M_2/M_1)	f'_c kgf/cm²	M_{cr} tonf·m	M_{1u} tonf·m	M_{1ucal} tonf·m	M_{1ucal}
U-0-0	0	0	261	0.98			
U-12.5-0	12.5	0	289	1.09	4.10	3.78	1.08
U-17.5-0	17.5	0	305	1.12	3.75	3.76	1.00
U-22.5-0	22.5	0	252	0.91	3.71	3.62	1.02
U-30-0	30	0	232	0.84	3.54	3.62	0.98
U-35-0	35	0	286	1.02	3.61	3.68	0.98
B-0-0.5	0	0.5	255	0.88	4.02	3.78	1.06
B-12.5-0.5	12.5	0.5	230	0.98	4.03	3.70	1.07
B-22.5-0.5	22.5	0.5	243	0.98	3.98	3.68	1.08
B-30-0.5	30	0.5	262	0.98	3.92	3.69	1.06
B-22.5-0.75	22.5	0.75	248	0.91	3.92	3.77	1.04

the averaged values = 1.03

coeffcient of variation = 4.8%

Table 1 Specimen properties, experimental and calculated value, and comparison of the both values

4. COMPARISON WITH OTHER APPROACHES

The analysis employed is a kind of sandwich method assuming a discrete layer between the compression zone and the tension zone. Some attempts have been made to analyze the behaviors of various members subjected to bending using sandwich models, and one of those was presented by Marti [4]. In his report, slabs were divided into an upper, a middle and lower layers; moment and axial force were resisted by the upper and the lower layers, while out-of-plane shearing force was resisted by the middle layer. In the model, the lever arm dv was assumed at 80% of thickness h. His proposed method of employing a truss mechanism in the middle layer is noteworthy. For the analysis of the upper and lower layers, compression field theory may be required. If the specimens used in this research were analyzed by the above methods, the ultimate strength obtained would show good agreement with test data. It is to be noted, however, that compatibility conditions in the upper and lower layers are not satisfied, those models will not be suited for the examination of serviceability limit state.

5. CONCLUSIONS

Here are the conclusions drawn from the study.

(1) Deformational behavior can be accurately traced by analyzing the lower layer of RC plates by compression field theory and by employing compatibility conditions of deformation between the upper and the lower layers.
(2) Accuracy of analysis can be enhanced by introducing the reduction coefficient β considering the influence of bidirectional cracks due to biaxial bending.
(3) Ultimate strength can be accurately represented by defining failure as a state with a strain of concrete's compressive edge of 0.35%, or as the yielding of reinforcing bars in y-axis direction.

The above results indicate that it is possible to unify the design process of RC plates under out-of-plane bending within the framework of the compression field theory as in the case of shear problems of RC plates under in-plane force.

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