Reliance upon concrete tensile strength

Reliance upon Concrete Tensile Strength

Prise en compte de la résistance à la traction du béton

Inanspruchnahme der Betonzugfestigkeit

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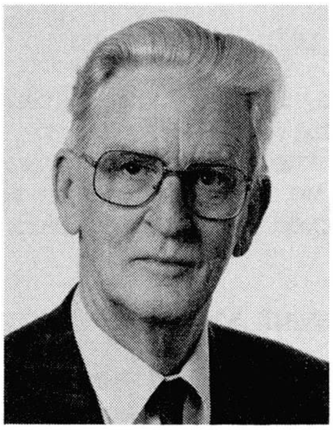

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SUMMARY

In designing concrete structures we rely on concrete tensile strength, although this is not clearly stated in the design rules. Examples of this are given. The tensile fracture process is described, the importance of toughness is demonstrated and the definition of toughness is discussed. Examples of analytical results are given. These show that there is a size dependency, in that the formal strength of a structure decreases as the size increases. Further analyses may lead to better design rules, resulting in a more constant safety factor.

RÉSUMÉ

Dans le cadre du dimensionnement des structures en béton, on tient compte de sa résistance à la traction bien que ceci ne soit pas clairement établi dans les règlementations le concernant. A titre d'illustration, des exemples sont présentés. Le processus de rupture en traction est décrit; on démontre l'importance de la ténacité du béton dont on discute la définition. Des résultats analytiques sont présentés: ils montrent que la taille de la structure est importante, car sa résistance formelle diminue lorsque ses dimensions augmentent. Des analyses futures peuvent mener à de meilleures règlementations aboutissant à un facteur de sécurité plus uniforme.

ZUSAMMENFASSUNG

Bei der Bemessung von Stahlbetontragwerken verlassen wir uns auf die Betonzugfestigkeit, obwohl dies aus den Bemessungsregeln nicht deutlich hervorgeht. Hierfür werden Beispiele gegeben. Es wird das Tragverhalten beim Zugbruch beschrieben, die Bedeutung der Zähigkeit demonstriert und ihre Definition diskutiert. Die Berechnungen einiger Beispiele zeigen, dass es einen Massstabseinfluss gibt, indem die Tragfähigkeit eines Tragwerks mit zunehmender Grösse abnimmt. Weitere Analysen können zu verbesserten Bemessungsregeln führen, die ein ausgeglicheneres Sicherheitsniveau ergeben.

1. CASES WHERE WE RELY ON CONCRETE TENSILE STRENGTH

When we teach the design of concrete structures, we often state that one basic assumption is that concrete does not carry tensile stresses, but that all tensile stresses have to be absorbed by reinforcement. As a matter of fact, this only applies to the design of tensile reinforcement.

The following are examples of cases where we rely on concrete tensile strength:

1.1 Bond and anchorage of reinforcing bars

Anchorage of deformed bars is to a large extent due to compressive forces between the lugs and the surrounding concrete. These forces act in a skew direction with respect to the bar and thus cause splitting tensile stresses in the concrete; see Fig. 1. We have to rely on the tensile strength of concrete for the anchorage of deformed reinforcing bars, and maybe also for plain bars.

1.2 Shear and punching without shear reinforcement

Shear stresses cause tensile stresses. When these stresses exceed the tensile strength, skew cracks appear. We rely on the tensile strength for preventing the formation of these skew cracks, or for preventing their growth from causing shear failure. In the latter case, the toughness associated with tensile fracture is important. The importance of toughness will be discussed later.

1.3 Splitting caused by dowel action

In connection with the formation of shear cracks, a rotation of the two parts of the beam or slab in relation to each other takes place around the compression zone above the crack; Fig. 2. This results in a vertical relative movement at the level of the tensile reinforcement. This movement is counteracted by dowel forces between reinforcement and concrete, which give rise to large local splitting stresses, tending to spall off the concrete cover. This type of failure may be of particular importance for punching failure. In this case the toughness is very important, as it may help in distributing the high local stresses over a larger area.

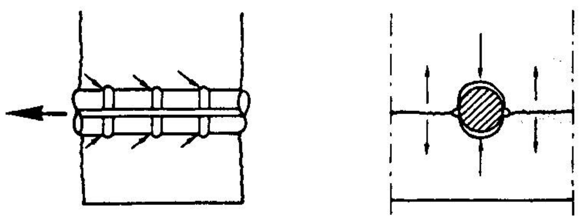

Fig.1 Splitting forces from reinforcement and possible tensile cracks

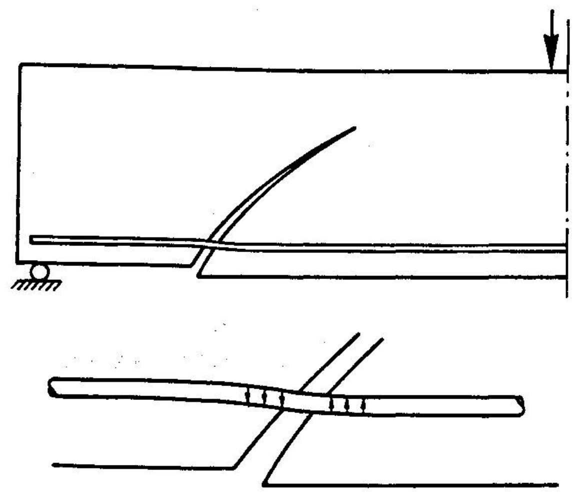

Fig.2 Dowel action causing risk of spalling of concrete cover.

