A consistent shear design model for concrete offshore structures

A Consistent Shear Design Model for Concrete Offshore Structures

Modèle de dimensionnement à l'effort tranchant des structures en mer

Schubbemessungsverfahren für Beton-Offshore Strukturen

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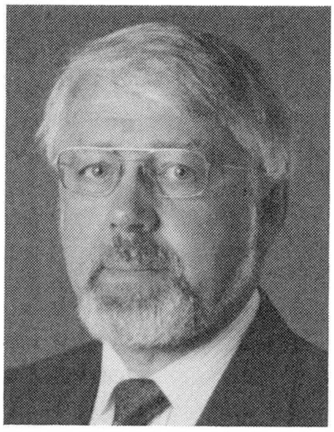

SUMMARY

A strain compatibility procedure for the sectional design of complex concrete structures is presented. The procedure, which is a generalization of the modified compression field theory for membrane elements, is capable of predicting the response of elements subjected to combined membrane forces, bending moments and transverse shear.

RÉSUMÉ

On présente ici une procédure basée sur la compatibilité des contraintes lors du dimensionnement de structures complexes tenant compte des efforts intérieurs. Cette méthode, généralisation de la théorie modifiée du champ de compression applicable aux éléments minces, est capable de prévoir la réponse d'éléments sollicités par des forces de membranes, des moments de flexion et des forces de cisaillement transversales.

ZUSAMMENFASSUNG

Diese Veröffentlichung beschreibt ein Kompatibilitätsverfahren für die Querschnittsberechnung von komplexen Betonstrukturen. Die Methode stellt eine Verallgemeinerung der modifizierten Druckfeld-Theorie für Membranelemente dar. Sie ermöglicht eine Berechnung des Elementverhaltens unter einer kombinierten Beanspruchung durch Membrankräfte, Biegemomente und Schub.

1. Introduction

Traditional sectional design procedures were developed for simple concrete structures such as buildings. The sectional forces (axial load, bending moment, and shear force) at various locations in a building frame are typically determined using a linear elastic analysis. In checking the ability of a particular section to resist the calculated stress resultants, the non-linear behaviour of concrete is taken into account. The response to axial load and bending moment is based on a strain compatibility approach, while shear design has traditionally involved empirical rules.

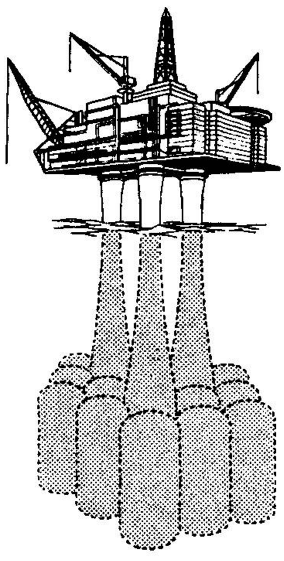

Fig. 1 Concrete Offshore Structure

The design of a more complex structure such as a concrete offshore structure (see Fig. 1) also involves determining the sectional forces at critical locations in the structure. Once again, linear elastic analysis is usually employed. However, the sectional forces for such a structure is considerably more complex. The loading demand at a particular location is expressed in terms of eight stress resultants, three membrane forces, N_x, N_y, N_{xy}, three bending moments, M_x, M_y, M_{xy}, and two trans verse shear forces, V_x, V_y, (see Fig. 2). The method currently used to design offshore structures for the three membrane forces and the three bending moments is a generalization of the strain compatibility approach used for beams, while for transverse shear the empirical beam shear design rules are used. This paper describes a procedure in which membrane forces, bending moments and transverse shear can be considered in a consistent strain compatibility approach.

