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Detection of Local Stiffness Changes of Buildings

Mesure des variations locales de rigidité dans des bâtiments

Messung örtlicher Steifigkeitsänderungen in Hochbauten

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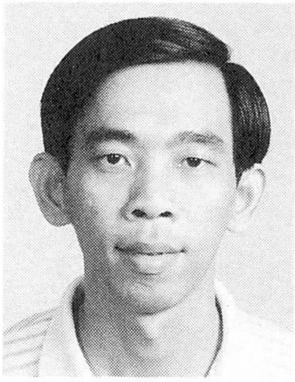

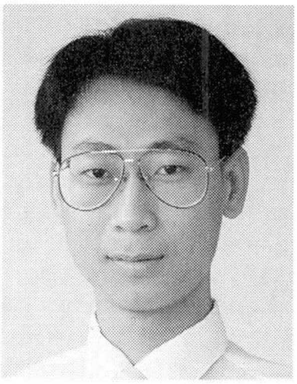

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SUMMARY

A method for identification of local structural changes in terms of storey stiffnesses of buildings is proposed here. Static condensation is applied to reduce the size of system for identification, while stiffness changes are determined recursively in a remedial model by the extended Kalman filter. The efficacy of this "improved condensation" method under various noise levels is illustrated numerically by an example of a twelve-storey plane frame building.

RÉSUMÉ

L'auteur présente ici une méthode pour identifier les faiblesses structurales locales des bâtiments. Il se base pour cela sur la détermination de la rigidité des étages par identification du système, appliquée à un modèle réduit par compression statique, en procédant par la méthode itérative de Kalman de filtrage non linéaire. Il démontre ainsi l'efficience de cette méthode de compression améliorée en l'appliquant à un bâtiment de douze étages à ossature plane en portique, sous niveaux différentiels de perturbations.

ZUSAMMENFASSUNG

Es wird eine Methode zur Auffindung örtlicher Tragwerksschwächungen in Hochbauten vorgestellt. Sie basiert auf Ermittlung der Stockwerkssteifigkeit durch Systemidentifikation an einem durch statische Kondensation reduzierten Modell, wobei iteraktiv mit einem erweiterten Kalman-Filter vorgegangen wird. Die Effizienz dieses verbesserten Kondensationsverfahrens wird an einem zwölfgeschlossigen ebenen Rahmentragwerk für unterschiedliche Störsignalpegel demonstriert.

INTRODUCTION

In recent years, application of system identification (SI) to damage assessment and safety evaluation of civil engineering structures has received considerable attention (e.g. Natke and Yao 1987; Agbabian et al. 1991). Based on input and output measurements of dynamically excited structures, structural parameters such as stiffnesses are determined and then compared with intended design values. In this manner, periodic monitoring of state of structures can be performed for detection of structural changes due to damage or deterioration. However, several problems have yet to be resolved before this methodology can become viable for actual structures.

One of the problems reported by some researchers is that current SI techniques have not been satisfactory in detecting local damages. Modal parameters as determined by frequency domain analysis are not sensitive to local damages, except for small structural systems or unless high modes are taken into account. The accuracy of high modes is, however, often difficult to achieve because of measurement noise. Hence, there is apparently a trend that researchers prefer time-domain SI approaches, among which the extended Kalman filtering (EKF) originally developed by Kalman and Bucy (1961) is perhaps most widely used. Nevertheless, it has been found that the change in stiffness matrix due to member stiffness changes in the locality tends to "spread out" or "diffuse" into adjacent structural members (herein referred to as "stiffness diffusion"), thereby making local damage identification difficult (Natke and Yao 1987).

In addition, from the viewpoint of structural safety evaluation, it is important to estimate the confidence level (or reliability) of identified parameters taking into consideration measurement noise as well as modeling errors. In this aspect, Agbabian et al. (1991) has applied least-squares approximation methods to successive time windows of input and output (I/O) time histories. In their numerical studies, the effects of I/O noise have been taken into account. However, to the authors' knowledge, modeling errors have thus far not been considered in the confidence estimate of identified parameters.