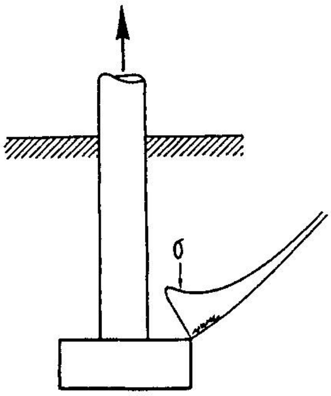

Fig.3 Stress concentration and possible formation of a crack where a bolt is anchored in concrete

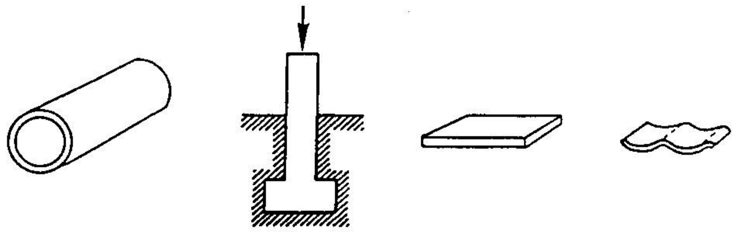

Fig.4 Examples of the use of unreinforced concrete

1.4 Anchorage of bolts

When a bolt is anchored in concrete, e.g. as in Fig. 3, a tensile force in the bolt causes tensile stresses in the surrounding concrete. It is not possible to anchor a bolt in concrete unless we rely on tensile strength.

1.5 Unreinforced concrete pipes, footings etc

Where unreinforced concrete is used in structures exposed to bending, it is quite obvious that we rely on tensile strength. Examples of such structures are concrete pipes of a moderate diameter, footings, prefabricated pavement slabs and concrete tiles; Fig. 4.

2. BRITTLENESS AND TOUGHNESS

Severe tensile stress concentrations may occur at, for example, the tip of shear cracks, near the crack surface in dowel action (Fig. 2), and near the bolt head at the anchorage of bolts (Fig. 3).

There are also stress concentrations in concrete due to internal cracking, which is always present. These cracks may e.g. be due to chemical shrinkage during the cement hydration. These cracks are small, but they nevertheless give rise to high theoretical stress concentrations at the crack tips.

Whenever the stress at a point exceeds the tensile strength, there is a risk that a crack may form, which has a tendency to propagate, as cracks in their turn give rise to stress concentrations. If the crack propagation is not prevented, it will lead to failure of the structure. As high stress concentrations are often present, it is of primary importance that mechanisms which prevent crack propagation exist.

There are two main mechanisms for the prevention of crack propagation. One is the possibility that the crack may propagate into a region where the tensile stress concentration at the tip disappears. The other is that the toughness of the material is great enough to absorb the released energy.

A bending crack in a reinforced beam is a typical point where an unstable crack may form and propagate as soon as the tensile strength is exceeded, but where it will be arrested and stay stable after the crack tip has reached the compression zone. This corresponds to the normal assumption for the design of a bent beam. In this case we may disregard the toughness of concrete in tension, even though it may have some influence on the beam stiffness and on the ultimate moment.

In many other cases, the formation of a crack and its propagation are only prevented by the toughness of concrete. If concrete were a perfectly brittle material, the stress concentrations would often lead to sudden brittle failures already at low external loads. Such concrete could hardly be used as a structural material. Toughness is therefore a very essential material property. For many structures it may be of the same importance as the strength.

In order to be able to take the toughness into account, it must be possible to quantify this property. This can be done by means of a suitable material model describing the tensile fracture behaviour.

3. TENSILE FRACTURE BEHAVIOUR

The tensile fracture behaviour can be described by means of a hypothetical tensile test on a plain bar of a constant cross section; Fig. 5. The test is made in deformation control, so that the total length of the specimen is slowly increased.

At first the stress increases as the deformation increases. At that stage the strains are the same all along the bar, provided that the material is homogeneous. Microcracks develop and grow, but in a stable way.

When the stress reaches the tensile strength, it cannot be increased any further. Instead, fracture phenomena start to develop at some section along the bar. A fracture zone starts to develop.

What happens at the fracture zone is that microcracks grow faster than at other points in the bar, and the microcracks also coalesce. These cracks would be unstable under constant stress conditions, but they may grow in a stable way in deformation control. As the cracks grow, the stress that can be transferred through this section decreases. The stress in the bar decreases as the deformation increases. The stress decrease outside the fracture zone corresponds to an unloading, with a corresponding strain decrease according to an unloading curve.

The fracture zone starts developing when the peak of the stress-deformation curve is passed. The part of the curve before the peak is reached is often called the pre-peak region or the ascending branch, whereas the part after the peak has been passed is called the post-peak region or the descending branch.

The pre-peak region can be described by a stress-strain curve, as the strain is equal all along the bar. This is not possible in the post-peak region, as we now have increasing deformations within the fracture zone, but decreasing deformations outside this zone. As there are two different types of behaviour, we need two different relations to describe the stress-deformation properties. This statement has general validity. Thus it is not possible to find a unique stress-strain curve which is also valid for the descending branch, either for tension or for compression.