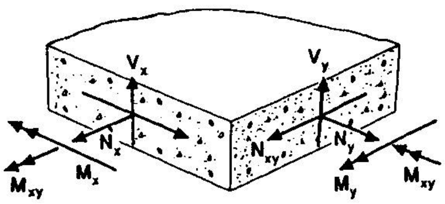

Fig. 2 Sectional Forces

2. Membrane Forces

The simplest "shear problem" is to predict the response of a reinforced concrete element subjected to only membrane forces, N_x, N_y, and N_{xy}. The problem involves relating the uniform biaxial strains ϵ_x, ϵ_y, γ_{xy} to the uniform biaxial stresses n_x, n_y, n_{xy}. A procedure for predicting the response of membrane elements was presented by Vecchio and Collins [1]. In this procedure, called the modified compression field theory, cracked concrete is treated as a new material with its own stress-strain characteristics. Rather than dealing with the variable local stresses (e.g., higher reinforcement stresses at a crack, lower away from the crack) the theory is formulated in terms of average stresses and average strains. In addition, the ability of the section to transmit the required forces across the cracks is specifically checked.

The biaxial stress-strain characteristics of cracked concrete were empirically determined from tests [1]. These tests showed that the principal compressive stress, f_{c2}, in cracked concrete is a function of not only the principal compressive strain, ϵ_2, but also of the co-existing principal tensile strain, ϵ_1. In addition, the tests showed that even severely cracked concrete stiffens the response of reinforcement. In the modified compression field theory this phenomenon is accounted for by assigning an average tensile stress to the concrete. After cracking, the average tensile stress in the concrete reduces as the strains increase (see Fig. 3).

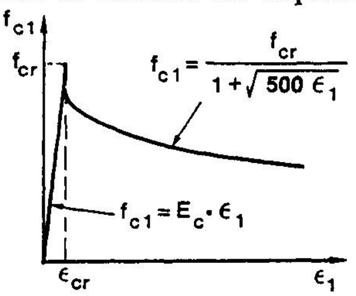

Fig. 3 Average Stress-Strain Relationship for Concrete in Tension

3. Membrane Forces and Bending Moments

A more complex problem exists when there are three bending moments, M_x, M_y, M_{xy}, in addition to the three membrane forces, because the stresses are now not uniform over the thickness of the element. The problem can be solved using a strain compatibility procedure which is a generalization of the plane sections theory. The membrane strains, ϵ_x, ϵ_y, γ_{xy}, are assumed to vary linearly over the thickness of the element, and therefore can be described by six variables. Given the six strain variables, the stresses in the concrete and the reinforcement can be determined from the biaxial stress-strain relationships. Integrating the stresses over the thickness of the element gives the six stress resultants. The concrete stresses are integrated numerically by dividing the thickness of the element into a number of membrane elements.

Finding the stress resultants which are associated with a given set of strains is a direct procedure. However, if the six stress resultants are given and it is desired to find the six strain variables, trial and error is required. Program SEP [2] was developed based on this approach. It incorporates the procedures of the modified compression field theory as biaxial stress-strain relationships.

4. Equivalent Beam Approach for Transverse Shear

Post-processing design programs based on strain compatibility procedures have been used to check sections of concrete offshore structures for the case of combined membrane forces and bending moments. However, to account for the influence of transverse shear, the empirical beam shear design rules are applied by using the concept of an "equivalent beam."

Consider the element shown in Fig. 4. An equivalent beam strip of unit width taken horizontally from this element would be subjected to the principal transverse shear, but no axial tension. However, the "beam" would be subjected to tension acting across its width. A beam strip taken vertically would be subjected to the highest axial tension, but would not be subjected to transverse shear along its length. The beam strip would be subjected to transverse shear on its "side faces." In neither case is the in-plane reinforcement parallel to the axis of the strip.

One procedure currently used to apply the beam shear design rules is as follows. For each beam strip any actions on the side faces are neglected, as is the influence of the orientation of the in-plane reinforcement. The transverse shear per unit width of beam strip is taken as

$$V = V_x \cos\alpha + V_y \sin\alpha$$

and the "axial force" per unit width of beam strip is taken as

$$N = N_x \cos^2\alpha + N_y \sin^2\alpha + 2N_{xy} \sin\alpha \cos\alpha$$

The required amount of stirrup reinforcement expressed in terms of stirrup area per unit area of concrete is determined for every possible beam strip direction using the beam shear equations. The largest amount of stirrup steel is taken as the amount which is needed.