IMPROVED CONDENSATION METHOD

The problem of local damage detection is aggravated by the large number of degrees of freedom (DOFs) in modeling an actual structure. When applied to a complete structural model involving all DOFs, the EKF and other time-domain SI approaches alike are often found to be numerically inefficient in terms of accuracy, convergence and computation speed. Alternatively, a "reduced" model with a smaller number of DOFs can be considered. For instance, if quantification of storey stiffness changes of a building suffice for the purpose of damage detection, a simple lumped mass model can be used to reduce DOFs. Unfortunately, as a result of considerable modeling errors, diffusion problem of storey stiffness into adjacent storeys would render local damage detection ineffective.

As an attempt to detect the location of damage and quantify the magnitude of damage in terms of stiffness reduction of a building, an improved condensation method (ICM) is proposed here. For illustration purpose, a single-bay n-storey plane frame building as shown in Fig. 1(a) is considered. Axial and shear deformations are assumed to be negligible. Static condensation is first conducted, reducing the complete structural model to a "condensed" model with a significantly smaller number of DOFs [Fig. 1(b)]. In this study, columns are considered to be the critical elements where damages are likely to occur and affect the overall performance of the building.

Non-critical elements (beams) are assumed to be undamaged and any difference in storey stiffness is solely due to changes in column flexural rigidities (*EI*). In order to narrow the gap between the condensed model and the actual structure (upon which I/O measurements are taken), a "remedial" (or correction) stiffness matrix $\mathbf{K}_R$ is derived based on a lumped mass model as shown in Fig. 1(c). By applying the EKF on a time window of data, the stiffnesses of this remedial model are identified and then used to update the condensed model. This process is repeated for a specified number of time windows (or until convergence).

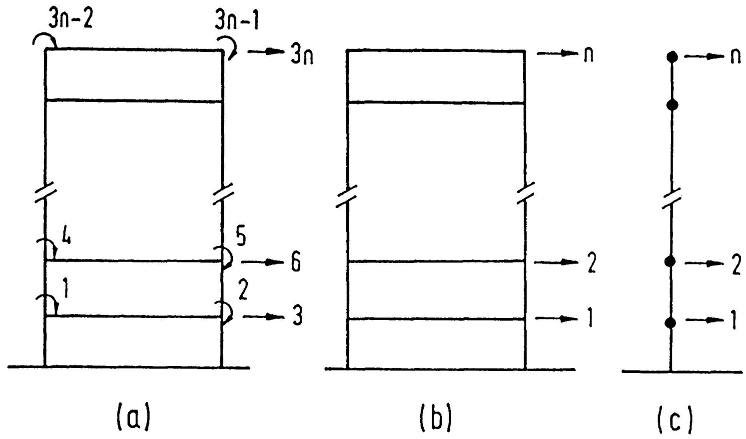

FIG. 1. (a) Complete Model; (b) Condensed Model; (c) Remedial model. (Numbers indicate DOFs).

The procedure of the ICM for damage detection of the building considered is described below.

(1) Divide the excitation and response time histories into time windows.

(2) Form mass matrix $\mathbf{M}$ ($3n$x$3n$) for the complete model based on known mass distribution. Given flexural rigidities of the "undamaged" building, form complete stiffness matrix $\mathbf{K}$ ($3n$x$3n$) encompassing all DOFs, i.e. two joint rotations and one horizontal translation per floor [Fig. 1(a)].

(3) Damping matrix $\mathbf{C}$ ($3n$x$3n$) is constituted by adopting Rayleigh damping , assuming that damping ratios of two vibration modes are known. (This assumption can be relaxed by including damping ratios as additional unknown parameters to be identified.)

(4) Perform static condensation to obtain condensed mass matrix $\mathbf{M}_C$,damping matrix $\mathbf{C}_C$and stiffness matrix $\mathbf{K}_C$, all of size nxn [Fig. 1(b)].

(5) The improved condensation model is derived by adding the remedial stiffness matrix $\mathbf{K}_R$ (as explained earlier) to the condensed stiffness matrix $\mathbf{K}_C$. Elements in $\mathbf{K}_R$ are the unknown parameters to be identified by the EKF, while $\mathbf{K}_C$ remains unchanged.