The total deformation ΔL on a gauge length L in the post-peak part can be

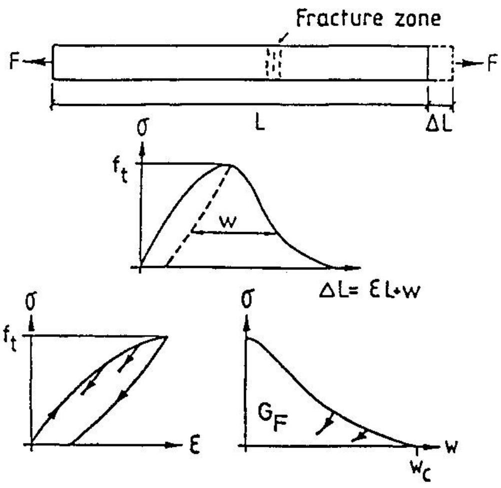

Fig.5 A tensile test and the description of the total elongation by means of two relations

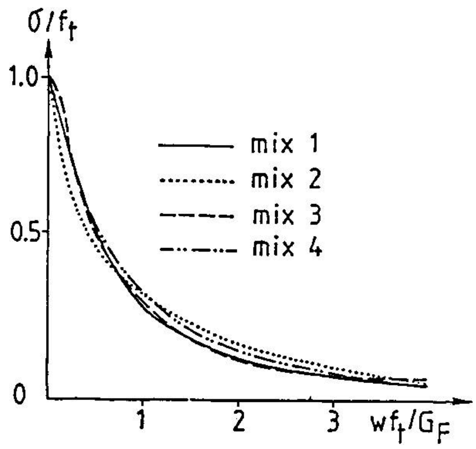

Fig.6 The shape of the σ-w-curve in a dimensionless form according to Petersson [6]

described by means of the following formula, referring to the two curves in Fig. 5

$$\Delta L = \epsilon L + w \tag{1}$$

For the descending branch of the total curve the value of ϵ should be taken from the unloading branch, starting from the peak in the σ-ε-diagram. The value w is the additional deformation within the fracture zone. It should be observed that w is a displacement, not a strain.

Equation (1) may be used as a general material description for stress-deformation properties in tension, and not only for the bar in Fig. 5. In the general case, the value of ϵ should be taken as the relevant value, which may be on the ascending branch or on an unloading curve starting from any point on the ascending branch. The value of w is zero if the peak has not been passed; otherwise it is taken from the σ-w-diagram, in some cases from an unloading branch in this diagram.

Concrete in a structure is exposed to a more complicated stress and strain situation than the bar in Fig. 5. There may be stresses and strains in directions perpendicular to the main tensile stress direction. If these are large compared to the strength at the moment when the peak is reached, they may influence the peak value and the shapes of the curves. Thus Eq. (1) can only be assumed to be a good approximation of the stress-deformation properties in tension if the stresses - tensile or compressive - in lateral directions are much lower than the correponding strengths.

One important property in this connection is the way in which the non-elastic deformations take place in the material. The non-elastic deformations in concrete are due to the opening of micro-cracks before the peak and of larger cracks after the peak. This type of deformation does not cause any major lateral stresses or strains. This is an important reason for assuming that Eq. (1) can be used as a general description for concrete. For metals, large non-elastic deformations are due to yielding, which causes high lateral deformations and stresses. Thus Eq. (1) with the corresponding diagrams in Fig. 5 may not be assumed to give a valid general description of the stress-deformation properties in tension for metals.

The toughness is due to the non-linear deformations. For concrete in tension, the non-linearity of the pre-peak curve is rather insignificant and does not contribute substantially to the toughness. The non-linear deformations and the toughness are mainly due to the post-peak part of the curve, which, in its turn, mainly depends on the σ-w-curve. As a reasonable simplification, the toughness of concrete in tension may be assumed to depend only on the properties of the σ-w-curve.

One important property of the σ-w-curve is the area below the curve. This area is denoted by G_F and it is defined as

$$G_F = \int_0^{w_c} \sigma dw \qquad (2)$$

From this formula it can be seen that G_F represents the amount of energy absorbed in one unit area of the fracture zone during the complete fracture process. G_F is thus the fracture energy per unit area. It is, as a rule, simply called fracture energy.

The properties of the σ-w-curve may be divided into three parts:

f_t

G_F

the shape, expressed in a dimensionless form

According to Petersson [6], Fig. 6 shows the shapes of the σ-w-curves for three different concrete mixes expressed in a dimensionless form. Petersson drew the conclusion that this shape is approximately the same for different concrete mixes. Later many σ-w-curves have been measured by different laboratories, and the results do not seem to contradict Petersson's conclusion. Thus as a reasonable approximation it will here be assumed that the shape is constant. This simplifies the analyses very much, as - in comparing different concrete qualities - the properties of the σ-w-curve are defined by only two values, f_t and G_F. The tensile strength f_t is a conventional property, and it may be assumed to be known. The only non-conventional value that has to be determined is G_F.

Of course it has to be remembered that the assumption that the shape of the σ-w-curve is constant is an approximation, which may be too rough for some types of concrete. It cannot, for example, be used for fibre-reinforced concrete. It is also possible that in some cases the slope of the first part of the curve is more important than the value of G_F. In the discussion below it is, however, assumed that the shape is constant.