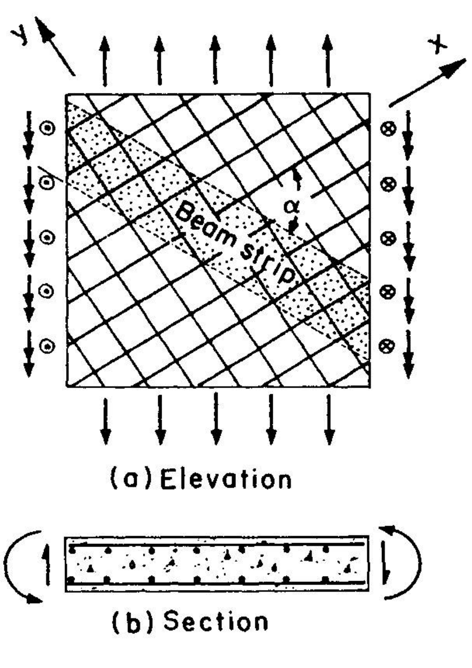

Fig. 4 Equivalent Beam Approach

5. Membrane Forces, Bending Moments and Transverse Shear

The case of combined membrane forces, bending moments and transverse shear is complicated by the need to deal with triaxial strains and triaxial stresses. A simple practical solution to the problem is possible by considering, the variation of biaxial strains over the thickness of the element, but triaxial strains at only the mid-plane of the element. The influence of the triaxial strains on the biaxial stresses is assumed to be uniform over the thickness of the element. That is, the transverse strains are assumed to influence only the membrane forces, not the bending moments.

The procedure is done as follows. From the applied bending moments, M_x, M_y, M_{xy}, and the applied membrane forces plus a "first guess" correction to account for the influence of the triaxial strains, $N_x+\Delta N_x^I$, $N_y+\Delta N_y^I$, $N_{xy}+\Delta N_{xy}^I$, the in-plane strains of the middle surface are calculated using the procedure described in the previous section. The in-plane concrete stresses f_{cx}, f_{cy}, v_{cxy} at the middle surface of the shell are first calculated from these biaxial strains neglecting any transverse strains, and then are recalculated considering the influence of the transverse strains (ie. considering triaxial strains). The differences between these two sets of stresses (ie. the additional concrete stresses due to transverse shear) are assumed to be uniform over the effective shear depth. The resultants of these additional concrete stresses, ΔN_x^{II}, ΔN_y^{II}, ΔN_{xy}^{II}, must equilibrate the membrane force corrections. For example, $N_x+\Delta N_x^I+\Delta N_x^{II} = N_x$ which means that $\Delta N_x^I+\Delta N_x^{II} = 0$. Several iterations are generally needed to determine the three membrane force corrections to account for transverse shear.

The triaxial stresses at the mid-depth of the shell are calculated by a trial and error procedure which adjusts the three additional strains ϵ_z, γ_{xz} and γ_{yz} until the required values of the transverse shear stresses v_{cxz} and v_{cyz} are obtained and the resultant normal stress on the z plane is zero (ie. the concrete compressive stress in the transverse direction equilibrates the tensile stress in the transverse reinforcement). The six concrete stresses, f_{cx}, f_{cy}, f_{cz}, v_{cxy}, v_{cxz}, and v_{cyz}, are determined from the six concrete strains, ϵ_x, ϵ_y, ϵ_z, γ_{xy}, γ_{xz} and γ_{yz}, using a three dimensional generalization of the modified compression field theory.