(6) Compute stiffness correction factor η_j for the j-th storey as follows:

$$\eta_j = \sum_{k=1}^{N_W} (EI_R)_j^{(k)} / (EI_U)_j \tag{1}$$

where U denotes the undamaged quantity, R denotes the remedial quantity and N_W is the current time window number. Since the remedial model is derived from a shear building, the remedial flexural rigidity is given by $(EI_R)_j=(K_R)_j\, l_j^3 / 12$, where $(K_R)_j$ is the corresponding storey stiffness in $\mathbf{K}_R$, and l_j is the column length. The updated flexural rigidities

$$EI_j = (EI_U)_j(1 + \eta_j), \qquad j = 1 , \dots , n \tag{2}$$

are then used to compute the complete stiffness matrix $\mathbf{K}$.

(7) Repeat step 3 to step 6 for all time windows considered. The severity of the damage in each storey is finally given by the end result of η_j.

ADAPTIVE FILTER

In SI techniques employing the EKF, uncertainties in terms of variances of identified parameters are supposedly reflected in the error covariance matrix. The error covariance is dependent on the output noise covariance and the system noise covariance in the EKF algorithm. The variances of input and output noise can be estimated from resolution and accuracy of instruments and data acquisition system. The main problem is the difficulty in estimating the variance of system noise which includes modeling errors as well as input noise. In application of the EKF, the uncertainty in the system noise causes the divergence phenomenon, especially when the input noise is small in comparison with the modeling errors.

An adaptive filter was developed by Jazwinski (1969) as an algorithmic attempt to control divergence in Kalman filtering of orbit determination problems. In this paper, this adaptive filter is modified to suit SI problems for the purpose of obtaining statistically consistent variances of identified parameters.

Determination of System Error Covariance Q

Consider a time window beginning with k-th time step. The predicted residual vector at p steps later (i.e. time t_{k+p}) is defined as

$$\mathbf{r}_{k+p} = \mathbf{y}_{k+p} - \mathbf{H}(t_{k+p})\hat{\mathbf{x}}(t_{k+p}|t_k) \tag{3}$$

where $\mathbf{y}_{k+p}$ is an observation vector (mx1), $\hat{\mathbf{x}}(t_{k+p}|t_k)$ is an expected state vector (nx1), and $\mathbf{H}(t_{k+p})$ is an observation matrix (mxn). The covariance of the predicted residual vector can be derived by means of the EKF algorithm as

$$E\{\mathbf{r}_{k+p}\mathbf{r}^T_{k+p}\} = \mathbf{H}(t_{k+p})\Phi(t_{k+p},t_k)\mathbf{P}(t_k|t_k)\Phi^T(t_{k+p},t_k)\mathbf{H}^T(t_{k+p})$$

$$+ \mathbf{H}(t_{k+p})\left\{\sum_{i=1}^{p}\Phi(t_{k+p},t_{k+i})\mathbf{Q}(t_{k+i-1})\Phi^T(t_{k+p},t_{k+i})\right\}\mathbf{H}^T(t_{k+p}) + \mathbf{R}_{k+p} \tag{4}$$

where $E\{\cdot\}$ denotes expectation operator, Φ is a state transition matrix (nxn), $\mathbf{P}$ is an error covariance matrix (nxn), $\mathbf{Q}$ is a system noise covariance matrix (nxn), and $\mathbf{R}$ is a measurement noise covariance matrix (mxm). $\mathbf{Q}$ is determined by ensuring consistency between the residuals and their statistics such that

$$\frac{1}{p}\sum_{i=1}^{p}\mathbf{r}_{k+i}\mathbf{r}^T_{k+i} \mathrel{\overline{\overline{\cdot}}} E\{\mathbf{r}_{k+p}\mathbf{r}^T_{k+p}\} \tag{5}$$