The toughness of the material may be characterized by a ratio between non-elastic and elastic deformations. G_F/f_t may be taken as a value for the non-elastic deformation; see Fig. 6. As a value for the elastic deformation, the strain at the tensile strength, f_t/E, where E is the modulus of elasticity, may be taken. The ratio between G_F/f_t and f_t/E is a length which is called the characteristic length l_{ch}

$$l_{ch} = EG_F/f_t^2 \qquad (3)$$

For ordinary concrete the value of l_{ch} is of the order 0.1-1.0 m, with an average value around 0.4 m.

The best way of measuring G_F has been much discussed. Since it is difficult to perform a stable tensile test in principle according to Fig. 5, this is not a

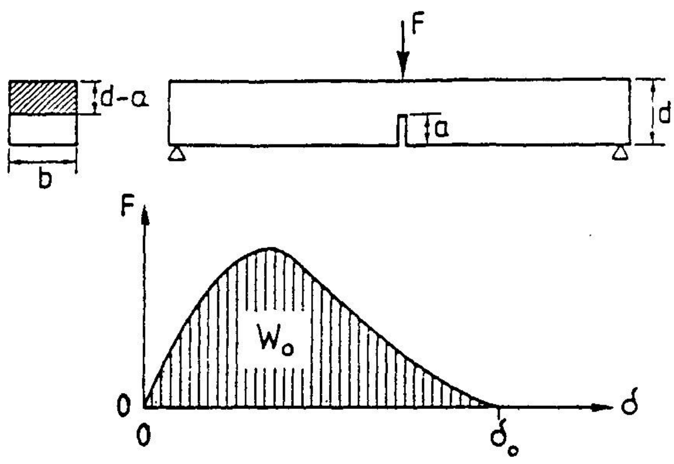

Fig.7 A stable bending test on a notched beam with registration of the load-deflection relation for the determination of G_F according to RILEM [7]

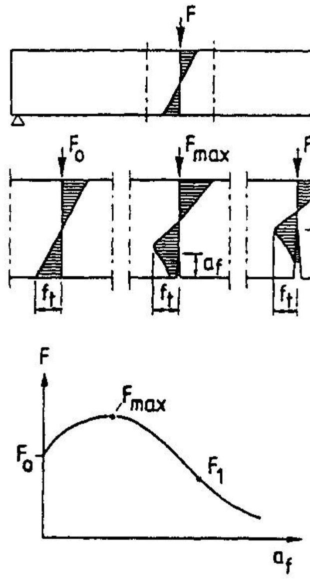

Fig.8 The principles of a numerical analysis of the formation and growth of a fracture zone

suitable standard test. A much simpler test is a stable bending test on a notched beam; Fig. 7. This type of test has therefore been proposed as a RILEM Recommendation [7]. It is assumed that all the energy released during the test is absorbed as fracture energy in the zone above the notch. The problems associated with the measurement of G_F will not be further discussed here.

The fracture energy G_F is a material property which has no direct connection with other properties, such as the compressive strength. It is quite possible that a change in concrete composition may lead to an increased strength but a decreased fracture energy. So far, we have too limited experience to know how to design a concrete mix in order to get a high fracture energy. We have some indications, e.g. that an increased maximum aggregate size may increase G_F, and that lightweight aggregates decrease G_F.

The characteristic length l_{ch} is a material property which can be said to define the toughness of the material. What is often more interesting is to have a value which characterizes the toughness of a structure. It can be shown that such a value is the ratio between l_{ch} and the size of the structure, defined e.g. by the depth of a beam. If the chosen size of the structure is denoted by d, the toughness is characterized by l_{ch}/d. As a rule, the inverse value d/l_{ch}, often called "brittleness number" [1] is used. It must be noted that the value of the brittleness number for a structure depends on the definition of d, which can be chosen arbitrarily, e.g. as beam depth, beam width, span, concrete cover, etc.

4. EXAMPLES OF RESULTS OF THEORETICAL ANALYSES

The description of tensile fracture given above can be used for analysing the behaviour of a structure which fails due to tensile stresses. Before failure, a fracture zone develops where the final failure takes place. Within the fracture zone, the σ-w-curve is valid for the relation between the transferred stress and the additional deformation. The σ-ϵ-curve is valid for all the material.

The fracture zone may be modelled as a zone which starts with zero width and takes all the displacement w, which means that its width is equal to w. It can then be looked upon as a "fictitious crack" with a width w, which transfers stresses σ according to the σ-w-curve. The model has therefore become known as the "fictitious crack model", although any reasonable small width of the fracure zone may be assumed in the numerical analyses without any noticeable influence on the results.

The analysis has to be numerically performed, as a rule by means of the finite element method. Fig. 8 demonstrates the principle of the analysis. When the tensile stress at the bottom of the beam reaches the tensile strength f_t, a fracture zone starts forming. When the beam deflection is increased, this fracture zone grows. At the same time, the stresses decrease close to the bottom of the beam, whereas they increase higher up in the beam. The stress at the tip of the fracture zone is constantly equal to f_t. As the fracture zone grows, the moment capacity, i.e. the load, at first increases due to a favourable redistribution of tensile stresses. At a later stage, it starts to decrease due to the decrease in tensile stresses at the bottom, associated with the increasing displacement w within the fracture zone. The moment capacity has a maximum, defining the flexural strength of the beam (modulus of rupture). In this way, the ratio between the flexural strength and the tensile strength can be determined; see below.

A number of structures have been numerically analysed by means of this method. Most of these analyses have been reported in [1]. Some significant examples will be discussed below.