The procedure involves first finding the principal strains, ϵ_1, ϵ_2, ϵ_3, and their directions. The principal concrete stresses, f_{c1}, f_{c2} and f_{c3}, are then found from triaxial concrete stress-strain relationships which are generalizations of the Vecchio and Collins biaxial relationships. In addition, a "crack check" is made to ensure that the loads resisted by the average triaxial stresses can be transmitted across the cracks. When significant transverse shear is present this check is often critical in determining the failure load.

6. Transmitting Forces Across Cracks

The forces resisted by the average stresses are transferred across cracks by a combination of increased reinforcement stresses at the crack and shear stresses on the crack interface. The magnitude of the shear stress depends on the relative increases in reinforcement stress in the various directions as well as the direction of the crack.

The ability of a crack to resist shear by aggregate interlock depends primarily on the crack width. It has been suggested [1] that if there are no compressive stresses on the crack interface the limiting value of the crack interface shear stress, v_{ci}, (in MPa) be taken as

$$v_{ci} \leq \frac{0.18}{0.3+\frac{24w}{a+16}}$$

where w is the crack width, a is the aggregate size, and f'_c is the cylinder compressive strength.

The width and direction of a crack can be estimated from the average stresses and average strains. The crack direction is assumed to be normal to the principal average tension direction defined by the three angles θ_x, θ_y, θ_z, while the crack width can be estimated as the product of the principal average strain ϵ_1 and a crack spacing parameter $s_{m\theta}$. That is, $w = \epsilon_1 s_{m\theta}$ where

$$\frac{1}{s_{m\theta}} = \left(\frac{\cos\theta_x}{s_{mx}} + \frac{\cos\theta_y}{s_{my}} + \frac{\cos\theta_z}{s_{mz}}\right)$$

s_{mx}, s_{my} and s_{mz} are indicators of crack control provided by the x, y and z reinforcement directions.

Once the crack direction is known and an assumption is made about the increases in reinforcement stress at a crack, the shear stress on the crack surface can be calculated from

$$v_{ci}=\sqrt{\xi_{xy}^2+\xi_{xz}^2+\xi_{yz}^2}$$

$$\xi_{xy} = (\rho_x\ \Delta f_{sx} - \rho_y\ \Delta f_{sy})\ cos\theta_x\ cos\theta_y$$
$$\xi_{xz} = (\rho_x\ \Delta f_{sx} - \rho_z\ \Delta f_{sz})\ cos\theta_x\ cos\theta_z$$
$$\xi_{yz} = (\rho_y\ \Delta f_{sy} - \rho_z\ \Delta f_{sz})\ cos\theta_y\ cos\theta_z$$

ρ_x, ρ_y and ρ_z are the reinforcement ratios in the x, y and z direction, and Δf_{sx}, Δf_{sy}, Δf_{sz} represent the differences between the average reinforcement stresses and the reinforcement stresses at a crack. Rather than using a strain compatibility approach to determine the reinforcement stresses at a crack, a lower bound approach is used in the modified compression field theory. The relative increases in reinforcement stress are chosen to minimize the required shear stress on the crack surface.

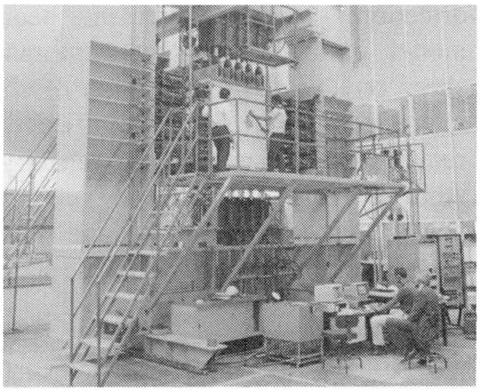

Fig. 5 University of Toronto Shell Element Tester

The increase in reinforcement stress at a crack may be limited by either yielding of the reinforcement or the inability of the crack to transmit the required shear. The average tensile stresses in cracked concrete is in turn limited by the increase in the reinforcement stress at a crack. That is,

$$f_{c1} = \rho_x\ \Delta f_{sx}\ cos^2\theta_x + \rho_y\ \Delta f_{sy}\ cos^2\theta_y + \rho_z\ \Delta f_{sz}\ cos^2\theta_z$$

7. Computer Program SHELL474

The procedures described in this paper have been incorporated into a computer program called SHELL474 [3]. The program operates in two modes. In SLS mode the program calculates the strains (e.g., steel strains, crack widths, etc.) associated with a specified set of eight sectional forces, while in ULS mode the program calculates the complete load-deformation response of a section including the maximum loads which the section can resist.