Equation 4 can thus be written as

$$\mathbf{H}(t_{k+p})\left\{\sum_{i=1}^{p}\Phi(t_{k+p},t_{k+i})\mathbf{Q}(t_{k+i-1})\Phi^T(t_{k+p},t_{k+i})\right\}\mathbf{H}^T(t_{k+p}) = \frac{1}{p}\sum_{i=1}^{p}\mathbf{r}_{k+i}\mathbf{r}^T_{k+i} - E\{\mathbf{r}_{k+p}\mathbf{r}^T_{k+p}|\mathbf{Q}(t_{k+p})=0\} \tag{6a}$$

where

$$E\{\mathbf{r}_{k+p}\mathbf{r}^T_{k+p}|\mathbf{Q}(t_{k+p})=0\} = \mathbf{H}(t_{k+p})\Phi(t_{k+p},t_k)\mathbf{P}(t_k|t_k)\Phi^T(t_{k+p},t_k)\mathbf{H}^T(t_{k+p}) + \mathbf{R}_{k+p} \tag{6b}$$

In the present formulation, only diagonal elements of $\mathbf{r}_{k+p}\mathbf{r}^T_{k+p}$ are considered whereas all off-diagonal elements are assumed to be zero (i.e. no cross-correlation between residuals). Let $\mathbf{Q}(t_{k+p})$ equal to $\mathbf{G}_{k+p}\mathbf{Q}_{k+p}\mathbf{G}^T_{k+p}$ where $\mathbf{G}_{k+p}$ is a distribution matrix. The left hand side of Eq. 6(a) can now be expressed as $\mathbf{A}_{k+p}\mathrm{diag}[\mathbf{Q}_{k+p}]$ where

$$\mathbf{A}_{k+p} = \sum_{i=1}^{p}\left[\mathbf{H}(t_{k+p})\Phi(t_{k+p},t_{k+i})\mathbf{G}_{k+i-1}\right]^2 \tag{7}$$

Hence, the diagonal terms of Eq. 6(a) can be obtained from

$$\mathbf{A}_{k+p}\mathrm{diag}[\mathbf{Q}_{k+p}] = \varepsilon_{k+p} \tag{8a}$$

where

$$\varepsilon_{k+p} = \mathrm{diag}\left[\frac{1}{p}\sum_{i=1}^{p}\mathbf{r}_{k+i}\mathbf{r}^T_{k+i} - E\{\mathbf{r}_{k+p}\mathbf{r}^T_{k+p} \,|\mathbf{Q}(t_{k+p})=0\}\right] \tag{8b}$$

It is reasonable to assume that $\mathbf{Q}$ remains constant in a time window of N sampling points, i.e. $\mathbf{Q}_k= \mathbf{Q}_{k+1}=...= \mathbf{Q}_{k+N}$. The system noise covariance in this time window is given by

$$\mathrm{diag}[\mathbf{Q}_{k,N}] = (\mathbf{A}^T_{k,N}\ \mathbf{A}_{k,N})^{-1}\mathbf{A}^T_{k,N}\ \varepsilon_{k,N} \tag{9a}$$

where $\mathbf{A}_{k,N}$ is an Nmxn matrix and $\varepsilon_{k,N}$ is an Nmx1 vector written as follows

$$\mathbf{A}_{k,N}= \{\mathbf{A}_k,\mathbf{A}_{k+1},...,\mathbf{A}_{k+N}\}^T, \qquad \varepsilon_{k,N}= \{\varepsilon_k,\varepsilon_{k+1},...,\varepsilon_{k+N}\}^T \tag{9b,c}$$

The main procedure of adaptive filter is schematically explained in Fig. 2. For purpose of discussion, we now let $\hat{\theta}$ denote a vector of parameters to be identified, such as unknown stiffnesses, and $\hat{\mathbf{x}}$ contains only response variables (displacements and velocities). Initially the EKF may be carried out once (not shown in Fig. 2) to obtain a better guess of $\hat{\theta}_0$. An adaptive filter cycle comprises two processes: (a) determination of $\mathbf{Q}$ by enforcing statistical consistency of residuals using N sampling points, and (b) determination of $\hat{\mathbf{x}}$, $\hat{\theta}$ and $\mathbf{P}$ by the EKF using M sampling points.