All the analyses are based on the simplifying assumption that the σ-ϵ-relation is a straight line. The shape of the σ-w-curve is simplified to either a straight line or a bi-linear relation; see Fig. 9. These simplifications do not have any appreciable influence on the general conclusions drawn from the analyses.

4.1 Bending of a plain unreinforced beam

Fig. 9 shows the theoretical ratio between the flexural strength f_f and the tensile strength f_t for a beam with a rectangular section. Two different assumptions are used for the shape of the σ-w-curve. It can be seen that the assumed shape of this curve does not significantly influence the result. The scale in mm corresponds to an assumed value l_{ch}=250 mm.

It should be observed that the ratio f_f/f_t is uniquely defined by the ratio d/l_{ch} if the shape of the σ-w-curve is known. This type of uniqueness can be shown to have a general validity for any ratio f/f_t, where f is the formal strength value for a structure.

It should also be noted that logarithmic scales are used in Fig. 9. This is convenient for many reasons. One such reason is that it facilitates the judgement of the sensitivity of the results to variations of different properties, as will be shown below.

In Fig. 9, a dashed curve shows how an assumed distribution of shrinkage strains influences the flexural strength. The distribution has been assumed to be parabolic in the vertical direction. The maximum value is of the order which can be expected in normal indoor conditions. Although the assumptions are rather approximate, the general conclusion can be drawn that beams with small depths are rather insensitive to the influence of shrinkage, whereas deep beams are very sensitive.

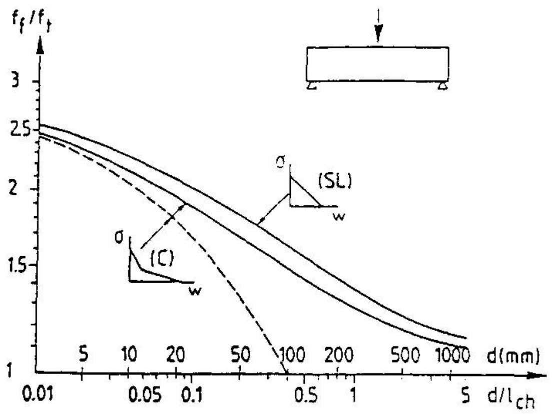

Fig.9 Theoretical ratio between flexural strength and tensile strength. The dashed curve includes influence of shrinkage

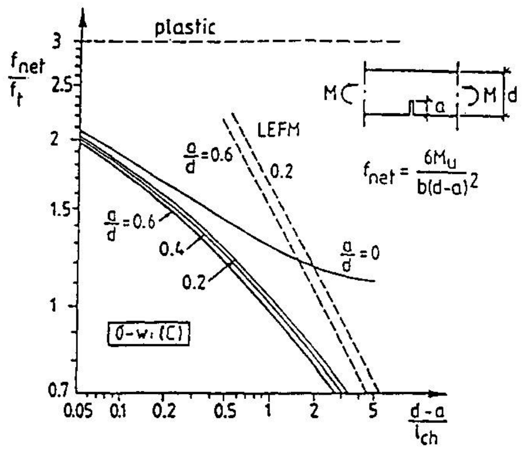

Fig 10 Theoretical ratio between net flexural strength and tensile strength for a notched beam

4.2 Bending of a notched beam

Fig. 10 shows how the net bending strength for the net area above a notch varies with the depth of the net section and with the relative notch depth a/d. A curve is also shown for a=0, i.e. a beam without a notch, the same as in Fig. 9. It can be seen that the relative notch depth has no significant influence as long as the notch is not very small. What is very interesting is that there is no major difference between the net bending strength of a notched beam and the strength of an unnotched beam for small beams, up to $(d-a)/l_{ch}$ about 0.5, which on an average corresponds to (d-a) about 0.2 m. Such small beams are said not to be "notch sensitive". The deeper the beams, the higher the notch sensitivity. It must be noted that the notch sensitivity depends both on the size of the structure and on a material property, l_{ch}. It is thus not a pure material property, which is sometimes claimed.

The dashed lines in Fig. 10 correspond to pure linear elastic fracture mechanics (LEFM), which can be seen not to be applicable to small beams, but to indicate the limiting case for large notched beams.

4.3 Unreinforced concrete pipes

Fig. 11 shows the results of an analysis of failure of unreinforced concrete pipes. Two different types of failure have been analysed. One type is a bending failure, with the pipe acting as a beam. The other type is called a crushing failure, with the forces acting along two opposite generatrices and the structure acting as a ring. In both cases, the formal flexural strength value is the maximum tensile stress at maximum load, calculated according to the theory of elasticity.

It can be seen from Fig. 11 that the formal flexural strength in crushing failure is higher than that in bending failure and that the size dependency is greater in crushing failure. One reason for these differences is that the active depth in the crushing failure is the wall thickness t, whereas in the bending failure it is the outer diameter. Another reason is that the structure is statically determinate in the bending failure, whereas in the crushing failure it is statically indeterminate, with a possible redistribution of moments before final failure. The ability to redistribute moments increases with a decreasing wall thickness.