8. Comparison with Experiments

A large testing machine capable of applying combined membrane forces, bending moments and transverse shear to reinforced concrete elements was constructed in 1984 at the University of Toronto. The machine uses sixty double acting hydraulic actuators to load 1.5 m high by 1.5 m wide elements of varying thickness (see Fig. 5).

Nine specimens were subjected to combined membrane forces, bending moments and transverse shear using the tester [4]. Seven of the specimens were 310 mm thick and had large amounts of in-plane reinforcement ($\rho_x=\rho_y=3.6\%$) but only a small amount of transverse shear reinforcement ($\rho_z=0.08\%$). The concrete (cylinder) strengths were approximately 52 MPa.

The experimentally observed interaction of transverse shear and membrane shear is shown in Fig. 6. Predictions are given from both program SHELL474 (strain compatibility model) and the equivalent beam model. The equivalent beam model gives a reasonable prediction for the pure transverse shear case, but is excessively conservative regarding the influence of membrane shear. This is because the equivalent beam model makes use of the traditional beam shear design rules, which are excessively conservative regarding the influence of membrane tension. Also, the equivalent beam model does not properly account for the in-plane reinforcement direction or the membrane shear direction. The equivalent beam model predicts that the membrane shear on the right hand side of the interaction (Fig. 6) significantly reduces the transverse shear capacity, while SHELL474 correctly predicts that in this case the membrane shear actually improves the transverse shear response.

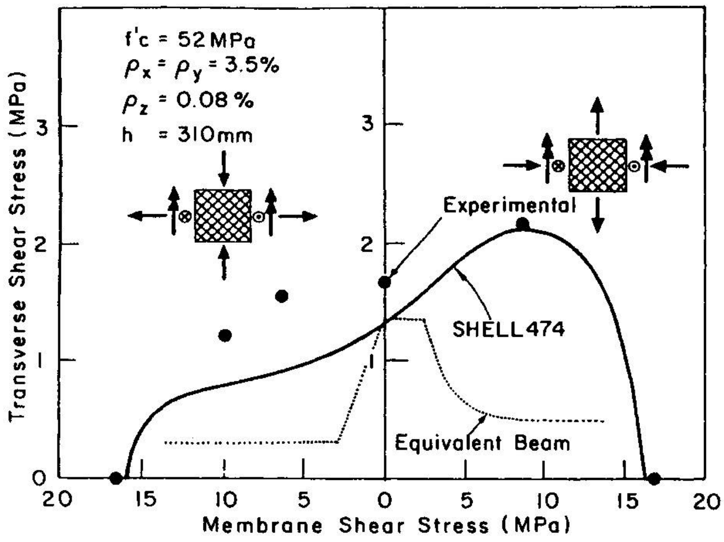

Fig. 6 Membrane Shear - Transverse Shear Interaction

9. Concluding Remarks

Traditional empirical shear design rules developed for simple building components are not appropriate for use in the design of complex concrete structures. If a consistent design approach is to be used for all types of structures then "*a more general approach is required for the (shear) design of structural concrete*" [Breen; Bruggeling].

A general approach to sectional design is possible by considering the following: (1) the compatibility of uniaxial, biaxial or triaxial strains; (2) realistic stress-strain relationships for the materials especially "cracked concrete," and; (3) equilibrium. As was demonstrated in this paper, such a general approach can be consistently applied to structural concrete.

Acknowledgements

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