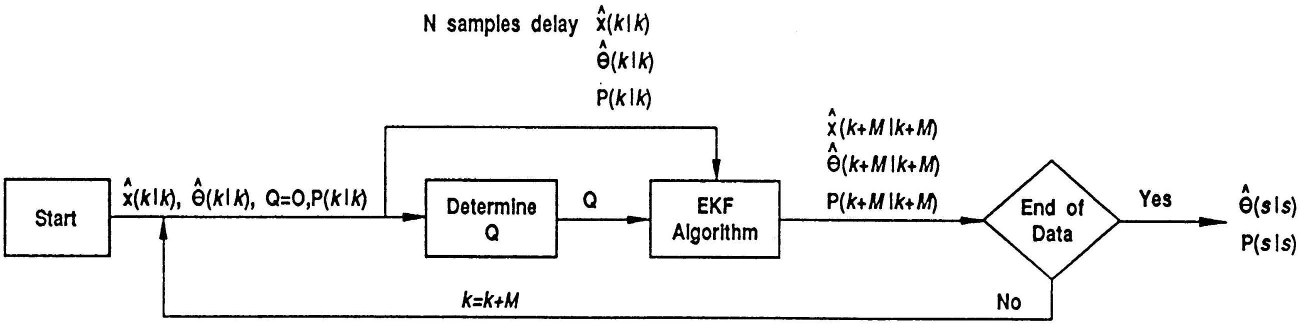

FIG. 2. The Adaptive Filter Procedure.

NUMERICAL EXAMPLES AND DISCUSSIONS

In our examples, the input is a force comprising several (five or more) harmonics of frequencies covering the first few significant vibration modes of the structure. Added to input and output time histories are independent Gaussian noises with zero mean and standard deviations equal to certain specified percentages of their respective unpolluted root-mean-square values.

Example 1 (ICM)

The procedure of ICM is illustrated by considering a 12-storey plane frame building. The "complete" plane frame model has a total of 36 DOFs (two joint rotations and one horizontal translation at each floor). The mass matrix and stiffness matrix for the undamaged building are derived from a small-scale steel laboratory model. The damping ratio is 0.5% as determined by free vibration tests of the laboratory model.

We now consider the building to be "damaged": the column stiffnesses in the first, fourth, eighth, ninth and eleventh storeys are reduced by 10, 15, 30, 20 and 25 per cents, respectively. An excitation force is applied at the top floor and horizontal responses at all floors are measured. Total observation time history of 2 s at a sampling rate of 0.0005 s is divided into 20 windows. All rotational DOFs are eliminated in the condensed model and the remedial model thus has 12 DOFs.

Following the procedure of the ICM described earlier, storey stiffnesses of the damaged building are identified without I/O noise and with different noise levels. The ratios of damaged storey stiffnesses to the corresponding values of the undamaged building are computed and summarized in Table 1. In the ideal case of zero I/O noise, the identified stiffnesses are almost exact (error < 1%) for all twelve storeys and the stiffness diffusion problem is negligibly small. In comparison, if the lumped mass model were used instead for the same conditions, the results (not shown in Table 1) would have been disastrous with error as high as 60% at some storeys. For an I/O noise level of 20% which is considerably high in practice, the identified results are remarkably good (with error ranging from 0.2% to 6% only) in view of the fairly large system for identification.

Story / I/O Noise	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	12th
0%	90.0	99.6	100.5	85.0	99.9	99.8	100.0	70.1	80.3	99.9	75.0	99.6
10%	89.7	97.2	103.4	87.6	100.7	99.6	99.8	72.8	83.6	97.7	76.8	100.3
20%	90.9	97.1	95.8	86.9	100.2	101.9	100.7	74.1	83.4	97.1	76.0	101.2
20%*	88.5	120.8	126.0	91.0	92.9	115.1	140.0	79.7	78.6	97.8	79.4	76.9
20%*+	90.6	99.6	105.5	86.9	97.5	104.5	101.0	72.3	81.0	97.5	78.4	94.2
Exact	90.0	100.0	100.0	85.0	100.0	100.0	100.0	70.0	80.0	100.0	75.0	100.0

* Six horizontal response measurements at alternate floors.
+ Averaged results based on twelve different time histories of excitation with same noise level

TABLE 1. Percentage Ratio of Damaged Storey Stiffness to Undamaged Storey Stiffness

In terms of computation time, the ICM requires only 20% more than the lumped mass approach in this example, whereas a complete structural identification with 36 DOFs would be very time consuming (easily ten times more) if convergence can be achieved at all. The ICM is hence a simple and yet effective approach to determine local structural changes with virtually no stiffness diffusion problem.