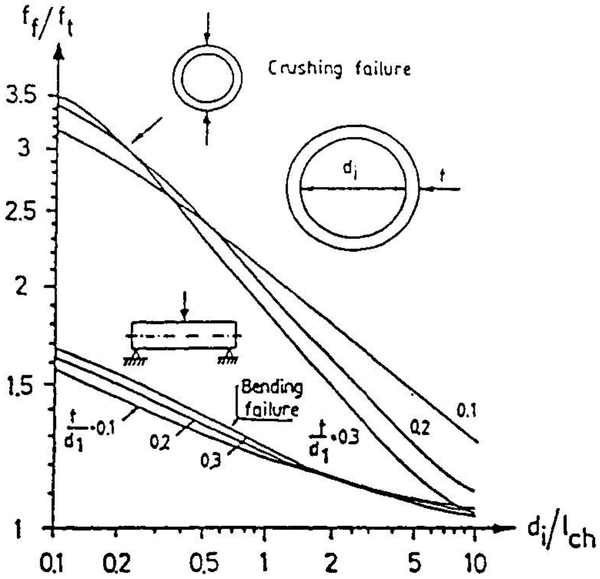

Fig.11 Theoretical ratio between a formal flexural strength and the tensile strength for two different types of failure of an unreinforced concrete pipe

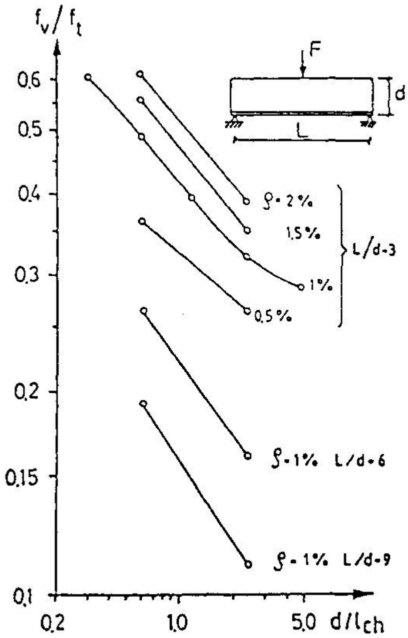

Fig.12 Theoretical ratio between a formal shear strength and the tensile strength of a beam with a tensile reinforcement ratio ρ

4.4 Shear failure

Gustafsson [2] performed an analysis of the shear failure of a beam with tension reinforcement, but without shear reinforcement. A number of assumptions and approximations had to be introduced in order to make the very laborious numerical analysis possible. No details will be given here, but the results alone, which are believed to give a reasonably accurate idea of the influence of different parameters. As the finite element programs develop, someone will hopefully perform a more advanced analysis. So far this is to the author's knowledge the only systematic analysis of shear failure which has been performed.

Fig. 12 summarizes the results. The points show calculated values: f_v is the formal shear strength, ρ is the percentage of reinforcement. The variables studied are the beam depth (d/l_{ch}), the reinforcement ratio and the span to depth ratio (L/d).

One thing which was not taken into account was the dowel action and the resulting possible splitting failure along the reinforcement. Nor were the shear stresses in the fracture zone ("aggregate interlock") taken into account in a realistic way.

5. INFLUENCING FACTORS

The formal strength f of a structure, expressed as e.g. a flexural strength or as a shear strength, is a function of the ratio d/l_{ch}, where d is a typical size of the structure. This function can be expressed in the general form (see

Figs. 9-12)

$$f/f_t = \varphi(d/l_{ch}) \quad (4)$$

In the diagrams, logarithmic scales are used. If we wish to study the sensitivity of f to small changes in other variables at a certain point in such a diagram, we may replace the curve by its tangent at this point. The equation of this tangent may be written in the following way:

$$\ln(f/f_t) = A - B\ln(d/l_{ch}) \quad (5)$$

Introducing $l_{ch}=EG_F/f_t^2$, the equation can be transformed into:

$$\ln f = A - B\ln d + B\ln E + B\ln G_F + (1-2B)\ln f_t \quad (6)$$

A differentiation of this equation gives:

$$df/f = -Bdd/d + BdE/E + BdG_F/G_F + (1-2B)df_t/f_t \quad (7)$$

This equation shows how the relative change in f is dependent on the relative change in each of the other variables. Thus an increase of G_F by 10% increases f by 10B%, and an increase of f_t by 10% increases f by 10(1-2B)%, where B is the negative slope of the tangent to the curve at the point in question.

The sensitivity of f with respect to changes in E or G_F can be said to be equal to B, and the sensitivity with respect to changes in f_t to (1-2B).

If different logarithmic scales are used on the axis, this has to be taken into account in the determination of B if Eq. (5) is to be fulfilled. In the diagrams above, the scale on the vertical axis is 4 times larger than that on the horizontal axis, which means that the slopes measured in the diagrams have to be divided by 4.

As to the flexural strength f_f according to Fig. 9, we find that, for ordinary beam sizes, B is approximately 0.15. The sensitivity with respect to changes in G_F is thus 0.15, and the sensitivitity with respect to changes in f_t 0.7. With all other factors unchanged, an increase in the tensile strength by 10% will only increase the flexural strength by 7%.

From the dashed curve in Fig. 9 it is evident that the value of the slope B is much higher when the beam is exposed to shrinkage. It may then easily reach values above 0.3, particularly for large beams. In that case the sensitivity of f_f with respect to changes in G_F becomes very important, at the same time as the sensitivity with respect to changes in f_t is reduced. In order to increase the flexural strength of large beams exposed to shrinkage, it may be important to try to increase G_F rather than f_t.

According to Fig. 10, the value of B is higher for a notched beam than for an unnotched beam. It is typically of the order 0.25, which means that the sensitivity of f_{net} with respect to changes in G_F is about 0.25 and with respect to changes in f_t about 0.5. With all other factors unchanged, an increase in the tensile strength by 10% will only increase the net bending strength by about 5%.

According to Fig. 11, the value of B for unreinforced concrete pipes is about 0.1 for bending failure and about 0.3 for crushing failure. This means that for crushing failure the sensitivity of f_f with respect to changes in G_F is about 0.3, and with respect to changes in f_t about 0.4. In order to increase the crushing strength, it may thus be more important to increase G_F, if this can be easily achieved, than to increase the tensile strength. For bending failure on the other hand, f_t is more important than G_F.

For shear failure according to Fig. 12 the average slope of all the lines corresponds to a value of B of about 0.25. The sensitivity of the shear strength with respect to changes in G_F is about 0.25, and with respect to changes in f_t about 0.5. The fracture energy is evidently an important factor for the shear strength of beams and slabs.

With B=0.25, the shear strength is inversely proportional to $d^{0.25}$, where d is the beam depth.

6. COMPARISONS WITH TEST RESULTS

6.1 Bending of plain unreinforced beams

Already in the very first published paper [3] on what later became known as the fictitious crack model, a comparison was made between the results of the theoretical analysis and of tests on the influence of the beam depth on the flexural strength. For beams without shrinkage, comparisons could be made with test results summarized by Meyer [5]. This comparison showed a good agreement, which encouraged a further development of the model. A comparison was also made with a small series of beams of different sizes with and without exposure to drying. In this case as well, a good agreement was found between theory and tests.

6.2 Unreinforced concrete pipes

Gustafsson [2] made a comparison of a few series of tests where equal pipes had been tested both for bending failure and for crushing failure. He was then able to show that the different formal flexural strengths calculated according to conventional formulae corresponded to the same tensile strength when the diagram in Fig. 11 was applied. As a matter of fact, the reason for his analysis was that the producers had difficulties in explaining why the formal bending strength calculated from a crushing test was more than 50% higher than that calculated from a bending test.

6.3 Shear failure

The analysis performed by Gustafsson [2] is the first purely theoretical analysis of shear failure. All design formulae have hitherto been based on extensive tests, often interpreted by means of some physical model, e.g. a strut and tie model, where empirical constants have been adjusted in order to achieve agreement with tests. A basic theoretical understanding of shear failure has been lacking. The analysis performed by Gustafsson is an important breakthrough, as it is the first time that some real theoretical understanding of shear fracture has been achieved. However, much more work remains to be done before shear fracture can be analysed and understood in detail.

One important factor which has been explained by means of theoretical analysis is the influence of beam depth. This influence has long been known from tests, and it is taken into account in concrete codes, e.g.according to CEB. No one has been able to explain it earlier by means of a theoretical analysis. From the test results the conclusion has been drawn that the shear strength is approximately inversely proportional to $d^{0.25}$ [4]. This is in agreement with the theoretical analysis.

As a basis for concrete codes, a great number of test results have been summarized regarding the influences of different factors: in the first place beam depth, reinforcment ratio and span to depth ratio. The theoretical relative influences of these factors can be estimated approximately by means of Fig. 12. In all cases, a reasonable agreement between the analytic and the empirical results has been found; see [1] or [2].

6.4 Fracture mechanics test specimens

A great number of fracture mechanics tests, i.e. tests on differently shaped notched or precracked specimens, have been theoretically analysed. In particular, Gustafsson [2] has reported many such analyses. As a rule, they show good agreement between tests and analyses.

6.5 Summary

The comparisons which have been made between theoretical analyses and tests all show that the fictitious crack model according to Fig. 5 gives a realistic description of tensile failure, and that it can be expected to give much more reliable predictions of failure loads than the conventional methods. A systematic study by means of this model of cases where we rely on tensile strength will lead to a safer basis for concrete design. The resulting consequences from the points of view of safety and economy motivate a strong support for this type of research.

7. SOME PRACTICAL CONCLUSIONS

Some practical conclusions can be drawn from the systematic analyses. A number of these will be commented upon.

Starting with Fig. 9, we can see that the flexural strength decreases with an increasing beam depth. The increase is particularly important where the structure is exposed to drying shrinkage. One consequence is that the flexural stress which causes cracking is lower for large beams than for small beams. This is also the case for reinforced beams, as the reinforcement has only a small influence on the cracking load. The influence of size ought to be taken into account when the cracking moment is used in connection with the calculation of the stiffness of a beam.

Another consequence is that the flexural strength should only be relied upon for structures with either a very small depth or with negligible drying shrinkage within the tensile zone. Examples of very thin structures are pavement slabs of about 50 mm thickness and concrete tiles. Footings are an example of a case with a negligible drying shrinkage. Fig. 9 thus explains why we may rely on concrete flexural strength in these cases.

Fig. 11 is already used by manufacturers for the design of unreinforced pipes. It is a good example of the practical applicability of the model for tensile fracture. As the wall thickness of these pipes is limited, stresses due to drying shrinkage can be estimated to have a limited influence, although some increasing influence with an increasing size may be expected.

The analysis of shear failure in Fig. 12 supports the empirical findings upon which the concrete codes are based, e.g. regarding the influence of beam depth.

The analysis may support some changes in concrete codes, resulting in a more even safety factor and a better economy.