Example 2 (Adaptive Filter)

In this example, the adaptive filter procedure is applied to an 1-DOF system to obtain error variances of identified parameters under the influence of I/O noise. The mass is known and has a value of 1 whereas the stiffness (K) and damping coefficient (C) are to be identified. Assuming independent system noises for all state variables, the distribution matrix G is simply a unit matrix. Initial conditions are: $x_1=y_0$, $x_2=y_0$, $x_3=100$, $x_4=1$, $P_{3,3}=400$ and $P_{4,4}=0.1$. The sampling numbers are $N=20$ and $M=3$. Total observation time is 22.5 s at a sampling rate of 0.075 s.

To evaluate the statistical consistency of estimation error, the following performance index is defined:

$$\psi_\gamma = \left\{ \left[\frac{1}{k} \sum_{i=1}^{k} [x_\gamma(t_i)-\hat{x}_\gamma(t_i|t_i)]^2 \right] \Big/ \left[P_{\gamma,\gamma}(k|k) \right] \right\}^{\frac{1}{2}} \qquad (12)$$

where γ =1, 2, 3 and 4 denote displacement, velocity, stiffness and damping, respectively. Due to randomness, the performance index fluctuates with k and it would be desirable to have the index averaging about one.

The performance of the adaptive filter is compared to that of the EKF with zero system noise(Q=0). The performance index for stiffness is shown in Fig. 3a for 10% I/O noise and in Fig. 3b for 30% I/O noise. It can be seen that the performance index diverges in the case of the EKF. This means that the error variance are statistically inconsistent and thus do not truly reflect the confidence level in the identified parameter. The performance indices in the case of the adaptive filter clusters around 1, which is an indication of good statistical consistency of error covariances determined in the SI process.