As the analytic results according to Fig. 12 are supported by test results regarding the influence of the beam depth, it may be assumed that the following general relation is valid:

$$f_v/f_t \sim (d/l_{ch})^{-0.25} \qquad (8)$$

Introducing $l_{ch} = EG_F/f_t^2$ and rearranging gives:

$$f_v \sim (EG_F f_t^2/d)^{0.25} \qquad (9)$$

As an approximation, it is often assumed that the tensile strength f_t is proportional to the square root of the compressive strength f_c. If we accept this assumption, we find:

$$f_v \sim (EG_F f_c/d)^{0.25} \qquad (10)$$

From this relation the conclusion can be drawn that the fracture energy G_F is of about the same importance as the compressive strength f_c for the shear strength of a reinforced beam. When shear tests on beams are performed, the concrete quality should be given, not only as a concrete strength f_c, but also as a fracture energy G_F.

From Eq (10) it may be expected that the shear strength of lightweight concrete beams is lower than that of normal concrete beams with the same tensile strength, as E and G_F are lower for lightweight concrete than for normal concrete.

Creep under long-term loading causes a reduction in the formal E-value by a factor $1/(1+\Phi)$, where Φ is the creep factor. Thus e.g. with $\Phi=2$, the shear strength may be expected to be reduced by a factor of $(1/3)^{0.25}=0.76$, provided that G_F and f_t are not time-dependent.

8. POSSIBLE FURTHER APPLICATIONS

In principle, the model for tensile fracture can be applied in analysing all the cases where we need to rely on the tensile strength of concrete. We are still far from a sufficient understanding of all types of tensile-induced structural failures. Some of them are very complicated to analyse. This is the case with, for example, shear and punching failure, including the dowel action in combination with spalling forces from the bond stresses between reinforcing steel and concrete. A major research effort is needed within this area in order to achieve better guidelines for design rules. Such a research effort may be expected to lead to a more even safety factor and large savings.

The anchorage of bolts is a case which can be theoretically analysed, although it is rather complicated, because not only tension, but also shear stresses act in the fracture zone, and several fracture zones may have to be taken into account. Test results [1] show that the fracture energy in this case is an important concrete property, whereas the tensile strength has a limited influence. In terms of sensitivity, it seems that in this case the value of B is close to 0.5. One consequence of this is also that the formal stress associated with pull out of a bolt is very size-dependent.

Anchoring reinforcing bars always gives splitting tensile stresses, which may cause tensile-induced failure in a structure. A number of different situations may be distinguished, depending on the type of anchorage, e.g. by bond or by end anchors, deformed or plain bars, prestressed or non-prestressed reinforcement, dowel action due to shear cracks etc. These different situations may all be theoretically analysed.

In order to obtain valid results for practical design purposes, the influence of shrinkage should also be analysed, as shrinkage may sometimes have an important unfavourable influence on the strength of a structure; see e.g. Fig.9.

9. CONCLUSIONS

The model for tensile fracture according to Fig. 5 (the "fictitious crack model") gives realistic predictions compared to test results. By means of this model it is possible to analyse tension-induced failures and to estimate to what extent the tensile strength of concrete may be relied upon.

From the analyses performed so far, the following general conclusions regarding tensile-induced structural failures may be drawn:

1. The formal strength (a formal stress value at maximum load) is size-dependent. The strength is proportional to d^{-B}, where d is a typical size of a structure and B normally varies between 0.15 and 0.5.

2. The formal strength depends on the tensile toughness of concrete. A material property closely connected with toughness is the fracture energy G_F. This material property should be determined and taken into account in the evaluation of test results and in design, particularly for less common types of concrete, such as high strength concrete and concrete with weak aggregates. The strength is proportional to G_F^B, with B as above.

3. The formal strength is proportional to E^B, where E is the elastic modulus and B has values as above. The ratio between long-term and short-term strength can be estimated from the formal reduction of E due to creep.

4. The formal strength is proportional to $f_t^{(1-2B)}$, where f_t is the tensile strength and B has values as above. In cases of high B values, the tensile strength may be a less important material property than G_F and E for a tension-induced failure.

5. Shrinkage may decrease the formal strength appreciably, particularly where the failure already has a brittle character.

6. Toughness (ductility) in tension is a property of the utmost importance for the reliance upon concrete tensile strength.

REFERENCES

1. ELFGREN L. (editor), Fracture mechanics of concrete structures. RILEM Report, Chapman and Hall, 1989.

2. GUSTAFSSON P J., Fracture mechanics studies of non-yielding materials like concrete. Report TVBM-1007, Div. of Building Materials, Lund Institute of Technology, 1985.

3. HILLERBORG A., MODÉER M., and PETERSSON P-E., Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. Cement and Concrete Research, Vol 6 (1976), pp. 773-782.

4. LEONHARDT F., Shear in concrete structures. Bulletin d'information CEB No. 126, Shear and torsion, Paris 1978, pp. 66-124.

5. MAYER H., Die Berechnung von Durchbiegung von Stahlbetongbauteilen. Deutscher Ausschuss für Stahlbeton, H 194, 1967, W Ernst & Sohn, Berlin.

6. PETERSSON P-E., Crack growth and development of fracture zones in plain concrete and similar materials. Report TVBM-1006, Div. of Building Materials, Lund Institute of Technology, 1981.

7. RILEM, Determination of the fracture energy of mortar and concrete by means of three-point bend tests on notched beams. Materials and Structures, Vol 18 (1985) pp. 285-290.