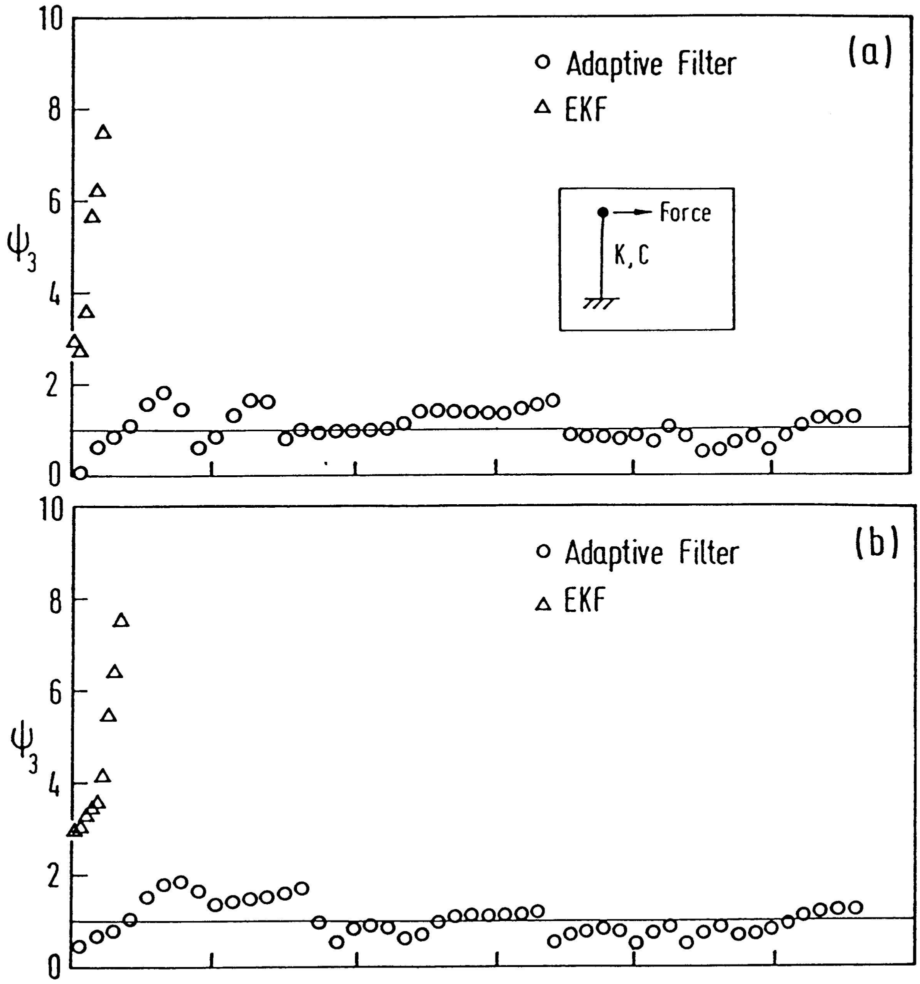

FIG. 3. Performance Indices for Stiffness (a) under 10% I/O Noise and (b) under 30% I/O Noise.

Example 3 (ICM with Adaptive Filter)

The last example demonstrates the combined application of the ICM and the adaptive filter for identification of local structural changes with confidence estimates of a three-storey plane frame building which has the same damping ratio, columns and beams as in Example 1. The excitation force is applied at the top floor and horizontal responses at all three floors are measured. Total observation time is 2 s at a sampling rate of 0.0005 s. In the application of the ICM, the observation time history is divided into 20 time windows.

For simulation of structural changes, we consider a damaged building with column stiffnesses at the first and third storeys reduced by 15% and 30%, respectively, and the second storey undamaged. These results in terms of percentage ratios relative to the "undamaged" values are summarized in Table 2. It can be seen that the effect of stiffness diffusion into the second storey is very small. Specifically, under a 0% noise level, the identified stiffness change of the supposedly undamaged second storey is only about 1%. The severity of damage in the first and third storeys is accurately reflected even for I/O noise level as high as 20%.

In the absence of I/O noise, the standard deviation of each identified stiffness ratio (in %) is about 0.8. The variability of identified results in this case is primarily attributed to the modeling errors. With 20% I/O noise, the standard deviation increases and thus the reliability of identified results is less. If randomness of an identified parameter is approximated by a Gaussian distribution, the reliability can be translated into a maximum likelihood range corresponding to a specified confidence level. As an illustration, 95% confidence ranges based on $\pm 1.96\sigma$ are presented in Table 3. Hence, with the determination of identified stiffnesses and their respective variances, the combined application of the ICM and the adaptive filter would be useful for the reliability analysis and safety evaluation of buildings.

I/O Noise	Storey	Stiffness Ratio	Standard Deviation	95% Confidence range
0%	1st	86.3	0.8	84.7 - 87.9
	2nd	101.2	0.8	99.6 - 102.8
	3rd	69.7	0.8	68.1 - 71.3
20%	1st	86.7	5.8	75.3 - 98.1
	2nd	95.1	5.8	83.7 - 106.5
	3rd	70.5	5.8	59.1 - 81.9
Exact	1st	85.0	-	-
	2nd	100.0	-	-
	3rd	70.0	-	-

TABLE 2. Percentage Ratio of Damaged Story Stiffness to Undamaged Storey stiffness and their Confidence Estimates

CONCLUSIONS

Two issues, namely (a) local damage detection and (b) confidence estimation of identified parameters, have been dealth with in this paper. Firstly, an "improved condensation" method is proposed to identify the locations and magnitudes of structural stiffness changes of buildings. Secondly, confidence levels in identified parameters are estimated by means of an adaptive filter which ensures statistical consistency of error covariances in the application of the EKF. The application of the proposed procedures to numerical examples have shown their potential as an effective tool to identify local structural changes of buildings with consistent confidence estimates.

APPENDIX I. REFERENCES

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